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- Perl language
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Thanks again for downloading my notes. I hope you enjoy them.

Sincerely,
- Eryk Wdowiak
Microeconomics is the study of the behavior of individual households, firms and industries as well as the supply and demand relationships between producers and consumers.

You might think of a household as a consumer, but households are also producers. For example, take a look at your kitchen: you take raw materials (meat, cheese, vegetables, eggs, salt and pepper) as well as capital (stove and frying pan) and your own labor to produce an omelet, which is demanded by you and members of your family. They may not pay you in money, but you’re compensated in other ways.

Cooking an omelet for your family is a very simple example of an economic problem. So what’s the point? The point is that economics isn’t “all about money.” It’s about life. It’s about human behavior. In fact, economic analysis can be applied to almost any problem imaginable.

For example, there is a branch of economics that studies the production of health and the demand for health. Notice that I wrote “health” and not “health care.”

To take another example, economic analysis can also be used to analyze the war on drugs without ever mentioning the word “price.” Instead what is important is “opportunity cost” – what we have to give up when we make a choice. In some American cities, police officers are so busy preparing prosecutions that they don’t have time to respond to 911 calls. Why? Because the politicians chose to put as many people in jail as possible, they actually had to forgo law and order.

Studying economics will help you understand the nature of trade-offs that you face in everyday life. If you spend more time studying economics, you’ll be less likely to make decisions that are as stupid as the ones our politicians have made and more likely to make rational decisions.
# Lecture Notes on the Principles of Microeconomics

Eric Doviak

3rd Edition, June 2005

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Helpful hints

- Economics doesn’t have to be difficult
- BUT... some people make it difficult for themselves.
- If a model is unclear, don’t try to think of an example from the $10 trillion US economy.
- Instead, apply the model to a small rural village.

- Most important part of any economic model are the: ASSUMPTIONS
- If you understand the assumptions of the model, you will understand the conclusions.
- You will NOT understand the conclusions, if you don’t understand the assumptions.
- WHEN READING, DON’T SKIP CHAPTERS!
Scope & Method of Economics

Why should I study economics?

• To learn a way of thinking! Hopefully, you’ll learn to use three key concepts in your daily lives:
  o efficient markets
  o marginalism and
  o opportunity cost

Efficient markets

• Profit opportunities are rare because everyone is looking for them.
• Efficient markets eliminate profit opportunities immediately.
• Ex. You’ll never find a good parking space, because if there was a good one, it would already be taken before you got there.

Marginalism

Average cost – total cost divided by quantity
• If I spend 300 hours preparing 30 lessons for you:
• You had better study!
• My average cost per lesson is 10 hours.

Sunk cost – costs that can no longer be avoided because they have already been “sunk”
• If I teach this class again next semester, I will have already sunk 300 hours into preparation.

Marginal cost – cost of producing one more unit
• Next semester I can recycle my notes, so my marginal cost per lesson will equal 75 minutes.
• Compare that with my current 10 hours!
Opportunity Cost

- We all face choices. **Resources are “scarce.”**
- We can’t spend more time or money than we have, so we have to give up one opportunity to take advantage of another.
- If I have a choice between earning $1000 per month by teaching this course OR earning $500 per month by working at McDonald’s, then:
  - It takes me one month to *produce* $1000 worth of teaching.
  - It takes me one month to *produce* $500 worth of burger flipping.
- **Q: What’s my opportunity cost of teaching?**
- **A: Half a burger flipping per unit of teaching.**

\[
\frac{\text{one month per $1000 of teaching}}{\text{one month per $500 of burger flipping}} = \frac{\text{one month}}{\text{one month}} \times \frac{\$1000 \text{ of teaching}}{\$500 \text{ of burger flipping}}
\]

\[
= \frac{\$500 \text{ of burger flipping}}{\$1000 \text{ of teaching}} = \frac{1}{2} \text{ burger flipplings per teaching}
\]

I’ll give a much, much better example in the next lecture.

---

**Math – tool of econ. analysis**

Point plotting (X,Y):
- the first point in a pair lies on the **X axis** (horizontal axis)
- the second point in a pair lies on the **Y axis** (vertical axis)

Let’s graph the following equation in red (square points):
\[
y = -5x + 20
\]

Connect points: (0,20), (1,15), (2,10), (3,5) & (4,0)

**y-intercept:**
- the value of y, when x = 0
- here it’s 20, because:
  \[20 = (-5*0) + 20\]

**slope:** (we’ll get back to that)

More examples:
\[
y = 4x + 5 \quad \text{(blue, round points)}
\]
\[
y = -2x + 15 \quad \text{(green, triangle points)}
\]
### What is SLOPE?

- **the change in y divided by the change in x**
  - \( y = -5x + 20 \)
  - If \( x \) increases from 1 to 2
  - \( y \) decreases from 15 to 10
  - Slope: \( \frac{10 - 15}{2 - 1} = -5 \)
- **positive slope**: \( x \) and \( y \) increase and decrease together
- **negative slope**: \( x \) and \( y \) increase and decrease inversely (when one rises the other falls)

### Math – tool of econ. analysis

- Why does curve slope up?
- When is avg. consumption greater than avg. income? How is this possible?

A statistical estimation of the relationship between avg. income and avg. consumption is:

\[
AC = 0.57 \times AI + 13,539
\]

where: \( AC = \text{avg. consumption} \) and \( AI = \text{avg. income} \)

- What’s the significance of the y-intercept ($13,539)?
- What’s the significance of the parameter next to the \( AI \)-variable (0.57)?

### Analyzing Graphs

The graph illustrates the relationship between average household income and average consumption expenditure. Along the 45 degree line, income equals expenditure.
AC = 0.57*AI + 13,539

marginal propensity to consume

I’m using an example from macroeconomics, because some of you have already taken a macro course. If you haven’t … Don’t worry. We’re just reviewing basic algebra.

• If your boss increased your income from $31,000 to $32,000, how much more would you consume?
  o On average, you would consume an extra $570 worth of goods.
  o Put differently, if you were an average person, your expenditure on consumption goods would rise from $31,209 to $31,779.
• Every $1000 increase in income raises consumption by $570. Why?
• marginal propensity to consume = 0.57 (NB: that’s the slope of the line!)

• What if you got fired? How much would you consume?
• Your income would fall to zero, but you’d still consume $13,539 worth of goods. After all, you’ve got to eat!

• When your income is less than $31,486 your expenditures on consumption goods exceed your income. (You run down your savings).
• When your income is more than $31,486 your income exceeds your expenditures on consumption goods. (You save some of your income).

---

A few more definitions

AC = 0.57*AI + 13,539

• **Model** – the formal statement of a theory, often presented using mathematical equations
• **Variable** – a measure that can change such as consumption or income
  o **Dependent variable**
  o **Independent variable**
  o In the example above, consumption depends on income.
• **Parameters** – values which remain constant in an equation (here: 0.57 and 13,539)

Y = C + I + G + (X–M)

• **Ceteris paribus** – “all else equal”
• How does an increase in investment, I, affect national income, Y?
• To answer this question we must hold all other variables constant, while we determine the effect of investment alone.
### Micro vs. Macro

**MICROeconomics**
- Study of the decision-making of individuals, households and firms
- Study of distribution of wealth

**MACROeconomics**
- Study of aggregates
- What factors affect:
  - Gross Domestic Product?
  - the price level?
  - the unemployment rate?

### Positive vs. Normative Economics

**Positive**
- No judgements
- Just asking how the economy operates

**Normative**
- Makes judgements
- Evaluates the outcomes of economic behavior
- Policy recommendations

### Economic policy

- **Positive** – economic policy starts with positive theories and models to develop an understanding of how the economy works
- Then economic policy evaluates (normative) on the basis of:
  - **Efficiency** – Is the economy producing what people want at the least possible cost? (quantifiable)
  - **Equity** – Is the distribution of wealth *fair*? Are landlords treating low-income tenants *fairly*? (non-quantifiable)
  - **Growth** – Increase in total output of the economy. Note: efficiency gains lead to growth (quantifiable)
  - **Stability** – steady growth, low inflation and full employment of resources – capital and labor (quantifiable)
- And recommends (normative) courses of action to policy-makers (presidents, congressmen, etc.)
Homework #1A

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
1. Graph these equations (placing Y on the vertical axis and X on the horizontal axis):
   - \( Y = 2X + 2 \)
   - \( Y = 4X + 2 \)
Comparing the two equations, which is different: the slope or the Y-intercept? How is it different? Are the lines parallel or do they intersect?

2. Graph these equations (placing Y on the vertical axis and X on the horizontal axis):
   - \( Y = 2 + 2X \)
   - \( Y = 2 - 2X \)
Comparing the two equations, which is different: the slope or the Y-intercept? How is it different? Are the lines parallel or do they intersect?

3. Graph these equations (placing Q on the vertical axis and P on the horizontal axis):
   - \( Q = 4 + 2P \)
   - \( Q = 2 + 2P \)
Comparing the two equations, which is different: the slope or the Q-intercept? How is it different? Are the lines parallel or do they intersect?

4. Graph these equations (placing Q on the vertical axis and P on the horizontal axis):
   - \( Q = 4 - 2P \)
   - \( Q = 2 + 2P \)
These two equations have different slopes and different Q-intercepts. Do the lines intersect? If so, can you find the value of P and Q at which they intersect?

5. Solve these two equations for \( P \). Then graph the new equations by placing P on the vertical axis and Q on the horizontal axis:
   - \( Q = 4 - 2P \)
   - \( Q = 2 + 2P \)
Do the lines intersect? If so, can you find the value of P and Q at which they intersect?

If demand curves slope down and supply curves slope up, then which of these two equations resembles a demand curve? Which resembles a supply curve?
6. The Law of Demand says that consumers purchase more of a good when its price is lower and they purchase less of a good when its price is higher. Can you give that statement a mathematical interpretation? (**Hint:** Does price depend on quantity purchased? or does quantity purchased depend on price?)

Is price an independent variable or a dependent variable? Is quantity purchased an independent variable or a dependent variable? What is the difference between a dependent variable and an independent variable?

On which axis (the vertical or horizontal) do mathematicians usually place the independent variable? On which axis do mathematicians usually place the dependent variable?

When economists draw supply and demand diagrams, they usually place price on the vertical axis and quantity purchased on the horizontal axis. Why is that “wrong”?

7. (A question about percentages) 0.750 = ______%

8. (A question about fractions) \( \frac{2}{3} = \) ______%
What’s the difference between Marginal Cost and Average Cost?

“Marginal cost is *not* the cost of producing the “last” unit of output. The cost of producing the last unit of output is the same as the cost of producing the first or any other unit of output and is, in fact, the average cost of output. Marginal cost (in the finite sense) is the increase (or decrease) in cost resulting from the production of an extra increment of output, which is not the same thing as the “cost of the last unit.” The decision to produce additional output entails the greater utilization of factor inputs. In most cases … this greater utilization will involve losses (or possibly gains) in input efficiency. When factor proportions and intensities are changed, the marginal productivities of the factors change because of the law of diminishing returns, therefore affecting the cost per unit of output.”


Let’s break Silberberg’s definition of marginal cost into its component pieces. First, he ascribes changes in marginal cost to changes in marginal productivities of factor inputs. (By factor inputs, he means factors of production, like labor and capital).

So what is the marginal product of labor and how is it affected by the law of diminishing returns?

Imagine a coal miner traveling deep underground to swing his pick at the coal face. The longer he swings his pick, the more coal he will produce, but it’s exhausting work, so if his boss were to require him to work a double shift, the miner wouldn’t double the amount of coal that he produces.

To be more specific, let’s assume that the miner produces an amount of coal equal to the square root of the number of hours he works. The tonnage of coal that he produces is his “total product of labor (TPL).” So if he puts in zero hours, he produces zero tons of coal. If he puts in one hour he produces one ton. If he puts in two hours, he produces $\sqrt{2} = 1.41$ tons of coal, etc.

If the miner increases the number of hours that he spends mining from one hour to two hours, his output of coal will increase by 0.41 tons. Increasing the miner’s hours from two to three hours only increases his output of coal by 0.32 tons however. Notice that the **additional coal he produces per additional hour that he works diminishes. This is the law of diminishing marginal returns.**

Notice also that the table lists the **ratio of the change in coal output to the change in the amount of hours worked.** That’s the slope of the total product function, or the “marginal product of labor.”
We don’t need to measure the changes in the miner’s coal output in one hour increments however. In fact, economists are usually more interested in a continuous rate of change. Those of you who have taken a course in calculus should know that the continuous rate of change in output is simply the first derivative of the total product function with respect to the number of hours worked.

For example, one second is $\frac{1}{360}$ of an hour or 0.002778 hours. If the miner works for exactly two hours, then he’ll produce 1.414214 tons of coal. If he works for exactly two hours and one second, then he’ll produce 1.415195 tons of coal. In other words, adding one second to a two hour workday increases his output of coal by 0.000982 tons. The marginal product of labor evaluated at two hours and one second is:

$$\frac{\sqrt{2.002778} - \sqrt{2}}{0.002778} = \frac{1.415195 - 1.414214}{0.002778} = \frac{0.000982}{0.002778} = 0.353431 \text{ tons per hour}$$

To see how successively smaller changes in units of time (by which output changes are measured) lead to closer and closer approximations to the first derivative, consider this graph of true marginal product (red), the change in output per half-hour change in work hours (green) and the change in output per one hour change in work hours (blue).

A table of the data points in the graph is given on the next page.

Now that you know what the “marginal product of labor” is, what do you think “marginal cost” is? It’s the change in total cost per unit change in output, calculated for an infinitesimally small change in output.

Just as the marginal product of labor measures the slope of the total product of labor function, marginal cost measures the slope of the total cost function.
Notice also what happens when we sum the last two columns of the table above (i.e. the columns of $\Delta \text{coal}/\Delta \text{hrs}$.). The column listing a time interval of one hour sums to 2, which is exactly how many tons of coal are produced. The area under the marginal product curve equals total product because increasing the number of hours from zero to one yields one additional ton of coal per hour, increasing the number of hours from one to two yields 0.41 additional tons of coal per hour, etc.

The column listing a time interval of half an hour sums to 4, which when multiplied by 0.5 hours also equals 2 (tons of coal), so once again the area under the marginal product curve equals total product.

The column listing the true marginal product of labor sums to 3.1 plus infinity, which at first glance seems to contradict the results above, but keep in mind that the true marginal product is calculated using infinitesimally small intervals of time. So we’d have to multiply infinity plus 3.1 by the infinitesimally small intervals of time that we used to obtain the true marginal product to obtain 2 tons of coal. (In mathematical terms: we could integrate the true marginal product of labor from zero hours to four hours with respect to the number of hours the miner works to obtain 2 tons of coal).

Now that we understand the law of diminishing returns and the concept of marginalism, let’s reexamine Silberberg’s quote. He says that to produce more output, a firm must hire more factors of production (like labor or capital) and/or use them more intensively, but such increased utilization reduces the efficiency of those factors of production (due to the law of diminishing returns) and raises the marginal cost of output.

For example, if the mining company only employs one miner and pays him a wage of $1 and that one miner is the only factor of production, then producing one ton of coal requires one hour of labor from the miner and costs a total of $1, but producing two tons requires four hours of labor and costs a total of $4. In this case, the marginal cost of increasing output from zero tons to one ton is $1 and the marginal cost of increasing output from one ton to two is: $4 – $1 = $3.

So let’s examine a hypothetical firm’s total, average and marginal costs by assuming that it faces a fixed cost of $10 and its variable cost is given by: $\text{VC} = X^3 – 3X^2 + 4X$, where $X$ is the amount of output that it produces. Total cost is equal to fixed cost plus variable cost, so: $\text{TC} = X^3 – 3X^2 + 4X + 10$.

In the specification above, the firm’s variable costs increase as the firm produces more output (and decrease as it produces less), therefore marginal cost reflects changes in variable cost. By definition, the firm’s fixed costs do not change when it increases or decreases the amount of output it produces, therefore marginal cost does not reflect changes in fixed cost – because there are no changes in fixed cost.
In the graph at right, I have drawn a total cost curve running from negative one units of output to four units of output.

Now it should be obvious to you that a firm would not produce a negative output.

I drew the total cost function from negative one to better show the shape of the curve and because I’ll use negative one to approximate the marginal cost at zero units of output.

The total cost curve depicted is everywhere increasing as output increases (i.e. is everywhere positively sloped), but it is not increasing at a constant rate. Initially total cost rises at a fairly rapid rate, but then the rate of increase slows, yielding a somewhat flat section. Finally, the rate of increase accelerates again. Since marginal cost is the rate of change in total cost (the slope of the total cost curve), the marginal cost curve will be U-shaped.

\[
TC = X^3 - 3X^2 + 4X + 10
\]

<table>
<thead>
<tr>
<th>X</th>
<th>TC</th>
<th>VC</th>
<th>AC</th>
<th>( \Delta TC/\Delta X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
<td>-8</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>0</td>
<td>infinite</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>2</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>4</td>
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<td>2</td>
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<tr>
<td>3</td>
<td>22</td>
<td>12</td>
<td>7 1/2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>32</td>
<td>10 1/2</td>
<td>20</td>
</tr>
</tbody>
</table>

If the firm faces a U-shaped marginal cost curve, then at low levels of output, it can increase the marginal productivity of its inputs by using them more intensively (a possibility I ruled out in the miner example), but at higher outputs, the firm confronts the law of diminishing returns and faces rising marginal cost.

At high levels of output, the firm’s average costs fall when it increases its production from zero to a moderate amount of output because its fixed cost is spread over a larger amount of output and (to a much lesser extent) because its average variable cost falls as inputs are used more efficiently (i.e. they yield a higher marginal product).

Finally, notice that the sum of the entries in the column containing the firm’s marginal cost equals $32 – the variable cost. (Ignore the marginal cost of $8 that occurs when the firm produces zero units of output because it was calculated by increasing output from negative one to zero). It equals $32 because marginal cost examines changes in variable cost, so the sum of the marginal costs must equal variable cost at that level of output.
Calculus is not a pre-requisite for this course. However, the foundations of economics are based on calculus, so what we’ll be discussing over the course of the semester is the intuition behind models constructed using calculus.

It’s not surprising therefore that the students who do better in economics courses are the ones who have a better understanding of calculus – *even when calculus is not a required part of the course*. So if you want to do well in this course, you should learn a little calculus.

Many times throughout the course, we’ll be discussing marginalism – e.g. marginal cost, marginal revenue, marginal product of labor, marginal product of capital, marginal utility, marginal rate of substitution, marginal rate of transformation, etc.

Whenever you see “marginal …” it means “the derivative of …”

A derivative is just a slope. So, for example, let’s say *labor is used to produce output*
- if TP stands for Total Production (quantity produced),
- if L stands for Labor input and
- if $\Delta$ denotes a change,

then if I write: $\frac{\Delta TP}{\Delta L}$ that’s the change in Total Production divided by the change in Labor.

- It’s the slope of the total production function.
- It’s the derivative of the production function with respect to labor input.
- It’s the marginal product of labor (MPL).

So if you understand derivatives, you’ll understand the course material much better.

---

**a few preliminaries – exponents**

You should recall from your high school algebra classes that when you see an exponent, it simply means multiply the number by itself the number of times indicated by the exponent.

\[
x^3 = x \cdot x \cdot x
\]

Now if you divide both sides of the above equation by $x$:

\[
\frac{x^3}{x} = \frac{x \cdot x \cdot x}{x} = x^2
\]

But what if you see the something like: $x^0$? Well, that’s simply equal to:

\[
x^0 = \frac{x^1}{x} = \frac{x}{x} = 1
\]

\[
\begin{align*}
2^3 &= 2 \cdot 2 \cdot 2 = 8 = \frac{16}{2} = \frac{2^4}{2} \\
2^2 &= 2 \cdot 2 = 4 = \frac{8}{2} = \frac{2^3}{2} \\
2^1 &= 2 = 2 = \frac{4}{2} = \frac{2^2}{2} \\
2^0 &= 1 = 1 = \frac{2}{2} = \frac{2^1}{2} \\
2^{-1} &= \frac{1}{2} = \frac{1}{2} = \frac{1}{2} = \frac{2^0}{2} \\
2^{-2} &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = \frac{1/2}{2} = \frac{2^{-1}}{2} \\
2^{-3} &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = \frac{1/4}{2} = \frac{2^{-2}}{2}
\end{align*}
\]
Similarly, $x^{-1} = \frac{x^0}{x} = \frac{1}{x}$ and $x^{-2} = \frac{x^{-1}}{x} = \frac{1}{x \cdot x} = \frac{1}{x^2}$.

But what about $x^{0.5}$? That’s the square root of $x$: $x^{0.5} = \sqrt{x}$. Ex. $16^{0.5} = \sqrt{16} = 4$

By the same logic as before: $x^{-0.5} = \frac{1}{\sqrt{x}}$. Ex. $9^{-0.5} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

---

**a few preliminaries – functions**

You may have seen something like this in your high school algebra classes: $f(x)$. This notation means that there is a function named “$f$” whose value depends on the value of the variable called “$x$.”

Some examples of functions in economics include:

- The quantity of output that a firm produces depends on the amount of labor that it employs. In such a case, we can define a function called “$TP$” (which stands for Total Production) whose value depends on a variable called “$L$” (which stands for Labor). So we would write: $TP(L)$.

- A firm’s total cost of producing output depends on the amount of output that it produces. In such a case, we can define a function called “$TC$” (which stands for Total Cost) whose value depends on a variable called “$Q$” (which stands for Quantity). So we would write: $TC(Q)$.

- A firm’s total revenue from selling output depends on the amount of output that it produces. In such a case, we can define a function called “$TR$” (which stands for Total Revenue) whose value depends on a variable called “$Q$” (which stands for Quantity). So we would write: $TR(Q)$.

---

**derivatives**

Now let’s return to the original purpose of these notes – to show you how to take a derivative.

A derivative is the slope of a function. For those of you who saw $f(x)$ in your high school algebra classes, you may recall taking a derivative called “$f$-prime of $x$,” $f'(x)$.

What you were doing was you were finding the slope of the function $f(x)$. You were finding how much the value of the function $f(x)$ changes as $x$ changes.

So let’s define the function: $f(x)=3x^2$ and let’s look at how the value of $f(x)$ changes as we increase $x$ by one unit increments.

Once again, let $\Delta$ denote a change.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$\frac{\Delta f(x)}{\Delta x}$</th>
<th>true $f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

The third column is our rough measure of the slope. The fourth column – entitled true $f'(x)$ – is the true measure of the slope of $f(x)$ evaluated at each value of $x$. The values differ greatly between the two.
columns because we are looking at “large” changes in x as opposed to the infinitesimally small changes described in the notes entitled: “What’s the Difference between Marginal Cost and Average Cost?”

Why does it make a difference whether we look at small or large changes? Consider the following derivation of the slope of \( f(x) \):

\[
f'(x) = \frac{\Delta f(x)}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

\[
= \frac{3(x + \Delta x)^2 - 3x^2}{\Delta x} = \frac{3(x + \Delta x)(x + 2\Delta x) - 3x^2}{\Delta x} = \frac{3(x^2 + 2x\Delta x + (\Delta x)^2) - 3x^2}{\Delta x}
\]

\[
= \frac{3x^2 + 6x\Delta x + 3(\Delta x)^2 - 3x^2}{\Delta x} = \frac{6x\Delta x + 3(\Delta x)^2}{\Delta x} = \frac{6x\Delta x + 3(\Delta x)^2}{\Delta x}
\]

\[
f'(x) = 6x + 3\Delta x
\]

If we look at one unit changes in the value of x – i.e. \( \Delta x = 1 \) – then the slope of \( f(x) \) evaluated at each value of x is equal to \( 6x + 3\Delta x \) which equals \( 6x + 3 \) since \( \Delta x = 1 \).

If we look at changes in x that are so small that the changes are approximately zero – i.e.: \( \Delta x \approx 0 \) – then the slope of \( f(x) \) evaluated at each value of x is approximately equal to \( 6x \) and gets closer and closer to \( 6x \) as the change in x goes to zero.

So if \( f(x) = 3x^2 \), then \( f'(x) = 6x \).

Since we’ll be looking at infinitesimally small changes in x, we’ll stop using the symbol \( \Delta \) to denote a change and start using the letter \( d \) to denote an infinitesimally small change.

---

**calculus tricks – an easy way to find derivatives**

For the purposes of this course, there are only three calculus rules you’ll need to know:

- the constant-function rule
- the power-function rule and
- the sum-difference rule.

**the constant-function rule**

If \( f(x) = 3 \), then the value of \( f(x) \) doesn’t change x as changes – i.e. \( f(x) \) is constant and equal to 3.

So what’s the slope? Zero. Why? Because a change in the value of x doesn’t change the value of \( f(x) \). In other words, the change in the value of \( f(x) \) is zero.

So if \( f(x) = 3 \), then \( \frac{df(x)}{dx} = f'(x) = 0 \).
the power-function rule

Now if the value of \( x \) in the function \( f(x) \) is raised to a power (i.e. it has an exponent), then all we have to do to find the derivative is “roll the exponent over.”

To roll the exponent over, multiply the original function by the original exponent and subtract one from the original exponent. For example:

\[
\frac{d}{dx} x^3 = 3 \cdot x^{3-1} = 3x^2 \\
\frac{d}{dx} 2x^2 = 2 \cdot 2x^{2-1} = 4x \\
\frac{d}{dx} x^4 = 4 \cdot x^{4-1} = 4x^3 \\
\frac{d}{dx} x^{1/2} = \frac{1}{2} \cdot 4x^{1/2-1} = 2x^{-1/2}
\]

the sum-difference rule

Now, say the function you are considering contains the variable \( x \) in two or more terms.

\[k(x) = 2x^2 - 3x + 5\]

if we define:

\[f(x) = 2x^2 \quad g(x) = -3x^1 = -3x \quad h(x) = 5\]

then:

\[k(x) = f(x) + g(x) + h(x) = 2x^2 - 3x + 5\]

Now we can just take the derivatives of \( f(x) \), \( g(x) \) and \( h(x) \) and then add up the individual derivatives to find \( k'(x) \). After all, the change in a sum is equal to the sum of the changes.

\[
\frac{d}{dx} k(x) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x) + \frac{d}{dx} h(x) \\
k'(x) = f'(x) + g'(x) + h'(x) = 2 \cdot 2x^{2-1} - 1 \cdot 3x^{1-1} + 0 = 4x - 3
\]
Example #1 – Total Revenue and Marginal Revenue

Total Revenue, denoted \( TR \), is a function of the quantity of output that a firm produces, denoted \( Q \), and the price at which the firm sells its output, denoted \( p \). Specifically, Total Revenue is equal to the amount of output that a firm sells times the price. For example, if the firm sells 20 widgets at a price of $5 each, then its Total Revenue is $100.

If a firm is in a perfectly competitive market, then the firm cannot sell its output at a price higher than the one that prevails in the market (otherwise everyone would buy the products of competitor firms). So we can assume that the price is constant.

So what is a firm’s Marginal Revenue? It’s Marginal Revenue, denoted \( MR \), is the derivative of Total Revenue with respect to a change in the quantity of output that the firm produces.

\[
TR(Q) = p \cdot Q \quad \rightarrow \quad MR = \frac{dTR(Q)}{dQ} = p
\]

Example #2 – Total Product and Marginal Product of Labor

If a firm produces output using “capital” – a fancy word for machinery – and labor, then the quantity of output that it produces – i.e. its Total Product, denoted by \( TP \) – is a function of two variables: capital, denoted by \( K \), and labor, denoted by \( L \).

So what is the Marginal Product of Labor, denoted \( MPL \)? Marginal Product of Labor is the change in Total Product caused by an increase in Labor input. Marginal Product of Labor is the derivative of Total Product with respect to Labor.

Notice that we’re looking solely at the change in Total Product that occurs when we vary the Labor input. We’re not changing the capital stock, so when we take the derivative of Total Product with respect to Labor, we’ll hold the firm’s capital stock is fixed – i.e. we’ll hold it constant.

\[
TP(K, L) = K^{0.5} \cdot L^{0.5} \quad \rightarrow \quad MPL = \frac{dTP(K, L)}{dL} = 0.5 \cdot K^{0.5} \cdot L^{-0.5}
\]
1. Find the derivative of each of the following functions:
   a. \( g(x) = 7x^6 \)
   b. \( k(y) = 3y^{-1} \)
   c. \( m(q) = \frac{3}{2}q^{-2/3} \)
   d. \( h(w) = -aw^2 + bw + \frac{c}{w} \)
   e. \( u(z) = 5 \)

2. The Total Product of a firm, denoted by \( TP \), depends on the amount of capital and labor that it employs. Denote capital by \( K \) and denote labor by \( L \).
The Total Product function is given by: \( TP(K, L) = K^{0.5} \cdot L^{0.5} \).
Throughout this problem, assume that the firm’s capital stock is fixed at one unit.
   a. Plot the Total Product function from zero units of Labor to four units of Labor.
      (Hint: Use graph paper if you have it).
   b. Now find the Marginal Product of Labor by taking the derivative of the Total Product function with respect to Labor.
   c. Plot the Marginal Product of Labor from zero units of Labor to four units of Labor.

3. The Total Cost function of a firm depends on the quantity of output that it produces, denoted by \( Q \).
The Total Cost function is given by: \( TC(Q) = Q^3 - 6Q^2 + 18Q + 6 \).
   a. Plot the Total Cost function from zero units of output to five units of output.
      (Hint: Use graph paper if you have it).
   b. Does the Total Cost function ever slope downward? Or is it strictly increasing?
   c. Now find the Marginal Cost function by taking the derivative of the Total Cost function with respect to the quantity of output that the firm produces.
   d. Plot the Marginal Cost function from zero units of output to five units.
   e. Does the Marginal Cost function ever slope downward? Or is it strictly increasing?
   f. If the Total Cost function never slopes downward, then why does the Marginal Cost function slope downward over some ranges of output?
The Economic Problem

- **What will be produced?**
  - **Basic needs:** food, clothing, shelter, etc.
  - **Non-essentials:** fish tanks, televisions, etc.
  - **Capital goods:** machinery, tools, human skills, etc. to produce more in the future

- **How will it be produced?**
  - What resources are available?
  - How should labor and capital be allocated the production of each of the various products?

- **Who will get what is produced?**
  - How should the products be allocated to the members of society – individuals, businesses, government, etc.?
Comparative Advantage

This is one of the very few economic principles which is undeniably true, but is not obvious to intelligent people.

- The US has a **comparative advantage** in the production of a good, if the opportunity cost of producing that good is lower in the US than it is in other countries.
  - **Opportunity cost** – how much of one good you have to give up in order to gain more of another.
  - **Unit labor requirement** – amount of labor needed to produce one unit of a good. Ex. If I type 2 pages of notes per hour, then my unit labor requirement (to type one page) is half an hour per page.

- Countries **gain from trade**, if they specialize in producing the goods in which they have a **comparative advantage**, although there may be **distributional effects** to consider.

![Comparative Advantage](image)

<table>
<thead>
<tr>
<th></th>
<th>Colleen</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>of cutting logs:</strong></td>
<td>$\frac{12 \text{ logs}}{1 \text{ day}} = \frac{10 \text{ bushels}}{12 \text{ logs}} = 0.83 \frac{\text{bushels}}{\text{log}}$</td>
<td>$\frac{5 \text{ logs}}{1 \text{ day}} = \frac{8 \text{ bushels}}{5 \text{ logs}} = 1.6 \frac{\text{bushels}}{\text{log}}$</td>
</tr>
<tr>
<td><strong>of gathering food:</strong></td>
<td>$\frac{10 \text{ bushels}}{1 \text{ day}} = \frac{12 \text{ logs}}{10 \text{ bushels}} = 1.2 \frac{\text{logs}}{\text{bushel}}$</td>
<td>$\frac{8 \text{ bushels}}{1 \text{ day}} = \frac{5 \text{ logs}}{8 \text{ bushels}} = 0.625 \frac{\text{logs}}{\text{bushel}}$</td>
</tr>
</tbody>
</table>

- **Colleen has a comparative advantage in cutting logs** because her opportunity cost of cutting logs is less than Bill’s.
- **Bill has a comparative advantage in gathering food** because his opportunity cost of gathering food is less than Colleen’s.
Colleen has a comparative advantage in cutting logs.
Bill has a comparative advantage in gathering food.
Colleen has an **ABSOLUTE advantage in the production of BOTH goods**, but she has a **COMPARATIVE advantage in the production of only ONE good** (cut logs).

Relative Price

- If Colleen and Bill valued logs and food equally, then they would trade logs for food at a one-to-one ratio.
- If you prefer to think in terms of dollar values:
  - let the price of logs be one dollar per log: $1/\text{log}$
  - let the price of food be one dollar per bushel: $1/\text{bushel}$
- so that:
  - the relative price of logs is one bushel per log: $\frac{1/\text{log}}{1/\text{bushel}} = 1 \frac{\text{bushel}}{\text{log}}$
  - the relative price of food is one log per bushel: $\frac{1/\text{bushel}}{1/\text{log}} = 1 \frac{\text{log}}{\text{bushel}}$

Colleen’s Specialization → Cutting Logs

- A person (country) **should** specialize in producing a good if its opportunity cost is less than the relative price of that good.
  - Colleen **should** specialize in logs because her opportunity cost of cutting logs is less than the relative price of logs.
  - By contrast, Bill **should not** cut logs because his opportunity cost of cutting logs is greater than the relative price of logs.

Colleen’s Gains from Trade

- By specializing in cutting logs and trading her logs for food, Colleen gains more food (per day of work) than if she gathered food herself.

$$\frac{12 \text{ logs} \times 1 \text{ bushel}}{1 \text{ day}} = 12 \text{ bushels} \frac{1 \text{ day}}{1 \text{ log}} > \frac{10 \text{ bushels}}{1 \text{ day}}$$
Bill’s Specialization → Gathering Food

- A person (country) should specialize in producing a good if its opportunity cost is less than the relative price of that good.
  - Colleen should not gather food because her opportunity cost of gathering food is greater than the relative price of food.
  - By contrast, Bill should gather food because his opportunity cost of gathering food is greater than the relative price of food.

<table>
<thead>
<tr>
<th>Colleen's opp. cost of food</th>
<th>rel. price of food</th>
<th>Bill's opp. cost of food</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 logs/bushel</td>
<td>1 log/bushel</td>
<td>0.625 logs/bushel</td>
</tr>
</tbody>
</table>

Bill’s Gains from Trade

- By specializing in gathering and trading his food for logs, Bill gains more logs (per day of work) than if he cut logs himself.

\[
\frac{8 \text{ bushels}}{1 \text{ day}} \times \frac{1 \text{ log}}{1 \text{ bushel}} = \frac{8 \text{ logs}}{1 \text{ day}} > \frac{5 \text{ logs}}{1 \text{ day}}
\]

Moral of the Story

- Even though Colleen can produce both goods more efficiently, she gains by specializing in logs (the good in which she has a comparative advantage) and trading her logs for food with Bill.
- Moral: America gains by trading with less developed countries.
- Even though Bill is less efficient at producing both goods, he gains by specializing in food (the good in which he has a comparative advantage) and trading his food for logs with Colleen.
- Moral: less developed countries gain by trading with America.

Lower Productivity → Lower Wage

- Recall the dollar prices of each good: $1/bushel and $1/log
  - Colleen produces 12 logs per day, so her wage is $12 per day.
  - Bill produces 8 bushels per day, so his wage is $8 per day.
- This is why the Malaysians who made your sneakers, receive a much lower wage than you do. They’re less productive.
### Trading Up

- Of the countries in the table above, the ones which have the highest levels of human development, are generally the ones that engage in more international trade.

- the Human Development Index is positively correlated with:
  - a country’s share of exports in GDP
  - a country’s share of imports in GDP

- the correlations are not perfect, but they are significant

### moral of the story

- Countries **gain from trade**, if they **specialize** in producing the goods
- in which they have a **comparative advantage**, although there may be **distributional effects** to consider:
  - workers who are not working in the sector where the country has a comparative advantage will be adversely affected by free trade
  - ex. in America steel workers, textile workers and farmers are adversely affected by trade
**Production Possibilities Frontier**

- **PPF** – all combinations of goods that can be produced if resources are used efficiently. One can produce at or below the PPF, but not above it.

  In one day, **Colleen** can:
  - cut 12 logs or gather 10 bushels
  - or produce a **combination**, such as: 6 logs and 5 bushels.

  In one day, **Bill** can:
  - cut 5 logs or gather 8 bushels
  - or produce a **combination**, such as: 2.5 logs and 4 bushels.

**Slope of the PPFs (above) is:** \(-1 \cdot \text{opp. cost of gathering food}\)

---

**Gains from Trade**

- Add a **red line** whose **slope** represents the **relative price**: \(\frac{\text{log}}{\text{bushel}}\)

  If **Colleen** specializes in cutting logs:
  - she can **trade** some of her logs for bushels of food and
  - consume a **combination** that exceeds any combination that she could produce on her own.

  If **Bill** specializes in gathering food:
  - he can **trade** some of his bushels of food for logs and
  - consume a **combination** that exceeds any combination that he could produce on his own.
Production Function

Quantity produced is a function of capital and labor:

\[ Q = f (K, L) \]

- If you have one unit of kapital (for example, one stove in a kitchen),
- and you keep increasing number of workers (labor) at that machine the quantity produced will increase
- but at a decreasing rate
  - because the workers start to get in each other’s way
  - “too many cooks in the kitchen”

This production function is drawn for a fixed amount of capital.

PPF represents:
- all the possible combinations of goods (and services)
- that can be produced,
- if resources are used efficiently.

Production possibilities are constrained by amount of labor and capital in the economy.

Cannot produce above PPF

If we shift labor from production of X and into production of Y,
- less X will be produced
- more Y will be produced

PPF summarizes opportunity cost of all such shifts.

If resources are not used efficiently
- labor unemployment,
- inefficient management
- the economy is producing at a point below the PPF.

Production Possibilities Frontier

Page 29
Cuba’s Ten Million Ton Sugar Harvest

- In the 1960s, Cuba produced about 6 to 7 million tons of sugar a year, which was sold primarily to countries in the Soviet bloc.
- Beginning in 1969, Cuban dictator Fidel Castro sent hundreds of thousands of urban workers into the fields in an effort to produce 10 million tons of sugar in 1970.
- Ultimately, Cuba missed its goal and only managed to produce 8.5 million tons – the largest harvest in Cuban history.

What were the effects on Cuban economy?

For simplicity, assume that before the plan:
- Cuba produced 6 million tons of sugar and 5 million tons of "everything else"
- relative price of sugar was one ton of everything else per ton of sugar,
- at a relative price of \( \frac{\text{everything else}}{\text{sugar}} \), Cuba traded 2 million tons of sugar for 2 million tons of everything else and
- consumed 4 million tons of sugar and 7 million tons of everything else

Massive disruptions in the Cuban economy

Since Cuba allocated all of its production to sugar, it produced at the "sugar corner" of its PPF. At that corner, the opportunity cost of producing sugar exceeds the relative price of sugar.

For simplicity, let's pretend that Cuba:
- succeeded in producing all 10 million tons of sugar, but didn't produce anything else
- at a relative price of \( \frac{\text{everything else}}{\text{sugar}} \), Cuba traded 6 million tons of sugar for 6 million tons of everything else and
- consumed 4 million tons of sugar and 6 million tons of everything else

So (in this example) Cubans consumed the same amount of sugar, but their consumption of everything else fell from 7 million tons to 6 million tons – a 15 percent decrease.
Q: Would a 15 percent decrease in consumption of everything else a massive disruption in the economy?
A: If you could consume the same amount of sugar that you did last year, but your consumption of everything else fell 15 percent, would you be happy?

\[\text{lesson from Cuba’s experiment}\]

- a country should produce at the point along its PPF, where the opportunity cost of producing a good (ex. sugar) equals the relative price of that good
- Cubans suffered because their country produced at a point where the opportunity cost of producing sugar exceeded the relative price of sugar
- Similarly, had Cuba allocated all of its resources to producing “everything else” and produced no sugar it also would have suffered
  - because at such a point, the opportunity cost of producing everything else would have been greater than the relative price of everything else
  - (from the opposite perspective…) because at such a point, the opportunity cost of sugar would have been less than the relative price of sugar

\[\text{Why did Bill and Colleen completely specialize in one good?}\]

- A country should completely specialize in the production of one good
  - ONLY if the relative price of that good is greater than the country’s opportunity cost of producing it at every point along the PPF
  - Bill and Colleen’s opportunity cost was constant all along their PPFs
- the PPF I drew for Cuba assumes increasing opportunity cost – i.e. Cuba’s opportunity cost of producing sugar increases as it produces more sugar
Homework #2

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Do this too! Suppose that the simple society of Greenville can produce rice and beans. Suppose also that the Greenville’s production possibilities frontier is given by the equation:

\[ \text{PPF: } \text{rice} = 18 - \frac{1}{2} \text{beans}^2 \]

a. Placing beans on the horizontal axis and rice on the vertical axis, graph Greenville’s PPF.

b. Suppose the relative price of beans is: \( \frac{\text{rice}}{\text{beans}} = 2 \). Using the Calculus Tricks you learned in the first lecture, find the quantities of rice and beans that Greenville should produce at that relative price.

c. Now suppose the relative price of beans rises to: \( \frac{\text{rice}}{\text{beans}} = 4 \). Should Greenville produce more or less rice? Should Greenville produce more or less beans? What quantities of rice and beans should Greenville produce at that relative price?

d. At what relative price of beans should Greenville specialize in the production of beans and produce no rice at all?

continued on the next page...
Do this too! In the story of Colleen and Bill on p. 28–29 of Case/Fair Principles…, there’s an error. The book says Bill and Colleen produce logs and bushels of food at the following rates:

<table>
<thead>
<tr>
<th>Production per day</th>
<th>Colleen</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>food (bushels)</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>fuel (logs)</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

The book also says that Bill and Colleen value bushels of food and logs equally, so that the price of one bushel equals the price of one log.

- Despite what is written,
  - Bill gains from trade with Colleen, but
  - Colleen doesn't gain from trade with Bill.
  - However, she doesn’t lose by trading with Bill.
- Why doesn’t Colleen gain from trade?
- Leaving opportunity costs unchanged, how can the story be rewritten, so that both Bill and Colleen gain from trade?

Hint: How does the assumption that Bill and Colleen value bushels of food and logs equally prevent Colleen from gaining from trade (given the production rates given above)?
How many TV sets do you have in your house?
One in the kitchen, one in the bedroom, one in the living room ....
Back in the 1950’s, most families only had one TV, if they had one at all.
Q.: Why do families have so many more TV sets today?
A. # 1: They’re cheaper.

If price of TV sets rises, then, ceteris paribus, the quantity of TV sets demanded will fall, and vice versa.
Demand

- Q.: Why do families have so many more TV sets today?
- A. # 2: Families’ real incomes are larger.
- An increase in income changes relationship between price and quantity demanded.
- Demand curve shifts out (up and right).

Movement along Demand Curve vs. Shift of Demand

**Movement along:**
- Only if there is change in the good’s price (shift of supply curve)

**Shift of demand,** due to changes in:
- Income
- Accumulated wealth
- Tastes and preferences (ex. fewer smokers)
- Prices of other goods
- Expectations (of future income, wealth and prices)

**Income** – *a flow* – sum of all earnings (wages, salaries, profits, interest payments, rents, etc.) in a given period of time

**Wealth** – *a stock* – total value of what household owns minus what it owes

**Substitutes** – when price of good X rises, demand for good Y rises (ex. cigs & rolling tobacco)

**Complements** – when price of good X rises, demand for good Y falls (ex. pasta & sauce)
Market Demand

Sum of all individual demand curves (sum of all individual quantities demanded at each price)

<table>
<thead>
<tr>
<th>price</th>
<th>quantity demanded by:</th>
<th>market demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4</td>
<td>4 + 0 + 4 = 8</td>
<td>$8</td>
</tr>
<tr>
<td>$2</td>
<td>8 + 3 + 9 = 20</td>
<td>$20</td>
</tr>
</tbody>
</table>

Supply

- If you were offered a job for $3 an hour, how many hours a week would you work?
- You wouldn’t take a job that pays so little.
- At $50 an hour, how many hours a week?
- 40, 50, 60 hours?
- At $100 an hour, how many hours a week?
- 40, 50, 60 hours? There’s a limit to how much you can work in a week.
- Ceteris paribus – ex. if there’s very high inflation, increase in wage reflect inflation and therefore will not increase output (hours worked)
- An increase in price – wage is the price of labor – will ceteris paribus increase the quantity of labor supplied and vice versa.
**Firm Supply**

**Total Cost:** \( TC = FC + VC \)
- **Fixed costs** (FC) – repayment of loans, lump sum taxes, etc.
- **Variable costs** (VC) – labor, raw materials, electricity, etc.

**Average Cost:** \( AC = AFC + AVC \)
- Average fixed cost (AFC) decreases as output increases
- Average variable cost (AVC) increases as output increases (at least at higher output levels)

**Marginal Cost** (MC):
- Rate of change in total costs from extra unit of output
- Is the supply curve when MC > AC

---

**Movement along Supply Curve vs. Shift of Supply**

**Movement along:**
- Only if there is a change in the good’s price (shift of demand curve)

**Shift of supply**, due to changes in:
- Costs:
  - wages
  - dividend payments
  - raw materials
- Technology
  - more productive machines
  - increased efficiency with which firm uses its inputs into its production
Market Supply

Sum of all individual supply curves (sum of all individual quantities supplied at each price)

<table>
<thead>
<tr>
<th>price</th>
<th>quantity supplied by:</th>
<th>market supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
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Market Equilibrium

Excess Demand

- The absolutely must-have Christmas present
- Parents bid up the price of the present, some parents drop out of the market
- Factories increase production and ask higher price
- Shortage eliminated – “price rationing”

At equilibrium there is no natural tendency for further adjustment.
Market Equilibrium

Excess Supply

- Car sales at the beginning of a recession
  - Think back to end-2001
  - Every other ad on TV/radio was a car commercial
- Buyers know of excess supply, offer lower prices and increase quantity demanded
- Automakers decrease production and accept a lower price

At equilibrium there is no natural tendency for further adjustment.

Changes in Equilibrium

Fall in Supply

- Some crops freeze
- Initially at equilibrium
- After freeze, market supply is more limited
- Supply curve shifts in
- Shortage at initial price
- Price bid up, some drop out of market
- Other farms harvest more of frozen crop
- New equilibrium
Scalpers

- How much would you pay for the best seats in the house?
- Limited supply
- Tickets priced below market equilibrium
- Excess demand at list price
- Arbitrage – whoever obtains tickets at list price, can profit by reselling them

Stupidity

If “pro–” is the opposite of “con–,” what’s the opposite of “progress?”

Price ceilings

- 1974 – OPEC imposed oil embargo on US
- Supply shock
- Congress imposed a price ceiling on the price of gasoline
- Result: long lines and shortage of gasoline

Ways to beat a ceiling
1. Queuing – be first in line
2. Favored customers – bribe the retailer
3. Ration coupons – buy coupons from friends
What is the Cost of Stupidity?

- Since a car owner has to wait in line to buy gas, he/she doesn’t just pay the fixed price of gas (denoted: $p$).
- For simplicity, we’ll assume that $\bar{p}$ is the price per tank of gas.
- He/She also faces a time cost. But what is the value of time?
- An hour of an individual’s time is equal to his/her hourly wage (denoted: $w$).
- So if he/she has to wait $t$ hours for a tank of gas
- effective price he/she pays per tank is: $\bar{p} + wt$.
- qty. demanded at effective price equals qty. supplied at fixed price

Excise Tax

- Now let’s say the government imposes a tax, $\tau$, on sales:
  - It could be a sales tax of 8.375%. If so: $\tau = p \cdot 8.375\%$
  - It could be a fixed dollar amount – e.g. $\tau = $2 per gallon of gas
  - Or it could be a FICA (Social Security) tax on wage income. If so, the wage is the “price” of labor and: $\tau = \text{wage} \cdot 15\%$
- Again, consumers pay an effective price, $p + \tau$, which:
  - is greater than market equilibrium price, $p^*$
  - is greater than price that producers receive, $p$
- Government collects revenue equal to the dollar amount of the tax times the quantity sold
Tariff

- When US imports good, domestic demand exceeds domestic supply
- Imposition of tariff drives a wedge between world price and domestic price
- After tariff imposed, domestic producers can sell more and charge higher price for their product
- Consumers however pay higher price and cut qty. demanded
- Government receives revenue from tariff
Homework #3

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Do this too! Suppose that the market demand for hamburgers is given by: \( Q_D = 10 - p \) and that the market supply is given by: \( Q_S = 2 + p \), where \( p \) is the price of a hamburger.

a. What is the equilibrium price of hamburgers? What is the equilibrium quantity of hamburgers supplied and demanded?

b. Solve the market demand equation and solve the market supply equation for price. This yields the inverse market demand function and the inverse market supply function.

c. Graph the inverse market demand and inverse market supply functions, placing quantity on the horizontal axis and price on the vertical axis. Do they intersect at the point corresponding to the equilibrium price and equilibrium quantity?

d. Now suppose that the government imposes an excise tax of $2 per hamburger. What is the new quantity of hamburgers supplied and demanded? Hint: At what quantity is the inverse supply curve $2 higher than the inverse demand curve?

e. What is the new effective price that consumers pay per hamburger? What is the new price that producers receive per hamburger?
I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
Shape of the Demand Curve

- When prices change, change in quantity demanded depends on shape of demand curve.
- **Consumer 1** has a very elastic demand curve.
- **Consumer 2** has a very inelastic demand curve.
- Elasticity often depends on the good in question:
  - Elastic: education, alcohol
  - Inelastic: gas, food, cigarettes, electricity

Diagram:
- When price rises to \( p_2 \), consumer 1 significantly reduces the quantity she demands.
- Initial quantity demanded by consumers 1 and 2 at \( p_1 \).
- When price rises to \( p_2 \), consumer 2 doesn’t reduce his quantity demanded very much.
Shape of the Supply Curve

- When prices change, change in quantity supplied depends on shape of supply curve
- **Producer 1** has a very **elastic** supply curve
- **Producer 2** has a very **inelastic** supply curve
- Elasticity often depends on the good in question:
  - Elastic: soft drink vendors on a hot day at the beach
  - Inelastic: housing, labor

Slope can be misleading

- Slope can be misleading because it depends on the units of measurement
- For example, if the price of a British Pound is 2 U.S. Dollars per British Pound, then the demand curve for milk is twice as steep if measured in dollars instead of pounds
- If we focus on a ratio of percentage changes, we can eliminate the confusion caused by differences in the units of measurement
What is elasticity?

- it’s a unit-free measure of responsiveness
- the own price elasticity of demand measures the ratio of a percentage change in quantity demanded of good X to a percentage change in the price of good X

\[ \varepsilon = \frac{\% - \text{age } \Delta Q}{\% - \text{age } \Delta P} = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \]

- notice that the component \( \frac{\Delta Q}{\Delta P} \) corresponds to the slope of a demand function such as: \( Q_d = 10 - 2P \), in which case: \( \frac{\Delta Q}{\Delta P} = -2 \)
- \( \frac{\Delta Q}{\Delta P} \) is also the inverse of the slope of the demand curve (when we plot price on the vertical axis and quantity of the horizontal axis)
- the component \( \frac{P}{Q} \) corresponds to the current price of the good and the quantity that consumers buy at that price

Milk Example – Dollars vs. Pounds

- As illustrated in the graphs of British and American demand for milk, the demand relationships are:
  \[ Q_{d, \text{America}} = 10 - 2P_{\text{Dollar}} \quad Q_{d, \text{Britain}} = 10 - 2P_{\text{Pound}} \]
  therefore:
  \[ \frac{\Delta Q_{d, \text{America}}}{\Delta P_{\text{Dollar}}} = -1 \quad \frac{\Delta Q_{d, \text{Britain}}}{\Delta P_{\text{Pound}}} = -2 \]
- If \( Q_{d, \text{America}} = Q_{d, \text{Britain}} = 6 \), then:
  \[ P_{\text{Dollar}} = 4 \quad \text{and} \quad P_{\text{Pound}} = 2 \]
- So the own-price elasticities of demand in each country must be:
  \[ \varepsilon_{\text{U.S.}} = \frac{\Delta Q_{d, \text{America}}}{\Delta P_{\text{Dollar}}} \cdot \frac{P_{\text{Dollar}}}{Q_{d, \text{America}}} \quad \varepsilon_{\text{Britain}} = \frac{\Delta Q_{d, \text{Britain}}}{\Delta P_{\text{Pound}}} \cdot \frac{P_{\text{Pound}}}{Q_{d, \text{Britain}}} \]
  \[ \varepsilon_{\text{U.S.}} = -2/3 \quad \varepsilon_{\text{Britain}} = -2/3 \]
- How much less milk would be demanded in each country if the price rose 1% in each country? 0.67% less
Why elasticity is so useful

$\varepsilon_{\text{U.S.}} = -\frac{2}{3}$  
$\varepsilon_{\text{Britain}} = -\frac{2}{3}$

- How much less milk would be demanded in each country if the price rose 1% in each country? **0.67% less**
- Now this result may seem trivial, but that’s the point!
- You want an easy way to make comparisons.
- You don’t want to have to convert U.S. Dollars into British Pounds or gallons into liters, etc.
- So for example, if I told you that:
  - own-price elasticity of demand for gasoline is $-0.5$ and
  - own-price elasticity of demand for restaurant meals is $-2.3$
- which is more responsive to changes in price? The demand for gasoline or the demand for restaurant meals?
- the demand for restaurant meals – because for a one percent increase in price, the quantity of restaurant meals demanded falls over four times more than the quantity of gasoline demanded does

Now let’s say you’re selling cream cheese …

- If there are many other companies selling cream cheese, then how responsive will the demand for your cream cheese be to the price?
  - very responsive
  - you’d face a very **elastic demand** – since small changes in the price of your cream cheese would induce consumers to buy your competitors’ cream cheese
- If you raised your price, your revenue would fall dramatically.
- Now let’s say you have few competitors in the cream cheese industry and a large group of people are cream cheese addicts.
  - demand for your cream cheese would not be very responsive to price
  - you’d face a very **inelastic demand**
- If you raised your price, the quantity of cream cheese that you sell would fall, but your revenue would not. Instead your revenue would increase due to the higher selling price and weak demand response.

**NB:** I have only mentioned revenue, not profit! Profit is the difference between your total revenue and your total cost.
Own-Price Elasticity of Linear Demand

\[ q = a - bp \]

\[ \epsilon = \frac{-\infty}{a/2b} \]

- \( \infty < \epsilon < -1 \)

**Elastic Demand:**
1% price increase reduces quantity demanded by more than 1%

- \( \epsilon = -1 \)

**Unit Elastic Demand:**
1% price increase reduces quantity demanded by 1%

- \( -1 < \epsilon < 0 \)

**Inelastic Demand:**
1% price increase reduces quantity demanded by less than 1%

---

... but what about butter?

- So you’re still selling cream cheese, but now the price of butter (a substitute for cream cheese) goes up.
- What’s going to happen to demand for your cream cheese?
- Since the goods are substitutes, an increase in the price of butter will increase the demand for cream cheese.
- The **cross-price elasticity of demand** for cream cheese with respect to a change in the price of butter is:

\[ e_{c.\text{cheese}, \text{butter}} = \frac{\Delta Q_{c.\text{cheese}}}{\Delta P_{\text{butter}}} \cdot \frac{P_{\text{butter}}}{Q_{c.\text{cheese}}} \]

- this elasticity will be positive since \( \Delta P_{\text{butter}} > 0 \) \( \Rightarrow \) \( \Delta Q_{c.\text{cheese}} > 0 \)
- so let’s say: \( e_{c.\text{cheese}, \text{butter}} = 0.5 \), then a one percent increase in the price of butter causes a 0.5 percent increase in the demand for cream cheese.
... and what about bagels?

- So you’re still selling cream cheese, but now the price of bagels (a complement to cream cheese) goes up.
- What’s going to happen to demand for your cream cheese?
- Since the goods are complements, an increase in the price of bagels will decrease the demand for cream cheese.
- The **cross-price elasticity of demand** for cream cheese with respect to a change in the price of bagels is:

\[
e_{c, \text{cheese, bagels}} = \frac{\Delta Q_{c, \text{cheese}}}{\Delta P_{\text{bagels}}} \cdot \frac{P_{\text{bagels}}}{Q_{c, \text{cheese}}}
\]

- this elasticity will be negative since \( \Delta P_{\text{bagels}} > 0 \Rightarrow \Delta Q_{c, \text{cheese}} < 0 \)
- so let’s say: \( e_{c, \text{cheese, bagels}} = -2 \), then a one percent increase in the price of bagels causes a 2 percent decrease in the demand for cream cheese.

So what will affect demand more?

- What will affect demand for cream cheese more?
  - a one percent increase in the price of butter
  - or a one percent increase in the price of bagels
- if \( e_{c, \text{cheese, butter}} = 0.5 \) and \( e_{c, \text{cheese, bagels}} = -2 \), then:

![Graph showing the effect of price changes on demand for cream cheese](image)
... and what about the income of consumers?

- So you’re still selling cream cheese, but now the income of your consumers goes up (prices held constant)
- What’s going to happen to demand for your cream cheese?
- Since your consumers now have more money to spend, an increase in the income of your consumers will probably increase the demand for cream cheese.

- The income **elasticity of demand** for cream cheese is:

\[
\eta_{c,\text{cheese}} = \frac{\Delta Q_{c,\text{cheese}}}{\Delta \text{Income}} \cdot \frac{\text{Income}}{Q_{c,\text{cheese}}}
\]

- this elasticity will be positive since \( \Delta \text{Income} > 0 \Rightarrow \Delta Q_{c,\text{cheese}} > 0 \)

- so let’s say: \( \eta_{c,\text{cheese}} = 0.75 \), then a one percent increase in the income of consumers causes a 0.75 percent increase in the demand for cream cheese

---

**Income Elasticities**

- **Normal Goods** can be broke down into:
  - **Income-Inelastic Normal Goods** \( \Rightarrow 0 < \eta < 1 \)
    - when income increases by 1% (prices held constant)
    - demand for such goods increases less than 1%
  - **Unit-Elastic Normal Goods** \( \Rightarrow \eta = 1 \)
    - when income increases by 1% (prices held constant)
    - demand for such goods increases by 1%
  - **Income-Elastic Normal Goods** \( \Rightarrow 1 < \eta \)
    - when income increases by 1% (prices held constant)
    - demand for such goods increases more than 1%
    - these goods are also called: **Luxury Goods**

- **Inferior Goods** \( \Rightarrow \eta < 0 \)
  - when income increases by 1% (prices held constant)
  - demand for such goods **FALLS**

- However, no good can be inferior over all ranges of income, otherwise it would never be consumed at all.
Income-Inelastic
Normal Good
\( 0 < \eta < 1 \)

Unit-Elastic
Normal Good
\( \eta = 1 \)

Income-Elastic
Normal Good
\( 1 < \eta \)
also called a Luxury Good

Inferior Good
\( \eta < 0 \)
As depicted, the income elasticity is initially positive, but becomes negative (inferior) after a certain income level is surpassed.
Supply Elasticity

- As alluded to at the beginning of the lecture, there’s also an elasticity of supply.
  
  \[ e_{\text{supply}} = \frac{\Delta Q_s}{\Delta P} \cdot \frac{P}{Q_s} \]

- The component \( \frac{\Delta Q_s}{\Delta P} \) corresponds to the slope of a supply function such as: \( Q_s = 3 + 2P \), in which case: \( \frac{\Delta Q_s}{\Delta P} = 2 \)

- \( \frac{\Delta Q_s}{\Delta P} \) is also the inverse of the slope of the supply curve (when we plot price on the vertical axis and quantity on the horizontal axis)

- The component \( \frac{P}{Q} \) corresponds to the current price of the good and the quantity that producers sell at that price.

Who Pays an Excise Tax?

- Legislators may fuss and worry over who should pay an excise tax (a tax on sales) – the producer or the consumer

- But economists know that the true burden of the tax falls more heavily on the one who has a lower elasticity of supply/demand

If the consumer’s demand is very inelastic and the producer’s supply is very elastic, then the consumer bears more of the tax burden.

If the consumer’s demand is very elastic and the producer’s supply is very inelastic, then the producer bears more of the tax burden.
Homework #4

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Do this too!  Bob is a shoemaker and an economist. He has estimated the following demand curve for his shoes:
\[ Q_D = 700 - 10P \]
\[ \text{a.} \quad \text{Calculate the price elasticity of demand at points A through F.} \]
\[ \text{b.} \quad \text{Find the price at which demand is unit elastic.} \]
\[ \text{c.} \quad \text{What happens to Bob’s total revenue (P*Q):} \]
\[ \begin{align*}
\text{if Bob increases the price from $20 to $30?} \\
\text{if Bob increases the price from $30 to $40?} \\
\text{if Bob increases the price from $40 to $50?}
\end{align*} \]
\[ \text{d.} \quad \text{How could you use the answers to a. and b. to predict the answers to c.} \]

Do this too!  Find the price and quantity where the price elasticity of demand equals one (unitary elasticity) for the following linear demand functions:
\[ \begin{align*}
Q_D &= 8 - 2P \\
Q_D &= 9 - 3P \\
Q_D &= 10 - 4P
\end{align*} \]

What would be the effect on revenue if the price rose from the level of unitary elasticity? If the price level fell? Why does revenue increase/decrease?

Do this too!  The market demand function for a certain good is given by: \[ Q_D = 100 - 5P. \]

Use that market demand function to answer following questions:
\[ \begin{align*}
1. \quad \text{What is the price elasticity of demand when the price is $5?} \\
2. \quad \text{What is the price elasticity of demand when the price is $10?} \\
3. \quad \text{What is the price elasticity of demand when the price is $15?} \\
4. \quad \text{Over what range of prices is demand for the good inelastic?} \\
5. \quad \text{Over what range of prices is demand for the good elastic?} \\
6. \quad \text{How does the concept of elasticity describe the way in which quantity demanded responds to changes in price?}
\end{align*} \]
Homework #4
(continued)

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
Homework #4
(continued)

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
Review for the Mid-term Exam

The first exam will cover Lectures 1, 2, 3 and 4. The exam will require you to define some of the terms listed below and require you to solve some problems. The problems given below were taken from exams that I have given in the past. They are a supplement to the homeworks. They are NOT a substitute for the homeworks.

Terms to know

- independent variable
- dependent variable
- ceteris paribus
- absolute advantage
- comparative advantage
- opportunity cost
- relative price
- gains from trade
- production possibilities frontier
- increasing opportunity cost
- price
- individual demand curve
- market demand curve
- quantity demanded
- individual supply curve
- market supply curve
- quantity supplied
- income
- substitutes
- complements
- total cost
- average cost
- marginal cost
- excess demand
- excess supply
- equilibrium
- price elasticity of demand
- elastic region of demand curve
- inelastic region of demand curve

Short Answer One

Dina and Jordan work for a state senator. Their responsibilities include clipping newspaper articles and writing happy letters. Dina can clip 10 articles per day and write 15 happy letters per day. Jordan can clip 4 articles per day and write 3 happy letters per day.

Who has a comparative advantage in clipping articles? Who has a comparative advantage in writing happy letters?

The senator is a very strange man. All of his happiness depends on the number of clipped articles and happy letters that Dina and Jordan produce.

The senator’s mood varies from day to day.
- When he’s in a good mood, he values clipped articles twice as much as he values happy letters.
- When he’s in a bad mood, he values happy letters twice as much as he values clipped articles.
- When he’s in a neutral mood, he values happy letters and clipped articles equally.

Suppose the senator is in a neutral mood. How can Dina and Jordan make the senator the happiest? In other words, who will clip articles and who will write happy letters? Or will Dina and Jordan both specialize in clipping articles or will they both specialize in writing happy letters? In your answer, be sure to explain why.

How can Dina and Jordan make the senator the happiest when he is in a good mood? How can Dina and Jordan make the senator the happiest when he is in a bad mood? In your answer, be sure to explain why.
Short Answer Two

If a business faces a production possibilities frontier (PPF) that exhibits increasing opportunity cost, why would the business want to produce at the point along the PPF where its opportunity cost equals the relative price?

Short Answer Three

Define “traffic jams” as the slow movement of vehicles on a road, causing long waiting times. Use that definition to answer the following questions (which I thought of while staring at the long line of cars ahead of me on the Long Island Expressway).

1. Using supply and demand curves for roadway capacity:
   a. Explain why traffic jams occur.
   b. What non-price rationing mechanism equates the quantity of roadway capacity supplied with the quantity of roadway capacity demanded?

2. Describe three policies that the government could use to reduce traffic jams.
   a. Illustrate each policy with supply and demand curves.
   b. If the demand curve were linear, how would each policy affect the elasticity of demand for roadway capacity?

Short Answer Four

Last year, Pres. George W. Bush and the Republicans in Congress added a prescription drug benefit to Medicare. While the senior citizens and the disabled will clearly pay less for prescription drugs after the reform is fully implemented, non-seniors and non-disabled will continue to pay the market price.

This problem requires you to use a highly simplified model to analyze the effect that the drug bill will have on the market price of prescription drugs.

Assume that there are only two groups of people – the young and the old and that before the Medicare was passed both the young and the old always paid the market equilibrium price for prescription drugs.

- The young’s demand for prescription drugs is given by: \( Q_{DY} = 400 - 15p \)
- The old’s demand for prescription drugs is given by: \( Q_{DO} = 600 - 5p \)
- The market supply of prescription drugs is given by: \( Q_{SM} = 200 + 20p \)

What is the market demand for prescription drugs? (Hint: market demand is the sum of all the individual demand curves. That is, market demand is the sum of the individual quantities demanded at each price).

Find the initial market equilibrium price and quantity. How many prescription drugs will the young buy? How many will the old buy?

For simplicity, assume that once the Medicare reform is fully implemented, the old will only pay a “co-payment” (a fixed price) which will not change due to shifts of market supply and demand. Medicare will pay the difference between the market price and the co-payment directly to the drug companies, so that the drug companies always receive the market price for each prescription drug that they sell.
If the government sets the co-payment at $6, then how many prescription drugs will the old buy?

What will the new market demand for prescription drugs be? What will the new market equilibrium price and quantity be? How many prescription drugs will the young buy? How many will the old buy?

How will the new Medicare prescription drug benefit affect the market demand for prescription drugs? How will the drug benefit affect the prices that non-seniors and the non-disabled pay for prescription drugs? Will they be helped or hurt by the new program? In your answer, be sure to explain why.

**Short Answer Five**

You are George Costanza. You work for the New York Yankees. George Steinbrenner wanted to make championship series tickets available to the public at a reasonable price, so he sold field box seat for tonight’s ALCS game against the Boston Red Sox for $225 at the box office.

Your telephone rings. You pick up the phone. It’s Mr. Steinbrenner and he is mad! He orders you to come into his office immediately. He just visited StubHub.com and saw that field box seats for tonight’s game are being resold for $2225.

Mr. Steinbrenner wants to know why people who slept on the street in front of Yankee Stadium to buy those tickets are now reselling them. Explain to Mr. Steinbrenner why they slept out for tickets, then explain to him why the tickets are being resold at a much higher price.

If Mr. Steinbrenner wants to maximize his profits, should he have sold field box seats for $2225? If there are 1000 field box seats at Yankee Stadium, how much higher would his profits have been?

**Short Answer Six**

You are the chief economic advisor to the president of Nacirema, a large country threatened by global terrorism. The terrorists seek to gain control of Iduas Aibara, the primary exporter of oil to Nacirema. The terrorists abhor Nacirema’s close relationship with Iduas Aibara and have publicly stated that if they gain control of Iduas Aibara’s oil, they will take revenge on Nacirema by cutting off its supply of oil.

Because Nacirema has very few supplies of domestic oil, Nacirema must prepare to deal with shocks to its oil supply. The president of Nacirema is particularly interested in a comparing his country’s dependence on foreign oil to Ecnarf’s.

Using the data available to you, you have estimated the demand curve in each country.

- The demand for oil in Nacirema is: \( D_N = 1000 - 10P \)
- The demand for oil in Ecnarf is: \( D_E = 2000 - 60P \)

If the world market price per barrel of oil is $15:

- How many barrels will Nacirema demand? What is Nacirema’s price elasticity of demand?
- How many barrels will Ecnarf demand? What is Ecnarf’s price elasticity of demand?

Answer those same questions for a price of $20 per barrel and for a price of $25 per barrel.

At what price levels is Nacirema’s demand elastic? At what price levels is Nacirema’s demand inelastic? At what price levels is Ecnarf’s demand elastic? At what price levels is Ecnarf’s demand inelastic?
Which country is more dependent on foreign oil? Which country is less dependent on foreign oil? Why?

Does the price elasticity of demand in each country explain each country’s dependence? Why or why not?

Provide a few reasons why one country might be more dependent on foreign oil than the other.

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<th>Quick answers</th>
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<td>Point of unity elasticity in percent</td>
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Quick answers

Price elasticity in percent

Barrels demanded in France

Barrels demanded in Mexico

Price elasticity in Germany

Which country is more dependent on foreign oil? Which country is less dependent on foreign oil? Why?

Which country is more dependent on foreign oil? Which country is less dependent on foreign oil? Why?

Does the price elasticity of demand in each country explain each country’s dependence? Why or why not?

Provide a few reasons why one country might be more dependent on foreign oil than the other.
Household Behavior

Perfect competition

- many firms, each small relative to the overall size of the industry, producing virtually identical products
- no firm is large enough to have any control over price

Perfect Knowledge

- households know the qualities and prices of everything available in the market
- firms have all available information concerning wage rates, capital costs and output prices

Households’ basic decisions in output markets

1. How much of each product to demand
2. How much labor to supply
3. How much to spend and how much to save
Household Choice in Output Markets

Determinants of Household Demand

- price of the product
- household’s income
- household’s wealth
- prices of related products
- household’s tastes & preferences
- expectations of future income, wealth and prices

The Budget Constraint

- limits imposed on choices by income, wealth and prices
- along and beneath the budget constraint lie all of the possible combinations of goods that a household can purchase

![Graph showing the budget constraint]

The Budget Constraint

When income allocated entirely towards only two goods, X and Y, income equals:

\[ I = x*p_x + y*p_y \]

Where:
- \( I \) = consumer’s income
- \( x \) = qty. of good X
- \( p_x \) = price of good X
- \( y \) = qty. of good Y
- \( p_y \) = price of good Y

Intercepts and Slopes

When \( x=0 \), only good \( y \) consumed and \( I = y*p_y \) and \( y = \frac{I}{p_y} \)

When \( y=0 \), only good \( x \) consumed and \( I = x*p_x \) and \( x = \frac{I}{p_x} \)

So when consumer moves from \( x=0 \) to \( y=0 \):

\[ \frac{\Delta y}{\Delta x} = \frac{I}{p_y} - \frac{I}{p_x} = -\frac{p_x}{p_y} = \text{slope of budget constraint} \]

Slope of budget constraint equals relative price of good X.
Change in real income
- if the consumer’s money income rises (holding prices constant), OR
- if prices fall (holding money income constant),
  - real income rises
  - budget constraint shifts outward

Change in relative price
- if the price of good X falls, the relative price of good X falls
- the maximum quantity of X he/she can consume increases
- budget constraint rotates outward

In the case depicted above real income also rises since one price falls while other remains constant.

Utility
- **Utility** is the satisfaction that a good or bundle of goods yield relative to their alternatives.
- The **Marginal Utility** of good X is the additional satisfaction gained by consuming one more unit of good X.
- **Diminishing Marginal Utility** – The more of one good consumed in a given period, the less satisfaction (utility) generated by consuming each additional (marginal) unit of the same good.

Why do demand curves slope down? ONE reason: Since marginal utility falls with each additional (marginal) unit consumed, people are not willing to pay as much for each additional (marginal) unit.
Indifference Curves

- Consumption of different combinations of goods X and Y yield different levels of utility.
- All points along indifference curve give the consumer the same level of utility.
  - Higher indiff. curves give the consumer more utility.
  - Lower indiff. curves give the consumer less utility.
- Why are indiff. curves convex to the origin? To keep consumer’s utility constant, you must compensate him/her with increasingly larger amounts of X for each additional unit of Y that you take from them – due to diminishing marginal utility.

slope of an indifference curve:

\[
\text{Marginal Rate of Substitution} = \frac{MU_X}{MU_Y}
\]

MU\textsubscript{X} = marginal utility derived from the last unit of X consumed
MU\textsubscript{Y} = marginal utility derived from the last unit of Y consumed

Allocating Income to Maximize Utility

Utility maximizing consumers spread out their expenditures until the following condition holds:

\[\frac{MU_X}{MU_Y} = \frac{p_X}{p_Y} = \text{relative price of good X}\]

Such a point represents the highest level of utility they can reach given their income and the prices of goods X and Y.

Graphically, consumers maximize their utility at the point where their indifference curve just touches the budget constraint.
Income & Substitution
Effects of a Price Change

- **Income Effect** – occurs when real income (purchasing power) changes holding the relative price constant.
- **Substitution Effect** – occurs when the relative price changes holding real income (purchasing power) constant.

Income Effect on Normal Goods

When a consumer's money income rises or falls, his/her purchasing power is obviously affected, but here we want to analyze the case of price changes, so we'll focus on the case where money income is being held constant.

If goods X and Y are both “normal goods,” then:

- when the price of X falls, the income effect encourages the consumer to use his/her higher real income to buy more of both X and Y.
- when the price of X rises, the income effect encourages the consumer to reallocate his/her lower real income by buying less of both X and Y.

Income Effect on Inferior Goods

If good X is an “inferior good,” then when the price of good X falls, the income effect encourages the consumer to use his/her higher real income to buy less of X and more of Y.

Substitution Effect on Net Substitutes

When there are only two goods, they **must** be net substitutes. We'll leave a discussion of net complements to a more advanced course.

Since goods X and Y are net substitutes:

- when the price of good X falls, it becomes more attractive relative to good Y, so the substitution effect encourages the consumer to buy more of X and less of Y.
- when the price of good X rises, it becomes less attractive relative to good Y, so the substitution effect encourages the consumer to buy less of X and more of Y.
**Gross Substitutes & Gross Complements**

If goods X and Y are both normal goods and if the price of good X falls, the income effect and the substitution effect would both encourage the consumer to buy more of good X, but the gross (combined) effect on good Y is ambiguous.

If the price of good X falls and if Y is a normal good, then:
- the **income effect** encourages the consumer to buy more of good Y
- the **substitution effect** encourages the consumer to buy less of good Y

So the gross effect depends on which effect is larger.

- If the substitution effect is larger than the income effect, then:
  - the consumer will buy less of good Y
  - and good X is a gross substitute for good Y.
- If the income effect is larger than the substitution effect, then:
  - the consumer will buy more of good Y
  - and good X is a gross complement to good Y.

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**Household Choice in Input Markets**

As in output markets, households face constrained choices in input markets.

Households’ basic decisions in input markets

1. Whether to work
2. How much to work
3. What kind of a job to work at

**These decisions are affected by:**

1. The availability of jobs
2. Market wage rates
3. The skill possessed by the household
The Price of Leisure

- The wage rate is the price – or opportunity cost – of the benefits of either unpaid work or leisure.
- The decision to enter the workforce involves a trade-off between wages and the goods and services that wages will buy on the one hand, and the leisure and the value of non-market production on the other.

Labor Supply Curve

- Labor supply curve shows quantity of labor supplied at different wage rates.
- Its shape depends on how households react to changes in the wage rate.

Substitution Effect of a Wage Change

- The substitution effect of a higher wage means that the relative price of leisure is now higher. Given the law of demand, the household will buy less leisure.
- When the substitution effect outweighs the income effect, the labor supply curve slopes upward.

Income Effect a Wage Change

- The income effect of higher wage means that households can now afford to buy more leisure.
- When the income effect outweighs the substitution effect, labor supply curve bends backward.
### Saving and Borrowing
#### Present vs. Future Consumption

Households can use present income to finance future spending (that is: save), or they can use future funds to finance present spending (that is: borrow).

#### Income & Substitution Effects of Higher Interest Rate

<table>
<thead>
<tr>
<th>Income Effect</th>
<th>Substitution Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>When the interest rate rises, households earn more on all previous savings, so the income effect encourages households to save less.</td>
<td>When the interest rate rises, the relative price of present consumption rises, so the substitution effect encourages households to save more.</td>
</tr>
</tbody>
</table>
Examples of Income and Substitution Effects

Case where Donuts are neither a Gross Complement nor a Gross Substitute for Coffee

I confess. I’m a Dunkin’ Donuts Junkie. I spend all of my income on coffee and donuts. I earn $100 per week and I spend it all on coffee and donuts. Every week I buy 25 donuts and 25 coffees.

But something strange is about to happen in Donut World. The price of donuts has always been $2 and the price of coffee has always been $2, but when I wake up tomorrow, the price of donuts will fall to $1.

What a wonderful day! My money income won’t change, but my real income (purchasing power) will be higher since I’ll now be able to purchase more coffee and more donuts.

The relative price of donuts will also fall from \( \frac{\$2/\text{donut}}{\$2/\text{coffee}} = 1 \) to \( \frac{\$1/\text{donut}}{\$2/\text{coffee}} = 0.5 \) and that will suit me just fine as I sink my teeth into a delicious Boston Cream and savor the fact that I now have to give up less coffee to eat more of my favorite donuts.

All I have to do now is figure out how much coffee and donuts I’ll consume after the price of donuts falls.

**substitution effect**

Had Dunkin’ Donuts held my real income (purchasing power) constant, by changing the price of donuts to $1.33 and the price of coffee to $2.67, I could have continued to consume 25 donuts and 25 coffees, but because the relative price of donuts would have fallen to \( \frac{\$1.33/\text{donut}}{\$2.67/\text{coffee}} = 0.5 \), I would have been offered an opportunity to relish more fluffy donuts than before.

To take advantage of that opportunity however, I’d have to drink a few less coffees, but I would have gladly given up those coffees, because powering myself up on 18.75 coffees and 37.5 donuts gives me more utility (satisfaction) than 25 coffees and 25 donuts.

**income effect**

Fortunately, Dunkin’ Donuts won’t hold my real income constant, so I’ll be able to consume more donuts and more coffees on my meager income.

Nonetheless, it was helpful to examine the effect of a change in the relative price while holding my real income constant, because now I know that – if both income elasticities equal one – I’ll always want to consume twice as many donuts as coffees at a relative price of \( 0.5 \). Since the new price of donuts will be $1 and the price of coffee will remain $2, I’ll consume 50 donuts and 25 coffees after the price change.

Therefore, by the income effect, I’ll increase my consumption of coffee from its substitution effect level of 18.75 to 25 and I’ll increase my consumption of donuts from its substitution effect level of 37.5 to 50.
Initial Budget Constraint and Indifference Curve

Initial and Substitution Effect

Initial, Substitution Effect and Income Effect
Demand Curves in the Case where Donuts are neither a Gross Complement nor a Gross Substitute for Coffee

Notice how a decrease in the price of donuts, increases the quantity of donuts that I demand. Specifically, when the price is $2, I demand 25 donuts, but when the price is $1, I demand 50. By connecting those two points on a graph we can sketch out the demand curve.

The price elasticity of demand for donuts is constant (all along this demand curve) and equal to negative one.

Since donuts are neither a gross complement nor a gross substitute for coffee, the demand curve for coffee doesn’t shift at all. It remains right where it was before the price of donuts fell. Therefore, the elasticity of demand for coffee with respect to the price of donuts equals zero.

Is this realistic? Maybe. But it seems more plausible to me that I would use some of my increased purchasing power to buy more coffee. In such a case, donuts would be a gross complement to coffee and we’d have to modify our analysis – as I’ll do on the next three pages.
Case where Donuts are a Gross Complement to Coffee

I’m still a Dunkin’ Donuts Junkie. I still spend all of my income on coffee and donuts. I still earn $100 per week. I still spend it all on coffee and donuts and I still buy 25 donuts and 25 coffees every week.

Once again, let’s assume that the price of donuts has always been $2 and the price of coffee has always been $2. Once again, let’s assume that when I wake up tomorrow, the price of donuts will fall to $1.

Once again, my money income won’t change, but my real income (purchasing power) will be higher since I’ll now be able to purchase more coffee and more donuts.

As before, the relative price of donuts will also fall from \( \frac{\$2/\text{donut}}{\$2/\text{coffee}} = 1 \) to \( \frac{\$1/\text{donut}}{\$2/\text{coffee}} = 0.5 \) so once again I’ll now have to give up less coffee to eat more of my favorite donuts.

The difference this time is that I’m going to increase my consumption of both coffee and donuts after the price of donuts falls.

substitution effect

Just like the previous case, had Dunkin’ Donuts held my real income constant, by changing the price of donuts to $1.33 and the price of coffee to $2.67, I could have continued to consume 25 donuts and 25 coffees, but because the relative price of donuts would have fallen to \( \frac{\$1.33/\text{donut}}{\$2.67/\text{coffee}} = 0.5 \), I would once again have been offered an opportunity to eat more donuts.

To take advantage of that opportunity, I’d have to drink a few less coffees, but I would have gladly given up those coffees, because powering myself up on 20.37 coffees and 34.26 donuts gives me more utility (satisfaction) than 25 coffees and 25 donuts.

Note that in this case my indifference curves are more L-shaped (than they were in the previous case), so my substitution effect consumption levels are 20.37 coffees and 34.26 donuts (as compared with 18.75 coffees and 37.5 donuts in the previous case).

income effect

Once again, Dunkin’ Donuts won’t hold my real income constant and I’ll be able to consume more donuts and more coffees on my meager income.

Nonetheless, it was helpful to examine the effect of a change in the relative price while holding my real income constant, because now I know that – if both income elasticities equal one – I’ll always want to consume 1.68 donuts per coffee at a relative price of \( \frac{0.5 \text{ coffee}}{0.5 \text{ donut}} \). Since the new price of donuts will be $1 and the price of coffee will remain $2, I’ll consume 45.68 donuts and 27.16 coffees after the price change.

By the income effect, I’ll increase my consumption of coffee from its substitution effect level of 20.37 to 27.16 and I’ll increase my consumption of donuts from its substitution effect level of 34.26 to 45.68.
Initial Budget Constraint and Indifference Curve

Initial and Substitution Effect

Initial, Substitution Effect and Income Effect
Demand Curves in the Case where Donuts are a Gross Complement to Coffee

Once again, a decrease in the price of donuts, increases the quantity of donuts that I demand. Specifically, when the price is $2, I demand 25 donuts, but when the price is $1, I demand 45.68. Once again, we can sketch out the demand curve by connecting those two points on a graph.

Does that demand curve look less elastic to you than the one in the case where donuts are neither a gross complement nor a gross substitute for coffee? It should. The price elasticity of demand for donuts at a price of $2 per donut is –0.875 and at a price of $1 per donut it’s –0.864.

Since donuts are a gross complement to coffee, my consumption of coffee rises from 25 to 27.16 when the price of donuts falls from $2 to $1. Graphically, the demand curve for coffee shifts outward when the price of donuts falls (because I’m demanding more coffee at a price of $2).

The elasticity of demand for coffee with respect to the price of donuts equals –0.125 at a price of $2 per donut and it equals –0.115 at a price of $1 per donut.
Case where Wine is a Gross Substitute for Beer

I’ll use beer and wine to illustrate gross substitutes, but the analysis will be identical to the previous cases.

This time I’m an alcoholic. I still earn $100 per week, but this time I spend it all on beer and wine and I buy 25 bottles of wine and 25 bottles of beer every week.

To preserve the similarity, let’s assume that the price of beer has always been $2 and the price of wine has always been $2. Once again, let’s assume that tomorrow the price of wine will fall to $1.

Once again, my money income won’t change, but my real income (purchasing power) will be higher since I’ll now be able to purchase more wine and more beer.

Similarly, the relative price of wine will fall from \( \frac{\$2/\text{wine}}{\$2/\text{beer}} = 1 \) beer/wine to \( \frac{\$1/\text{wine}}{\$2/\text{beer}} = 0.5 \) beer/wine so once again, I’ll now have to give up less beer to drink more of my favorite wine.

Compared with the previous cases, the difference this time is that I’m going to increase my consumption of wine and reduce my consumption of beer after the price of wine falls.

**substitution effect**

Just like the previous cases, had the liquor store held my real income constant, by changing the price of wine to $1.33 and the price of beer to $2.67, I could have continued to consume 25 wines and 25 beers, but because the relative price of wine would have fallen to \( \frac{\$1.33/\text{wine}}{\$2.67/\text{beer}} = 0.5 \) beer/wine I would once again have been offered an opportunity to drink more wine.

To take advantage of that opportunity, I’d have to drink a few less beers, but I would have gladly given up those beers, because inebriating myself up on 12.5 beers and 50 wines gives me more utility (satisfaction) than 25 beers and 25 wines.

Note that in this case my indifference curves are flatter (than they were in the previous cases), so my substitution effect consumption levels are 12.5 beers and 50 wines (as compared with 20.37 coffees and 34.26 donuts in the previous case where donuts are a gross complement to coffee).

**income effect**

Once again, the liquor store won’t hold my real income constant and I’ll be able to consume more wines and more beers on my meager income.

Nonetheless, it was helpful to examine the effect of a change in the relative price while holding my real income constant, because now I know that – if both income elasticities equal one – I’ll always want to consume four wines per beer at a relative price of 0.5 beer/wine. Since the new price of wine will be $1 and the price of beer will remain $2, I’ll consume 66.67 wines and 16.67 beers after the price change.

By the income effect, I’ll increase my consumption of beer from its substitution effect level of 12.5 to 16.67 and I’ll increase my consumption of donuts from its substitution effect level of 50 to 66.67.
Demand Curves in the Case where Wine is a Gross Substitute for Beer

Once again, a decrease in the price of wine, increases the quantity of wine that I demand. Specifically, when the price is $2, I demand 25 wines, but when the price is $1, I demand 66.67. Once again, we can sketch out the demand curve by connecting those two points on a graph.

Does that demand curve look more elastic to you than the ones in the previous cases? It should. The price elasticity of demand at a price of $2 is $-1.5$ and at a price of $1$ it’s $-1.33$.

Since wine is a gross substitute for beer, my consumption of beer falls from 25 to 16.67 when the price of wine falls from $2$ to $1$. Graphically, the demand curve for beer shifts inward when the price of wine falls (because I’m demanding less beer at a price of $2$).

The elasticity of demand for beer with respect to the price of wine equals 0.5 at a price of $2$ per wine and it equals 0.667 at a price of $1$ per wine. NB: In this case, the demand curve shift is larger (in absolute value) than the shift in the previous case. The difference in shift size is reflected in the larger cross price elasticities.
Homework #5

This set of questions requires you to derive a consumer’s demand function. Remember that the demand curve shows the relationship between the price of one good and the quantity of that good demanded when the consumer’s income and the price of other goods are held fixed.

Specifically, we’ll look at how a certain economics professor’s consumption of beer is affected when the price of beer falls from $2 to $1.

The economics professor spends his entire income on two goods: beer and pizza. The cruel university only pays him $30 per week. The initial price of beer is $2 and the initial price of a slice of pizza is $1.

If he were to spend all of his income on beer, how many beers could he buy? If he were to spend all of his income on pizza, how many slices of pizza could he buy? Placing pizza on the Y-axis and beer on the X-axis, draw the professor’s initial budget constraint. (When drawing the graph, do yourself a favor by drawing it very large).

Draw the professor’s indifference curve by assuming that he initially buys 10 beers and 10 slices of pizza. (When drawing the indifference curve, do yourself a favor by drawing the indifference curve with a very small degree of curvature).

Now, let’s examine a substitution effect. Assume that the price of beer falls to $1 and the professor’s income falls to $20 (while the price of pizza remains constant at $1). If he were to spend all of his income on beer, how many beers could he buy (after the income and price changes)? If he were to spend all of his income on pizza, how many slices of pizza could he buy (after the income and price changes)? Could he continue to consume 10 beers and 10 slices of pizza?

On the same graph that you drew the initial budget constraint and initial indifference curve, draw the new budget constraint. What do you notice about the position of the new budget constraint in relation to the initial indifference curve? Do you think the professor will consume more beer or less? Do you think the professor will consume more pizza or less? Will he be on a higher indifference curve? Draw the new indifference curve.

Next, let’s examine an income effect. Assume that the price of beer is $1 and the professor’s income is $30 (the price of pizza continues to remain constant at $1). If he were to spend all of his income on beer, how many beers could he buy now? If he were to spend all of his income on pizza, how many slices of pizza could he buy now?

On the same graph, draw the new budget constraint. What do you notice about the position of the new budget constraint in relation to the second indifference curve that you drew? Do you think the professor will consume more beer or less? Do you think the professor will consume more pizza or less? Will he be on a higher indifference curve? Draw the new indifference curve.

Congratulations! You just examined the combined income and substitution effects. Notice that you:

- started at a point where:
  - the professor’s income is $30,
  - the price of pizza is $1 and
  - the price of beer is $2.

- ended at a point where:
  - the professor’s income is $30,
  - the price of pizza is $1 and
  - the price of beer is $1.
In the end, the only thing that changed was the price of beer. The substitution effect holds the professor’s real income constant. The income effect holds the relative price of beer constant.

Now, let’s examine the professor’s consumption of beer before and after the price change. Does the substitution effect allow him to consume more beer? Does the income effect allow him to consume more beer?

On a new graph, draw the professor’s demand curve by connecting two points – one point should represent initial price of beer and quantity of beer demanded and the other point should represent the new price of beer and quantity of beer demanded.

Finally, let’s examine the professor’s consumption of pizza before and after the price of beer changes. Does the substitution effect allow him to consume more pizza? Does the income effect allow him to consume more pizza?

Notice that – in the case of beer – the income and substitution effects move in the same direction. Notice that – in the case of pizza – the income and substitution effects move in opposite directions.

In other words, the professor will consume more pizza (after the price of beer falls), if the income effect dominates the substitution effect. Here beer is a gross complement to pizza and the professor’s demand curve for pizza would shift out when the price of beer falls.

On the other hand, the professor will consume less pizza, if the substitution effect dominates the income effect. Here beer is a gross substitute for pizza and the professor’s demand curve for pizza would shift in when the price of beer falls.

Cyclones vs. Mets

The scenarios below describe how I alter my purchases of tickets to Brooklyn Cyclones’ games and tickets to New York Mets’ games when my income changes and/or when the price of a ticket to the games of one or both teams changes. For each scenario, say whether my ticket purchases are affected by:

- an income effect,
- a substitution effect,
- a combined income and substitution effect
- or no effect.

and explain why!

Each scenario is independent of the previous ones and independent of successive ones. In each scenario, I initially buy 25 Cyclones’ tickets and 25 Mets’ tickets. My initial income is $1000. The initial price of a Cyclones’ ticket is $20 and the initial price of a Mets’ ticket is $20.

1. The price of a Cyclones’ ticket falls to $10. I increase my purchases to 40 Cyclones’ tickets and 30 Mets’ tickets.
2. The price of a Cyclones’ ticket and the price of a Mets’ ticket both fall to $10. I increase my purchases to 50 Cyclones’ tickets and 50 Mets’ tickets.
3. The price of a Cyclones’ ticket and the price of a Mets’ ticket both fall to $10. My income falls to $500. I continue to purchase 25 Cyclones’ tickets and 25 Mets’ tickets.
4. The price of a Cyclones’ ticket falls to $10. The price of a Mets’ ticket rises to $30. I increase my purchases of Cyclones’ tickets to 40 tickets and decrease my purchases of Mets’ tickets to 20 tickets.

(continued on the next page)
Demand Curves for Mets’ Tickets and Cyclones’ Tickets

For the scenario above where my purchases of Mets’ tickets and Cyclones’ tickets are affected by a combined income and substitution effect:

5. Draw the initial budget constraint and indifference curve. Place Mets’ tickets on the vertical axis and Cyclones’ tickets on the horizontal axis.

6. Draw the substitution effect. Be sure to LABEL that substitution effect.

7. Draw the income effect. Be sure to LABEL that income effect.

8. Are Cyclones’ games (tickets) a gross substitute for Mets’ games? OR are Cyclones’ games a gross complement to Mets’ games?

9. Draw the demand curve for Mets’ tickets and the demand curve for Cyclones’ tickets. If a demand curve shifts, then illustrate that shift. If there is movement along a demand curve, then illustrate that movement.
I know you all think I’m crazy for proposing that the City of [redacted] designate a time and place where drug traffic could be conducted without fear of arrest, but I guess that’s the price you pay for being the only level-headed pragmatist in the office. 

I’m not saying that we should accept crime. I’m saying that we should recognize our limitations. There is no way to eliminate all drug-trafficking from [redacted], but we can make [redacted] neighborhoods drug-free.

After visiting Mr. [redacted]’s home and developing an understanding the aggravation that the adjacent crackhouse has been causing him for the last four months, I was startled that the [redacted] Police have not been able to permanently shut down the crackhouse earlier.

Why have Mr. [redacted] and his neighbors had to endure this intolerable situation for four months? Why are the [redacted] Police unable to arrest the occupants and put an end to the problem?

This memo will demonstrate that the lawlessness in Mr. [redacted]’s neighborhood persists because our government (broadly defined) places a relatively high value on prosecution, which requires police officers to spend large amounts of time preparing for court and prevents them from maintaining law and order.

This memo will also demonstrate that adding more officers to the police force will reduce the level of disorder in our neighborhoods, but the reductions will be small. Great reductions in disorderly behavior will only occur if we reduce the volume of criminal prosecutions.

This memo makes use of microeconomic theory – but relax! I’ll keep it simple. I promise.

A microeconomic theory of policing

The critical assumption of this model is that people do not knowingly commit crimes if they think that there is a good chance of being arrested. In other words, I’m assuming that no one would commit a crime if they knew a police officer was present.

They only commit crimes if they think that they will not be caught, so prosecutions do not deter crime. Considering the fact that the drug dealers in Mr. [redacted]’s neighborhood have already been arrested and charged and are currently awaiting prosecution, this seems like a very reasonable assumption. The only deterrent to crime and lawlessness is putting more police officers on patrol.

Since it is impossible to simultaneously file paperwork and walk the beat, there is a trade-off between the two. Every hour that the police spend on paperwork is one less hour that they can spend walking the beat.

The figure below illustrates the “police budget constraint.” The police budget constraint depicts the different combinations of time that the police can allocate to paperwork and “walking the beat” (i.e. responding to complaints, car patrols, etc.). The point is that the police have to budget their time.
The police force has a limited number of officers who divide their time between filing paperwork and walking their beat.

The police budget constraint reflects the tradeoff between paperwork and walking the beat at the maximum level of police activity.

A police force that spends most of its time filing paperwork spends few hours on patrol.

Whereas a police force that spends little time filing paperwork can spend more time on patrol.

In order for a police force to spend most of its time filing paperwork, it cannot spend much time walking the beat. Conversely, a police force that spends most of its time walking the beat cannot spend much time filing paperwork.

At the point where the budget constraint intersects the “paperwork” axis, the police spend all of their time filing paperwork and cannot spend any time on patrol or walking the beat. Similarly, at the point where the budget constraint intersects the “beat walks” axis, the police spend all of their time on patrol and cannot spend any time filing paperwork that lead to criminal prosecutions.

So who determines how much time the police devote to each activity? And what is the optimal amount of time that the police should devote to each activity? The answer lies in the preferences of the mayor, who oversees the police department.

The “indifference curves” depicted in the next figure represent different levels of satisfaction that the mayor derives from the different combinations of paperwork and beat walks. Obviously, the mayor would like all criminals to be prosecuted and he would also like to have his police officers on the beat all the time to prevent disorderly behavior from occurring in the first place.

This assertion is illustrated by the location of the indifference curves relative to the origin. Indifference curves which are further from the origin contain combinations of paperwork and beat walks which give him higher levels of satisfaction (“utility”) that crime is being fought.

The mayor is willing to accept certain trade-offs however. All of the different combinations of paperwork and beat walks along an indifference curve give the mayor the same level of satisfaction. Not all trades are equal however.

If the initial ratio of paperwork to beat walks is relatively high (point B), then – in order to preserve the same level of satisfaction – the mayor requires a large increase in paperwork to compensate for the small decrease in beat walks (movement along the indifference curve from point B to point A).
If more paperwork leads to more criminal prosecutions and if more beat walks lead to lower crime rates, then the mayor of the city would prefer both more paperwork and more beat walks. So the mayor prefers all points on indifference curve 2 to any point on indifference curve 1.

The mayor is indifferent however between points A, B, C and D.

In order to remain indifferent between points A and B however, the mayor would require a very large increase in paperwork to offset the small decrease in beat walks.

Conversely, in order to remain indifferent between points C and D, the mayor would require a very large increase in beat walks to offset the small decrease in paperwork.

In other words, at point B the mayor demands a large increase in prosecutions to compensate for the increase in disorderly behavior that results from fewer beat walks.

Conversely, if the initial ratio of paperwork to beat walks is relatively low (point C), then – in order to preserve the same level of satisfaction – the mayor requires a large increase in the number of beat walks to compensate for the small reduction in paperwork (movement from point C to point D).

In other words, at point C the mayor demands a large increase in beat walks and more order to compensate for the small reduction in prosecutions that result from less paperwork being filed.

So what combination of paperwork and beat walks will the mayor accept? The mayor will choose a point along the police budget constraint since he prefers more paperwork and more beat walks to less.

But what point along the police budget constraint will he choose? The mayor will choose the point where his indifference curve just touches the budget constraint, so that he can obtain the maximum level of satisfaction given the constraint on the police’s time (that is, given the police budget constraint).

He would not choose a point where his indifference curve crosses the budget constraint because at such a point, he could still gain more satisfaction (at a higher indifference curve) by choosing a different combination of time that the police spend filing paperwork and walking the beat.

**How should the mayor respond to complaints about drug trafficking?**

A mayor who wants to address constituent concerns about drug trafficking in his city frequently responds by expanding the police force. Such an expansion will increase both the amount of paperwork that the police can file and the number of patrols that the police can undertake.

If however, the mayor places such a high value on prosecutions that the police are forced to spend the vast majority of their time filing paperwork, then the vast majority of the additional police capacity will be
Politicians frequently respond to constituents complaints about crime by promising to enlarge the police force.

Adding more officers to the police force shifts the police budget constraint outward and leads to both an increase in justice and an increase in order.

Notice however, that if the mayor’s preferences remain unchanged, then the increase in justice will be much larger than the increase in order.

 allocated to filing paperwork, not patrolling the neighborhoods. Consequently, there will be a large increase in the justice that crime victims receive from prosecutions, but the reduction in disorderly behavior (increase in order) will be minimal.

In fact, because our government (broadly defined) places such a high value on prosecution relative to patrolling, the police spend most of their time filing paperwork and very little time maintaining order in our neighborhoods.

Every time the Police make a drug bust, they have to spend a week compiling all of the evidence and assisting the prosecution. Then when the case goes to trial, the officers are spending time in court and are not walking the beat. In the meantime, another crackhouse pops up somewhere else.

Because the Police are so deluged with paperwork, they have completely stopped responding to Mr. ’s complaints. 911 has told him not to call. He and his neighbors are frustrated.

I don’t think Mr. and his neighbors are upset about the drug-trafficking per se, I think they’re upset about the anarchic state of their neighborhood.

They’re tired of car horns blowing when someone wants a quick pick-up. They’re tired of cars driving in and out all night. They’re tired of being afraid that a drug-addict might rob them. They’re tired of the drug dealers threatening them. The cold, hard fact of the matter is that the dealers can do anything they want without any regard to the effect that their activities have on the neighborhood.

Given what they’ve been through over the past four months, they certainly want the dealers arrested, but if they had to choose between having a peaceful neighborhood without prosecution and having the dealers prosecuted but without an elimination of the drug trafficking, I guarantee you that they would choose peace and quiet over prosecution.
Mayors A and B face the same police budget constraint, but they have different preferences (as illustrated by the different positions of their indifference curves).

Mayor A is willing to tolerate a higher level of lawlessness in his city, so that his police force can prosecute more criminals.

Mayor B prefers to prosecute fewer criminals, so that his police force can maintain a higher level of order.

A larger police force would be helpful, but what we really need is police who respond quickly to disorderly behavior in our neighborhoods. What we need is for the mayor to change his preferences.

If the mayor accepted some limited drug-trafficking in a designated area far from all residential neighborhoods, but also vowed to swiftly arrest and prosecute anyone who buys, sells or uses illegal drugs in a residential neighborhood, then police could spend less time filling out paperwork and more time maintaining order in our neighborhoods.

Even better, the police could watch the traffic in that parking lot to make sure that the traffic stays respectable. No violence. No loud music. No driving away under the influence of mind-altering drugs. And make the users clean up after themselves.

The risk of course is that the designated area will become a major distribution center for the whole [town name] and surrounding regions. But if the police are on the scene, they can still take notes about what is occurring there, which will enable them to make prosecutions elsewhere.

Would this solve the drug problem in [town name]? Most certainly not! But it would:
- keep drug-trafficking away from small children,
- keep drug-trafficking out of our neighborhoods and
- allow Mr. [name] and his neighbors to sleep peacefully at night.

Questions to explore

- Are officers required to report all drug activity when they respond to a complaint? If so, that may be one of the reasons why they cannot afford to respond to Mr. [name]'s calls.
- Are police officers legally obligated to report any and all drug activity that they see? If so, then they would be breaking the law themselves if they allowed drug traffic to occur in such a designated area.
- Would the City of [town name] be liable if someone overdosed in the designated area?
Intro to Firm Behavior

- production – process by which inputs are combined, transformed, and turned into outputs
- firm – person or a group of people that produce a good or service to meet a perceived demand
- we’ll assume that firms’ goal is to maximize profit

Perfect Competition

- many firms, each small relative to overall size of the industry, producing homogenous (virtually identical) products
- no firm is large enough to have any control over price
- new competitors can freely enter and exit the market

Competitive Firms are Price Takers

- firms have no control over price
- price is determined by market supply and demand
What does it mean to be a price taker?

- Each firm has an upward sloping supply curve, but:
  - price is determined by market supply and demand
  - so shifts of one household’s demand curve do not affect the market price
- Each household faces infinitely elastic (horizontal) supply

Households

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<td>S</td>
<td>D</td>
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</table>

- Each household has a downward sloping demand curve, but:
  - price is determined by market supply and demand
  - so shifts of one firm’s supply curve do not affect the market price
- Each firm faces infinitely elastic (horizontal) demand

Firms

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</table>

Firms’ Basic Decisions

1. How much of each input to demand
2. Which production technology to use
3. How much supply

Short-Run vs. Long-Run

In the short-run, two conditions hold:
1. firm is operating under a fixed scale of production – i.e. at least one input is held fixed (ex. it may be optimal for a firm to buy new machinery, but it can’t do so overnight)
2. firms can neither enter nor exit an industry

In the long-run:
- there are no fixed factors of production, so firms can freely increase or decrease scale operation
- new firms can enter and existing firms can exit the industry
Profit-Maximization

(economic) profit = total revenue – total (economic) cost

**total revenue** – amount received from the sale of the product (price times number of goods sold)

**total (economic) cost** – the total of:
1. **out of pocket costs** (ex. prices paid to each input)
2. **opportunity costs**:
   a. **normal rate of return on capital and**
   b. **opportunity cost of each factor of production** – ex. if an employee in my firm could earn $30,000 if he/she worked for another firm, then I’d have to pay him/her at least $30,000, otherwise he/she would leave. (In reality, you might work in your parents’ firm and not be paid. In such a case, the accounting profit of their firm would be higher than the economic profit of their firm).

**normal rate of return on capital** – rate of return that is just sufficient to keep owners and investors satisfied (ex. stock dividends or interest on bonds)

- nearly the same as the interest rate on risk-free government bonds for relatively risk-free firms
- higher for relatively more risky firms

Production Process

optimal method of production minimizes cost

**production technology** – relationship betw/n inputs & outputs
- labor-intensive technology relies heavily on labor instead of capital
- capital-intensive technology relies heavily on capital instead of labor

**production function** – units of total product as func. of units of inputs

**average product** – average amount produced by each unit of a variable factor of production (input)

\[
\text{avg. product} = \frac{\text{total product}}{\text{total units of labor used}}
\]

**marginal product** – additional output produced by adding one more unit of a variable factor of production (input), ceteris paribus

\[
\text{marg. product} = \frac{\Delta \text{total product}}{\Delta \text{units of labor used}}
\]
Total, Average and Marginal Product

- **diminishing marginal returns** – when additional units of a variable input are added to fixed inputs, the marginal product of the variable input declines (in the graph, diminishing marginal returns set in after point A)

  - **Marginal Product** is the slope of the total product function

  - **Point A** – slope of the total product function is highest; thus, marginal product is highest

  - **Point B** – total product is maximum, the slope of the total product function is zero, and marginal product is zero

Production with Two Inputs

- Ex. cooks (Labor) and grills (Kapital)
- If you hire more cooks, but don’t add any more grills, the marginal product of labor falls (too many cooks in the kitchen)
- If you hire (rent) more grills, but don’t add any more cooks, the marginal product of capital falls (grills sit idle).

Given the technologies available, a profit-maximizing firm:

- hires labor up to the point where the wage equals the price times the marginal product of labor (MPL)
- hires kapital up to the point where the rental rate on kapital equals the price times the marginal product of kapital (MPK)

\[
\text{Profits } (\Pi) = \text{Total Revenue} - \text{Total Cost} = p_x X(K,L) - wL - rK
\]

at profit maximum:

\[
w = p_x \cdot MPL
\]

\[
r = p_x \cdot MPK
\]
Profit Maximization with Two Inputs

- In a perfectly competitive industry:
  o firms are price takers in both input output markets,
  o firms cannot affect the price of their product, nor can they affect the price of inputs (wages, rental rate on kapital).

- Assume that a firm produces X with kapital and labor and that its production function is given by:

\[ X = K^{2/3}L^{1/3} \]

- Assume also that:
  o price of the firm’s output is $1
  o wage rate and rental rate are both $0.53

- So its profits are given by:

\[ \Pi = p_x X - rK - wL \]
\[ = $1* K^{2/3}L^{1/3} - $0.53*K - $0.53*L \]

- and firm should hire labor until \( w = p_xMPL \)

Profit Maximization with Two Inputs

- If the firm has 20 units of kapital on hand (this is the short-run):
  o how much labor should it hire?
  o how much should it produce?
  o how much profit will it make?

<table>
<thead>
<tr>
<th>Output of X</th>
<th>Kapital</th>
<th>Labor</th>
<th>MPL</th>
</tr>
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<tbody>
<tr>
<td>14.74</td>
<td>20</td>
<td>8</td>
<td>0.61</td>
</tr>
<tr>
<td>15.33</td>
<td>20</td>
<td>9</td>
<td>0.57</td>
</tr>
<tr>
<td>15.87</td>
<td>20</td>
<td>10</td>
<td>0.53</td>
</tr>
<tr>
<td>16.39</td>
<td>20</td>
<td>11</td>
<td>0.50</td>
</tr>
<tr>
<td>16.87</td>
<td>20</td>
<td>12</td>
<td>0.47</td>
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</tbody>
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- In the case depicted, the firm:
  o would hire 10 labor units
  o would produce 15.87 units of X
  o would make **zero profit** … (I’m foreshadowing Lecture 8 a little here)

\[ \Pi^* = $1*15.87 - $0.53*20 - $0.53*10 = 0 \]

- Hiring more labor or less labor would lower profit:

\[ \Pi_8 = $1*14.74 - $0.53*20 - $0.53*8 = -$0.14 \]
\[ \Pi_9 = $1*15.33 - $0.53*20 - $0.53*9 = -$0.07 \]
\[ \Pi_{11} = $1*16.39 - $0.53*20 - $0.53*11 = -$0.03 \]
\[ \Pi_{12} = $1*16.87 - $0.53*20 - $0.53*12 = -$0.06 \]
Recall the production function I used above:

\[ X = K^{2/3} \times L^{1/3} \]

and solve for \( K \):

\[ K = X^{3/2} \times L^{-1/2} \]

This second equation shows us how much we need to increase use of kapital as labor inputs decrease in order to hold output constant.

This relationship gives us an “isoquant.”

**Isoquant** – shows all combinations of kapital and labor that can be used to produce a given level of output.

**Slope of isoquant: marginal rate of technical substitution**

\[ \text{slope} = \frac{\text{MPK}}{\text{MPL}} = \frac{\Delta K}{\Delta L} \]

Recall that the wage rate and the rental rate were both equal to $0.53

So what’s the relative wage?

You can trade one unit of labor for one unit of capital at a one-to-one ratio, so the relative wage is:

\[ \frac{w}{r} = \frac{$0.53/\text{unit of labor}}{$0.53/\text{unit of capital}} = 1 \frac{\text{unit of kapital}}{\text{unit of labor}} \]

The relative wage gives us the slope of the isocost line.

**Isocost line** – shows all possible quantities of labor and kapital which yield the same total cost.

The optimal employment levels of kapital and labor are given by at the point where the isoquant is tangent to (just touches) the isoquant.

Note that the isocost line looks just like a budget constraint.
Note that the isoquant curve looks just like an indifference curve.
Application to Land Value Taxation

As you’ll read in my memo (“economic thought on Land Value Taxation”), I was asked for my opinion on how shifting a city’s property tax burden from the combined value of the land and building, to the value land primarily would affect investment in the city’s existing stock of buildings.

If you think of kapital (buildings) and land (abbreviated T for “Terra”) as inputs into the production of asset returns and if you think of tax rates as the input prices, then you can analyze the question with isoquants and isocosts.

They had hoped that by taking the tax burden off of the building’s rental value, property owners would have a greater incentive to renovate their properties – renovation is a form of investing in capital. (They were seeking to rid the city of unsightly abandoned properties).

The effective price of an asset equals \( \frac{p_{\text{asset}}}{1-\tau} \), where \( p_{\text{asset}} \) is the market price of the asset and \( \tau \) is tax on the return to the asset.

Under the proposal, the overall tax burden would remain constant, so the only effect would be a substitution effect. If the isoquants have a nice curved shape, then property owners would substitute out of land and into kapital and their asset returns would be higher.

I argued that the isoquants may be L-shaped due to the complementarity between land and buildings. Consequently, the proposal would have no effect on the optimal holdings of land and kapital.
Eric Doviak

To: 
Date: 
Subject: economic thought on Land Value Taxation

I did some research into the economic thought underlying the Land Value Taxation idea and I found a solid piece written by Martin Feldstein back in 1977. (Feldstein was Pres. Reagan’s chief economic advisor. He now teaches at Harvard and is the president and CEO of the widely-respected, non-partisan National Bureau of Economic Research).

His article suggests that there are a few problems with the LVT proposals that you are considering.

**the basic argument for land value taxation**

Before I discuss his paper, let’s review the basic argument for land value taxation. Specifically, how would LVT affect you personally?

As I recall, you just bought a rental property.

To keep the math simple, if the value of your new property is $100,000, if you can rent it for $10,000 and if the tax rate is 10 mills (or one percent), then you would pay $1000 in taxes, so your annual net return would be $9000.

Under the current property tax system, any improvement on the property that increases the rent you can earn from the property would be subject to tax. So if the improvement increases the property value to $200,000 and the rent to $20,000, then you would pay $2000 in taxes (after reassessment) and your annual net return would be $18,000.

Now here’s the important part ... By making the improvement, your annual net return would increase by $9000.

Under a highly simplified LVT system – where the land is taxed and the property value is not taxed at all – making a capital improvement would not increase your tax burden at all. So if the land value tax was a flat $2000, then your net return before the improvement would be $8000 and your net return after the improvement would be $18,000.

The important part once again ... By making the improvement, your annual net return would increase by $10,000.

So comparing the current system with the simplified LVT system, your return from the improvement would be $1000 higher under LVT. Consequently, in this simplified example, you would no longer be “punished” $1000 for making improvements on your property. Instead you would receive the full rental value of the improvements that you made.

This line of reasoning suggests that LVT would send a positive signal to property owners that they will not be penalized for making improvements on their property.
portfolio allocation

Now let’s discuss a simple theory of optimal allocation of assets in a portfolio. Such theories state that the returns on assets of equal risk should be equal at the margin to maximize profit.

I’m going to discuss stocks, but to keep things simple let’s assume that the share price remains constant at $1. The share price never goes up and it never goes down.

The only reason why you would buy such a stock is if it paid a dividend each year. I’m going to assume that the dividend is set at a constant percentage of the share price and call it “the return.”

So let’s say you’re talking with your broker about two different stocks that are equally risky. One has a return of 10 percent per year and the other has a return of 5 percent a year. Which stock would you buy? The one with the higher return, right? You’d be foolish if you didn’t.

Now let’s say you had to pay a tax of 50 percent on the return to the stock with a 10 percent return, but no tax on the stock with a 5 percent return. Which stock would you buy? Net of taxes, both stocks now have the same return, so you’d buy a little of each.

That was simple, right?

As an introduction to Feldstein’s article, I’m going to change the example and introduce a concept that I will refer to later as “complementarity.”

Let’s say one stock has a 10 percent return and the other has a 5 percent return, but this time assume that for every share of stock with a high return that you purchase, you MUST also buy one share with a low return. (The stocks are “complements.”)

If you bought 100 shares of each stock (at $1 each), then you’d receive $10 from the stock with a 10 percent return and $5 from the stock with a 5 percent return. At the end of the year, you’d be $15 richer.

As before, let’s say you had to pay a tax of 50 percent on the return to the stock with a 10 percent return, but no tax on the stock with a 5 percent return. So net of taxes, you’d receive $5 from each stock and at the end of the year you would be $10 richer.

The story doesn’t end there however. Notice that because you have to buy the stocks in equal quantities, the tax on the returns to the 10-percent return stock imposes an effective tax on the return to the 5-percent return stock.

The effective tax on the returns to each stock is 33.3 percent (one-third).

To see that more clearly, assume that you had to pay a 33.3 percent tax on the returns to each stock. Your after tax return on the stock with a 10 percent return would be $6.67 and your after tax return on the stock with a 5 percent return would be $3.33.

At the end of the year, you would be $10 richer – exactly the same as if you only paid a tax of 50 percent on the return to the stock with a 10 percent return.
The point of this exercise was to show that when two different assets are “complements,” imposing a tax on the returns to one asset imposes an effective tax on the returns to the other, so part of the tax burden is shifted from the one asset to the other.

**relevance of portfolio allocation to land value taxation**

By this point, you’re probably wondering why I went into a lengthy explanation of the theory of optimal allocation of assets in a portfolio. You’re probably wondering: “how could this possibly be relevant to land value taxation?”

Remember that returns on assets of equal risk should be equal at the margin to maximize profit.

Remember that when two assets are complements, a tax on the return to one asset imposes an effective tax on the return to both assets – some of the tax burden is shifted.

The Feldstein article (that I keep alluding to) suggests that there are problems with the LVT proposals that you are considering. Feldstein demonstrated that when a government taxes the value of land, some of the tax burden will be shifted onto capital. There’s no way to tax land only – a “land-only” tax would still impose an effective tax on capital.

Feldstein considered a case where two assets enter an individual’s portfolio – land and capital. (“Capital” basically means machinery or buildings).

Unlike my examples above, which assumed that each asset has a constant return, Feldstein assumes that the returns on an asset decrease as you employ more of that asset (while holding the other constant).

The assumption that assets have diminishing marginal returns is similar to the “too many cooks in the kitchen” problem.

If you are cooking a big Thanksgiving dinner, it’s helpful to bring a second cook into the kitchen because the two of you will be able to produce more in the same amount of time. If there are four cooks in the kitchen, the cooks will get in each other’s way and you won’t necessarily be able to produce much more than two cooks can produce.

The same is true of land and capital.

If you build a two family home on a property, you might be able to rent each apartment for $1000 a month, but if you build a building with 50 apartments on that same property, you might only be able to rent each apartment for $500 each. If you build a building with 1000 apartments, you will have so massively increased the housing supply in that you won’t be able to rent each apartment for much more than $200 a month.

Consequently, capital (in this case, the apartments) has diminishing marginal returns. The rent decreases as you build more and more apartments on a single property.

Similarly, if you restore an old building, you will increase its rental value, but each additional improvement brings smaller and smaller returns. Replacing the wiring and plumbing and repairing damage to the walls will greatly increase its rental value (which, of course, is the return on capital). If, after installing new wiring and plumbing, you line the walls with 14 carat gold, you might still increase the rental value, but don’t expect a much higher return!
Land also has diminishing marginal returns. If you buy an acre of virgin land in [BLANK], you might be able to rent it to the City as park space for $10,000 a year. If however you bought all of the land in [BLANK] Township, don’t expect to be able to rent it for $10,000 an acre!

Now let’s return to Feldstein’s discussion of optimal allocation of assets in a portfolio, where the assets face diminishing marginal returns.

If the City of [BLANK] increases the tax on land and decreases the tax on capital, the net return on land will fall, while the net return on capital will rise.

If individuals optimally allocate assets in their portfolios of land and capital, they will increase their holdings of capital and decrease their holdings of land until the net returns on each assets are once again equal. To increase their capital holdings they will renovate their buildings, increasing their rental value – the return on capital.

### the problem with land value taxation

Feldstein’s contribution is that there is some complementarity between land and capital. Farming is a classic example of complementarity between land and capital. You need both land and tractors to produce wheat. There is of course some complementarity between land and capital in a city as well. After all, a building can’t float in the sky. It has to sit on land.

Due to the complementarity, some of the burden of land taxation would be shifted onto capital. The degree of tax shifting depends on the degree of complementarity between land and capital. If land and capital are “perfect complements” (they must be employed in rigidly fixed proportions), then the shifting will be large. If, on the other hand, land and capital can be substituted for one another (the opposite of complementarity), then the shifting will be small.

Now here’s the problem with Land Value Taxation. Zoning regulates the type of building that can be built on a given piece of property. If you own a single family home, you can’t simply tear it down and build a skyscraper.

Zoning enforces near perfect complementarity between land and capital, so the shifting of the tax burden would probably be large under the proposal that you are considering.

Consequently, a two-tiered property tax system might encourage renovation of existing buildings within the city as the owners of property add to their capital stock by renovating existing buildings, but due to zoning and tax shifting, these effects will be very, very small.

### conclusion

If the City Council passes LVT, it’s not going to hurt, but it’s definitely not the magic bullet you’re looking for.

More flexible zoning laws would increase the effectiveness of LVT by reducing the mandated degree of complementarity between land and capital.
As a practical matter however (and thinking particularly of what used to be the [redacted] neighborhood), if you are trying to restore the look and feel of a vibrant downtown shopping and residential neighborhood, then you may want to impose more stringent zoning requirements, such as: maximum setbacks from the street and adjacent properties, mandated mixed-use commercial and residential (i.e. apartments above the shops), etc.

After all, the level of economic activity in a city depends on the number of businesses within the city. The overwhelming majority of businesses in a vibrant urban area are downtown merchants and downtown merchants cannot survive without a client base.

To attract (and retain) businesses and families, you need good urban planning. In the long run, good urban planning will be much more effective than tinkering with the tax code.
Notes on Isoquants, Isocosts and the Memo on Land Value Taxation

In Lecture 6, I used isoquants and isocosts to analyze profit-maximization. This is not strictly correct. The point of tangency between an isoquant and an isocost line illustrates the point where cost is minimized for a given level of output. It is not necessarily the point where a firm maximizes its profit.

Obviously, a firm cannot be maximizing its profit unless it’s minimizing cost, but the reverse is not necessarily true. The difference occurs because:

- **cost minimization** – occurs when the firm minimizes the cost of producing a given level of output
- **profit maximization** – occurs when the firm minimizes the cost of producing the level of output which maximizes its profit.

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**asset returns – the case of perfect substitutes**

Imagine that you have the opportunity to buy shares of two equally risky stocks.

- the price of stock A is $1
- the price of stock B is $1
- the return on stock A is 5%
- the return on stock B is 10%

**Isocost** – if you have **$1000 to invest**, how much of each stock can you buy?

- If you don’t buy any shares of stock B, then you can buy 1000 shares of stock A.
- If you don’t buy any shares of stock A, then you can buy 1000 shares of stock B.

**Isoquant** – how much of each stock would you have to buy to get a **return of $100**?

- If you invest entirely in stock A, then 2000 shares of stock A would give you a $100 return.
- If you invest entirely in stock B, then 1000 shares of stock B would give you a $100 return.

Note that the isoquant is a straight line. Isoquants are always straight lines when the two inputs (in this case: stocks) are perfect substitutes. The same is true of a consumer’s indifference curves.

Now let’s examine the case where the government imposes a 50% tax on the return to stock B. There are two ways of looking at the change:

- **Method #1** – The return to stock B (net of taxes) has fallen to 5%, so the slope of each isoquant changes. Given the same isocost line, you end up on a lower isoquant since it’s no longer possible to earn a $100 return. Instead you can only earn a $50 return.
• **Method #2** – It is now more costly to hold stock B, so you can think of the tax as increasing the price of stock B from $1 to $2, while leaving the return unchanged*. The slope of each isocost line changes. Once again, you end up on a lower isoquant, since you can no longer earn a $100 return. You can only earn a $50 return.

Even though method #1 more accurately describes the case of taxing asset returns, I’ll use method #2 in this analysis because my goal is to eventually describe the taxation of land and capital.

---

**asset returns – the case of perfect complements**

Once again, imagine that the price of stock A and the price of stock B are both $1 and that you have $1000 to invest, so that the initial isocost line is drawn from 1000 shares of stock A on the vertical axis to 1000 shares of stock B on the horizontal axis.

Now assume that you have to buy stocks A and B in equal quantities – stocks A and B are perfect complements. This is not as absurd as it may seem. For example, if you’re buying a house, you have to buy the plot of land it sits on too.

In this case, the isoquant becomes L-shaped.

- You could buy one house and two plots of land, but your return (in terms of the rent you can charge) will be no greater than if you had only bought one house and one plot of land. After all, who’s going to pay to live on a vacant plot of land?
- Similarly, if you bought one plot of land and two houses, but the second house had no land to sit on (imagine that the second house was just an unassembled pile of bricks and mortar), then your return (in terms of the rent you can charge) will be no greater than if you had only bought one house and one plot of land.

Since you have to buy the shares in equal quantities:

- you would buy 500 shares of stock A and 500 shares of stock B
- your 5% return on stock A would give you $25 and
- your 10% return on stock B would give you $50.

So your initial isoquant is drawn for a $75 return.

---

* The effective price is equal to \( \frac{p_{\text{share}}}{1 - \tau} \), where: \( p_{\text{share}} \) is the market price per share and \( \tau \) is the tax rate on returns to that stock.
If the government imposes a 50% tax on the return on the return to stock B – via method #2: the effective price of stock B rises from $1 to $2 (effectively, a $1 tax per share) – then your isocost line would rotate inward in a clockwise direction.

Since you have to purchase stocks A and B in equal quantities:
- you would now buy 333 shares of stock A and 333 shares of stock B
- your 5% return on stock A would give you $16.67 and
- your 10% return on stock B would give you $33.33.

So your new isoquant is drawn for a $50 return.

---

**the substitution effect – the case of perfect complements**

In the case of an individual’s consumption of two goods, the pure substitution effect contains a change in relative price, but compensates the consumer for the relative price change by enabling him to consume the same initial bundle of goods. That is: there is a relative price change, but the consumer’s initial real income (purchasing power) is left unchanged.

To draw the pure substitution effect for the case of stocks A and B, we’ll rotate the isocost line through the initial portfolio – so that the initial allocation of stocks is still possible.

So once again, imagine that the initial price of stock A and the initial price of stock B are both $1 and that you have $1000 to invest. The initial isocost line is drawn from 1000 shares of stock A to 1000 shares of stock B.

Once again imagine that you have to buy stocks A and B in equal quantities, so you initially buy 500 shares of A and 500 shares of B. Your initial return is still $75.

Now imagine that the government imposes 25% tax on the returns to stock B – raising the effective price to $1.33 tax per share of stock B – and gives a 50% subsidy on the returns to stock A – lowering the effective price to $0.67 per share of stock A.

Your new isocost line runs from 1500 shares of stock A to 750 shares of stock B. Notice that you can still purchase 500 shares of stock A and 500 shares of stock B (500*$1.33 + 500*$0.67 = $1000).
Will you change the allocation of stocks in your portfolio? No, you won’t. You cannot earn a higher return by changing your allocation because the stocks are perfect complements. Any reallocation would leave you on a lower isoquant – your returns would fall.

But what if the stocks were not perfect complements?

✦✦✦

**the substitution effect – the “normal” case**

Rarely are two goods or two stocks are perfect complements or perfect substitutes. Normally, they’re somewhere in between. For review:

- **perfect substitutes** – indifference curves/isoquants are straight lines
- **perfect complements** – indifference curves/isoquants are L-shaped
- **somewhere in between** – indifference curves/isoquants are curved (convex to the origin).

For example, two stocks are not perfect substitutes when one is more risky than the other. Land and capital are not perfect complements because on a given plot of land you can build a single-family home or a 10-story apartment complex.

When zoning is present however, land and capital are “near perfect complements,” but not “perfect complements.” After all, a rental property can be in good condition or poor condition.

If you read my memo on land value taxation, you’ll notice that the councilman who advocated land value taxation ignored the issue of zoning, so he thought that you could substitute land for capital more easily. That is: he thought that the isoquant was curved (convex to the origin).

Let’s explore his idea in more detail. He proposed that the city should raise the tax on land and lower the tax on capital (buildings) in such a way that the overall tax burden would be left unchanged. That is he proposed a “pure substitution effect.”

If not for zoning, his idea would have encouraged owners of property to reduce their holdings of land and increase their holdings of capital – via renovation or other improvements – in order to earn a higher return on their portfolio of land and capital assets.

Their higher return is illustrated by their ability to reach a higher isoquant when they sell off some of their stock of land (abbreviated with a T) and purchase capital (abbreviated with a K).

In the presence of zoning however, the near perfect complementarity between land and capital would have given owners little incentive to substitute land for capital (as depicted in the previous section), since the isoquants would have been nearly L-shaped.
Pam owns a public relations firm that produces flyers, press releases and websites. For simplicity, assume that she out-sources the printing, so that we can ignore inputs of printing machinery and paper.

Pam’s firm produces public relations material using two inputs: computers (a form of capital) and the efficiency with which her workers (labor) use those computers. Computers and the efficiency of labor are perfect complements in Pam’s production process.

For the past few years, Pam has not noticed any changes in either the labor that she hires or the computers that she rents. Lately however, her workers have been attending Prof. Doviak’s classes and, as a result, they have become much more productive!

Pam pays the same rental rate for her computers and the same wage rate to her workers, but now each worker produces twice as much public relations material as they did before, using the same equipment (computers) that they did before.

1. The slope of Pam’s isocost and the slope of Pam’s isoquant are both defined in terms of two “inputs.” What are those two inputs?
2. How will the increase in the efficiency of labor affect the relative rental rate for computers?
3. How will the substitution effect alter Pam’s optimal employment of computers and labor efficiency?
4. If Pam continues to produce at the same total cost, can she now produce more public relations material, less public relations material or the same amount as before?
5. If Pam continues to produce at the same total cost, how will her optimal employment of computers and labor efficiency change?

Julia is a college student. Her utility depends on her consumption of two goods: hamburgers and cocaine. Assume that Julia’s Marginal Rate of Substitution (MRS) between hamburgers and cocaine is constant and equal to six hamburgers per line of cocaine, so that hamburgers and cocaine are perfect substitutes in Julia’s utility function.

The price of a hamburger is $5. The price of one line of cocaine is $10. Since Julia is a student, an allowance from her father is her sole source of income. Her father gives her $120 per week.

1. If Julia did not consume any cocaine, how many hamburgers could she consume?
2. If Julia did not consume any hamburgers, how many lines of cocaine could she consume?
3. Placing hamburgers on the vertical axis and cocaine on the horizontal axis, draw Julia’s budget constraint.
4. On the same graph that you drew Julia’s budget constraint, draw her indifference curve using the assumption that Julia’s MRS is constant and equal to six hamburgers per line of cocaine. Be sure to indicate the point at which her utility is maximized.
5. Will Julia consume both hamburgers and cocaine? Will she consume only hamburgers and no cocaine? Or will she consume only cocaine and no hamburgers? Explain your answer.

(continued on the next page)
Now Julia graduates college. Since she has graduated, her father no longer gives her an allowance. To fund her purchases of hamburgers and cocaine, Julia now needs to work.

She gets a job at a bank and receives $5 for each hour that she works. She is expected to work 24 hours per week, but when she consumes cocaine she arrives at work late the next day and is unable to make up the hours that she missed (resulting in a loss of income). Specifically, the number of hours that she misses is equal to the square of the number of lines of cocaine that she snorts:

\[
\text{missed hours} = c^2 \quad \text{where: } c \text{ is the number of lines of cocaine that she snorts.}
\]

In other words, if she snorts one line she misses one hour of work. If she snorts two lines, she misses four hours of work. If she snorts three lines, she misses nine hours of work, etc.

6. If Julia did not consume any cocaine, how many hamburgers could she consume now?

7. If Julia did not consume any hamburgers, how many lines of cocaine could she consume now?


9. On the same graph that you drew Julia’s new budget constraint, draw her new indifference curve using the assumption that Julia’s MRS is constant and equal to six hamburgers per line of cocaine. Be sure to indicate the point at which her utility is maximized.

10. Will Julia consume both hamburgers and cocaine? Will she consume only hamburgers and no cocaine? Or will she consume only cocaine and no hamburgers? Explain your answer.

Answer the following questions about how working affects Julia’s consumption decisions and utility levels.

11. Has Julia’s optimal consumption of hamburgers changed since she graduated? Why or why not?

12. Was Julia’s level of utility higher or lower before she graduated? Explain your answer.

   Hint #1: When thinking about this last question, you may want to look at a graph that contains Julia’s old and new budget constraints and Julia’s old and new indifference curves.

   Hint #2: No. You do not have to graph the income and substitution effects.
Decisions Facing Firms

<table>
<thead>
<tr>
<th>Decisions</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quantity of output to supply</td>
<td>1. The price of output</td>
</tr>
<tr>
<td>2. How to produce that output (which technique to use)</td>
<td>2. Techniques of production available*</td>
</tr>
<tr>
<td>3. Quantity of each input to demand</td>
<td>3. The price of inputs*</td>
</tr>
<tr>
<td></td>
<td>* Determines production costs</td>
</tr>
</tbody>
</table>

Costs in the Short Run

The short run is a period of time for which two conditions hold:

1. Firm is operating under a fixed scale (fixed factor) of production and
2. Firms can neither enter nor exit an industry.

In the short run, all firms have costs that they must bear regardless of their output. These kinds of costs are called fixed costs.
Costs in the Short Run

**Fixed cost:**
- any cost that does not depend on the firm’s level of output. (The firm incurs these costs even if it doesn’t produce any output).
- firms have no control over fixed costs in the short run. (For this reason, fixed costs are sometimes called sunk costs).
  - obvious examples: property taxes, loan payments, etc.
  - not-so-obvious example: firm must pay “rent” to hired capital. If that level of capital cannot be adjusted immediately (“fixed factor”), then rental payments are a fixed cost in the short-run

**Variable cost:**
- depends on the level of production
- derived from production requirements and input prices
  - variable cost rises as output rises because firm has to hire more inputs (kapital and labor) to produce larger quantities of output

<table>
<thead>
<tr>
<th>Costs – Total vs. Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC, VC, FC</td>
</tr>
<tr>
<td>VC</td>
</tr>
<tr>
<td>FC</td>
</tr>
<tr>
<td>Q</td>
</tr>
<tr>
<td>TC=VC+FC</td>
</tr>
</tbody>
</table>


TC = FC + VC

\[
AC = \frac{TC}{Q} = \frac{F C}{Q} + \frac{V C}{Q}
\]
Marginal Cost

Marginal cost:
- increase in total cost from producing one more unit of output (the additional cost of inputs required to produce each successive unit of output)
- only reflects changes in variable costs
  - fixed cost does not increase as output increases
  - marginal cost is the slope of both total cost and variable cost

Shape of the Marginal Cost Curve

In the short run, the firm is constrained by a fixed input, therefore:
1. the firm faces diminishing returns to variable inputs and
2. the firm has limited capacity to produce output

As the firm approaches that capacity it becomes increasingly costly to produce successively higher levels of output. Marginal costs ultimately increase with output in the short run.

Marginal Product of Labor (MPL) is the additional product produced by each successive unit of labor.

Slope (change in VC per unit change in Q) always positive (VC increasing), but over some ranges the slope is greater than it is over other ranges. MC is simply the slope of VC at each level of Q.
Marginal Cost is NOT Average Variable Cost

Average Variable Cost – the (Total) Variable Cost divided by total quantity produced. (It’s a simple fraction).

Marginal Cost – the increase in (Total) Variable Cost incurred by producing one additional unit of output. (It’s a derivative).

Marginal Cost curve intersects the Average (Total) Cost and Average Variable Cost curves at their minimum points.

Short-Run Average and Marginal Cost

- If a firm’s capital stock is fixed in the short-run, then the rental payments that the firm makes on its capital stock is a fixed cost.
- We can use that assumption to derive short-run average and marginal cost curves.
- So start by assuming that a firm’s production function is given by:
  \[ X = K^{2/3}L^{1/3} \]
  
  Since the firm’s capital stock is fixed (by assumption) we can solve the production function for labor to find the amount of labor needed to produce various levels of output:
  \[ L = \frac{X^3}{K^2} \]

- Its total costs are given by:
  \[ TC = rK + wL \]
  \[ TC = rK + w \frac{X^3}{K^2} \]
**Short-Run Average and Marginal Cost**

To find Short-Run Average Cost simply divide Total Cost by X:

\[ AC = \frac{TC}{X} \]

Thus:

\[ AC = AFC + AVC = \frac{rK}{X} + \frac{wX^2}{K^2} \]

To find Short-Run Marginal Cost take the derivative of Total Cost with respect to X:

\[ MC = \frac{d}{dX} \left( \frac{TC}{X} \right) \]

\[ MC = 3w \frac{X^2}{K^2} \]

So if the wage and rental rate are both equal to $1 and the capital stock is equal to 10 units, then:

\[ w = $1 \]
\[ r = $1 \]
\[ K = 10 \]

---

**Output: Revenues, Costs and Profit Maximization**

- In the short run, a competitive firm faces an infinitely elastic demand curve (which corresponds to the market equilibrium price).
- (A monopolist faces the downward-sloping market demand curve).

- Each household has a downward sloping demand curve, but:
  - price is determined by market supply and demand
  - so shifts of one firm’s supply curve do not affect the market price
- Each firm faces infinitely elastic (horizontal) demand
Total Revenue and Marginal Revenue

- **Total Revenue** – total amount that firm receives from sale of its output
- **Marginal Revenue** – additional revenue that a firm takes in when it increases output by one additional unit.

If the market demand curve is given by: $D_M = 10 - p$
and if the competitive industry equilibrium price = $8$

<table>
<thead>
<tr>
<th>p</th>
<th>qty. dem.</th>
<th>TR</th>
<th>MR</th>
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<tbody>
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<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
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<td>1</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
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<td>8</td>
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<td>8</td>
<td>3</td>
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</tr>
<tr>
<td>8</td>
<td>7</td>
<td>56</td>
<td>8</td>
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</table>

Comp. Firm’s MR = p*

<table>
<thead>
<tr>
<th>p</th>
<th>qty. dem.</th>
<th>TR</th>
<th>MR</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
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<td>6</td>
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<td>6</td>
<td>24</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>21</td>
<td>-4</td>
</tr>
</tbody>
</table>

Monopolist’s MR

Note that: $Q = 10 - p \Rightarrow p = 10 - Q \Rightarrow TR = p \cdot Q = (10 - Q) \cdot Q = 10Q - Q^2$

therefore: $MR = \frac{dTR}{dQ} = 10 - 2Q$ in the case of a monopoly

Profit-Maximization

Profit-maximizing level of output for all firms is the output level where firms’ MR = MC

- **Perfectly competitive firm’s** MR = p*, so it will produce up to the point where p* = MC.
- **Monopolist** produces up to the point where MR = MC, but this occurs at a lower output level than would occur if the industry were perfectly competitive (and monopolist sells at a price that that exceeds MR and MC)
- The key idea here is that firms will produce as long as marginal revenue exceeds marginal cost.
Short-Run Supply Curve

- At any market price, the marginal cost curve shows the output level that maximizes profit.
- Thus, the marginal cost curve of a perfectly competitive profit-maximizing firm is the firm’s short-run supply curve.

\[ MC = S \]

Market level

- \( D_1 \)
- \( D_2 \)
- \( D_3 \)

Competitive firm

- \( D_1 = MR_1 \)
- \( D_2 = MR_2 \)
- \( D_3 = MR_3 \)
Why does a Firm Maximize its Profit where Marginal Revenue equals Marginal Cost?

If a firm is operating in a competitive industry, then its total revenue is simply equal to the market price times the quantity it produces, so we can depict Total Revenue as a linear function of output (a straight line) in the graph on the next page (i.e. $TR = pQ$).

In the graph, I’ve assumed that the firm’s Total Cost is increasing at an increasing rate (due to diminishing marginal product of labor).

Notice that if the firm produces a very low level of output (quantity produced), it will not be profitable. If it produces too much, its costs will once again exceed its revenues and it will not be profitable.

Over the range of output where the firm’s total revenue exceeds its total cost, the firm is making positive profit (in the short-run anyway).

The firm maximizes its profit in the middle of that range, but at what point specifically?

In the range of output where the slope of the Total Revenue curve is greater than the slope of the Total Cost curve, the firm could increase its profit by producing more output. In the range of output where the slope of the Total Revenue curve is less than the slope of the Total Cost curve, the firm could increase its profit by producing less output.

When the slope of the Total Revenue curve is equal to the slope of the Total Cost curve, the firm’s profit is maximized.

Since Marginal Revenue is the slope of the Total Revenue curve and since Marginal Cost is the slope of the Total Cost curve, the point at which the firm maximizes its profit corresponds to the point where Marginal Revenue equals Marginal Cost.

Since we’ve assumed that the firm is operating in a competitive industry, the firm’s Marginal Revenue is simply equal to the market price over all ranges of output because it faces an infinitely elastic (horizontal) demand curve.

Because the firm produces up to the point where Marginal Revenue equals Marginal Cost (in order to maximize its profit), the Marginal Cost curve is the firm’s Supply curve.
Total Revenue and Total Cost

\[ \frac{dTR}{dQ} > \frac{dTC}{dQ} \]
\[ \frac{dTR}{dQ} = \frac{dTC}{dQ} \]
\[ \frac{dTR}{dQ} < \frac{dTC}{dQ} \]

in this range, the slope of the Total Revenue curve is greater than the slope of the Total Cost curve

in this range, the slope of the Total Revenue curve is less than the slope of the Total Cost curve

Profit

\[ \Pi = TR - TC \]

A firm’s Profit is equal to Total Revenue minus Total Cost

A firm maximizes its Profit by producing up to the point where the slope of the Total Revenue curve is equal to the slope of the Total Cost curve. At this point, the slope of the Profit curve is Zero.

Marginal Revenue and Marginal Cost

\[ MR = MC \]

\[ MC = \frac{dTC}{dQ} \]

Marginal Cost is the slope of the Total Cost curve

\[ MR = \frac{dTR}{dQ} \]

Marginal Revenue is the slope of the Total Revenue curve
Homework #7

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
Profit-Maximization

(economic) \( \text{profit} = \text{total revenue} - \text{total (economic) cost} \)

**total revenue** – amount received from the sale of the product (price times number of goods sold)

**total (economic) cost** – the total of:
1. out of pocket costs (ex. prices paid to each input)
2. opportunity costs:
   a. normal rate of return on capital and
   b. opportunity cost of each factor of production – ex. if the firm I own pays me $30,000, but I could only earn $10,000 if I worked for another firm, then the “best alternative I forgo” when I work for my own firm is $10,000

In contrast to the examples in Lecture 6, here I’m earning MORE than my opportunity cost.

I’m giving an example of economic profit.
Profit-Maximization

To maximize profit, a firm sets the level of output to the point where marginal revenue equals marginal cost.

But what if the point where MR = MC, causes the firm to lose money?

In that case, it has to minimize its losses.

- **Total profit** (or loss) = TR – TC = TR – VC – FC
- **Operating profit** (or loss) = TR – VC

Note: operating profit is greater than total profit when FC > 0

If revenues exceed variable costs, operating profit is positive and can be used to offset fixed costs (thus reducing losses), and it will pay the firm to keep operating – in the short-run.

Profitability

- When total revenue exceeds total cost (p > AC), firm makes positive profits.
- When total cost exceeds total revenue, but revenues exceed variable cost (AC > p > AVC), firm suffers losses, but its operating profit is still positive. It continues operating in the short-run, but exits industry in the long-run.
- If revenues are less than variable costs (p < AVC), firm suffers operating losses. Total losses exceed fixed costs. To minimize losses firm shuts down.

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<table>
<thead>
<tr>
<th>Short Run Condition</th>
<th>Short Run Decision</th>
<th>Long Run Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>TR &gt; TC P = MC: operate</td>
<td>Expand: new firms enter</td>
</tr>
<tr>
<td>Losses</td>
<td>TR &gt; VC fixed costs)</td>
<td>Contract: firms exit</td>
</tr>
<tr>
<td></td>
<td>POP = MC: operate</td>
<td>Shut down: (losses variable costs)</td>
</tr>
<tr>
<td></td>
<td>TR &gt; TC</td>
<td>Losses minimized by shutting down (TR &lt; VC)</td>
</tr>
<tr>
<td></td>
<td>OP = MC: operate</td>
<td>Shut down: (losses + fixed costs)</td>
</tr>
<tr>
<td></td>
<td>$8000</td>
<td>Shut down: (losses + fixed costs)</td>
</tr>
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<td>Total Rev (Q = 0)</td>
<td>$3600</td>
<td>Shut down: (losses + fixed costs)</td>
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<tr>
<td></td>
<td>$2000</td>
<td>Shut down: (losses + fixed costs)</td>
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<tr>
<td></td>
<td>$0</td>
<td>Shut down: (losses + fixed costs)</td>
</tr>
</tbody>
</table>

**Firm Decisions in the Short and Long Run**

Loss Minimization

- When TR > VC, the firm’s total operating loss is lower when it continues.
- When TR < VC, the firm’s total operating loss is lower when it shuts down.
Short-Run Supply Curve of a Perfectly Competitive Firm

- In Lecture 3, I wrote that the marginal cost curve is the firm’s supply curve when MC > AC. That simplification is not strictly correct.
- The short-run supply curve of a competitive firm is the part of its MC curve that lies above its AVC curve.
- In the long-run, MC must exceed AC or firm will exit the industry.

Entry and Exit from the Industry

- In the long run, firms can enter and exit.
- They enter the industry in response to profit opportunities:
  - shifting out the market supply curve
  - and lowering the market price.
- They exit when they make losses:
  - contracting the market supply curve
  - and raising the market price.

Long-Run Costs: Returns to Scale

- In the short run, firms have to decide how much to produce in the current scale of plant (factory size is fixed).
- In the long run firms, have to choose among many potential scales of plant (they can expand the factory).

- Increasing returns to scale (or economies of scale), refers to an increase in a firm’s scale of production, which leads to lower average costs per unit produced.
- Constant returns to scale refers to an increase in a firm’s scale of production, which has no effect on average costs per unit produced.
- Decreasing returns to scale (or diseconomies of scale) refers to an increase in a firm’s scale of production, which leads to higher average costs per unit produced.
Long-Run Average Cost Curve

- The Long-Run Average Cost (LRAC) curve shows the different scales on which a firm can operate in the long-run. Each scale of operation defines a different short-run.
- The Long-Run Average Cost curve of a firm:
  - is downward-sloping when the firm exhibits increasing returns to scale.
  - is upward sloping when the firm exhibits decreasing returns to scale.
- The optimal scale of plant is the scale that minimizes long-run average cost.

![LRAC, SRAC, SRMC diagram](image)

Long-Run Adjustments to Short-Run Conditions

- In the long run, firms expand when increasing returns to scale are available (and contract when they face decreasing returns to scale).
- In the long run, the market price will be driven down to the minimum point on the LRAC curve and profits go to zero.

![LRAC, SRAC, SRMC diagram with price](image)
Long-Run Adjustment Mechanism

The central idea behind the discussion of entry, exit, expansion and contraction is:

- In efficient markets, investment capital flows toward profit opportunities.
- Investment – in the form of new firms and expanding old firms
  - will over time tend to favor those industries in which profits are being made,
  - and over time industries in which firms are suffering losses will gradually contract from disinvestment.
Notes on the Zero-Profit Result

In Lecture 6, I gave an example of a firm operating in a competitive industry that makes zero profit in the short-run, but I didn’t explain when such a situation would arise.

Firms in competitive industries may make positive profits in the short-run, but – if there is free entry into the industry and firms face constant returns to scale over some range of output – then in the long-run, the firms’ profits will be driven to zero.

**one reason why profits go to zero in the long run**

Suppose that there is free entry into the industry in which my firm operates, that the technology I use is widely available and that my firm is making positive profits in the short run. Since I’m making positive profits, another person (call him John) will use the same technology that I am using to produce output.

John will enter the industry, increase the market supply and lower the market price. This will reduce my profit, but if John and I are still making a positive profit, then yet another person (call her Jane) will enter the industry and use the same technology to produce output. Jane’s output will further increase market supply and lower the market price. This process will continue until there are no more profits to be made.

Eventually, John, Jane and I will all be producing output at the minimum point along our long-run average cost curves – this corresponds to the point where our firms all face constant returns to scale.

The table shows that the firm maximizes profit by employing labor up to the point where the wage equals the price times the marginal product of labor. (Here, capital is held fixed since this is the short-run).

The graph corresponds to the table and shows that the firm maximizes its profit by producing up to the point where price (marginal revenue) equals marginal cost.

<table>
<thead>
<tr>
<th>X</th>
<th>K</th>
<th>L</th>
<th>MPL</th>
<th>p*MPL</th>
<th>profit</th>
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<table>
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<td><strong>6</strong></td>
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<td><strong>-4.39</strong></td>
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<td>8</td>
<td>0.61</td>
<td>0.43</td>
<td>-4.50</td>
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returns to scale

As we saw in Lecture 8, it’s assumed that firms have a U-shaped long-run average cost curve. At low output levels, firms face increasing returns to scale. At high output levels, they face decreasing returns to scale. At the minimum point on the long-run average cost curve, they face constant returns to scale.

If a firm facing constant returns to scale doubles all of its inputs, then its output will exactly double. For example, if a firm’s production function is given by:

\[ X = K^{2/3}L^{1/3} \]

then if it doubles its inputs of capital and labor, its output doubles.

\[
2X = (2K)^{2/3}L^{1/3} = 2^{2/3}K^{2/3}L^{1/3} \\
2X = 2K^{2/3}L^{1/3}
\]

Similarly, you can easily see that when a firm doubles all of its inputs, its output:

- more than doubles when it faces increasing returns to scale
- less than doubles when it faces decreasing returns to scale.

increasing returns to scale:

\[
X = K^{2/3}L^{2/3} \\
2X < (2K)^{2/3}(2L)^{2/3} < 2^{2/3}K^{2/3}L^{2/3} \\
2X < 2^{4/3}K^{2/3}L^{2/3}
\]

decreasing returns to scale:

\[
X = K^{1/3}L^{1/3} \\
2X > (2K)^{1/3}(2L)^{1/3} > 2^{1/3}K^{1/3}L^{1/3} \\
2X > 2^{2/3}K^{1/3}L^{1/3}
\]

two more reasons why profits go to zero in the long run

Since a firm facing constant returns to scale can double its output by doubling each of its inputs, then if it doubled its inputs and output, its profits would also double.

\[
\Pi = pX - wL - rK \\
2\Pi = p(2X) - w(2L) - r(2K)
\]

But if the firm can double its profit by doubling its inputs and output, then it could also quadruple its profit by quadrupling its inputs and output. So when exactly would a firm ever maximize its profit? The only reasonable assumption to make therefore is that the firm’s profits go to zero in the long run. (Two times zero is zero).

There are three reasons why the profits of firms operating in competitive industries go to zero in the long run. The first reason was discussed above – as firms enter they drive down the market price to the point where no firm profits.

Another reason is because at very high levels of output, a firm may encounter logistical difficulties. For example, it may have difficulty coordinating the activities of all of its plants. Coordination difficulties at high levels of output are simply an example of decreasing returns to scale.

Finally, if the firm were lucky enough to face increasing returns to scale over all ranges of output, then it could always make ever higher profits by growing ever larger. Such a firm would eventually dominate its industry and become a monopolist – thus, the firm no longer operates in a competitive industry.
I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.
Review for Final Exam

The final exam will cover Lectures 5, 6, 7 and 8 as well as the memos and notes that I gave you. The exam will require you to define some of the terms listed below and require you to solve some problems. The problems given below were taken from exams that I have given in the past. They are a supplement to the homeworks. They are NOT a substitute for the homeworks.

Terms to know

- perfect competition
- perfect knowledge
- budget constraint
- indifference curve
- isoquant
- isocost
- gross complements
- gross substitutes
- perfect complements
- perfect substitutes
- utility
- relative price
- real income
- income effect
- substitution effect
- short run
- long run
- price taker
- total cost
- fixed cost
- variable cost
- average cost
- marginal cost
- total revenue
- marginal revenue
- economic profit
- accounting profit
- total profit/loss
- operating profit/loss
- shutdown point
- entry into industry
- exit from industry
- returns to scale
- long-run average cost
- short-run average cost

Short answer questions on social “profit”

The Americans with Disabilities Act (ADA) requires that institutions like Brooklyn College accommodate individuals with disabilities in such a manner that everyone will have equal access to all facilities. Issues of accessibility include physical entrances to buildings and classrooms as well as the technology that students use (e.g. computers must be fitted with audio output for students with visual impairments).

Some interpret the ADA to mean that the disabled have a civil right to full parity in access to resources. Others complain about the costs.

<table>
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<tr>
<th>degree of parity</th>
<th>total benefit</th>
<th>total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10 %</td>
<td>36</td>
<td>39</td>
</tr>
<tr>
<td>20 %</td>
<td>64</td>
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<td>123</td>
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<tr>
<td>100 %</td>
<td>120</td>
<td>135</td>
</tr>
</tbody>
</table>

1. Does the total benefit of parity exhibit diminishing marginal returns? Explain your answer.
2. When does the benefit of parity exceed the cost of parity?
3. To find Brooklyn College’s optimal degree of parity, what condition should you look for?
4. What is Brooklyn College’s optimal degree of parity?
5. In the context of this model, why shouldn’t Brooklyn College strive for full parity?
Short answer questions on household behavior and consumer choice

The scenarios below describe how my consumption of beer and wine changes when my income changes and/or when the price of one or both goods changes. For each scenario, say whether my consumption is affected by:

- an income effect,
- a substitution effect,
- a combined income and substitution effect
- or no effect.

and explain why!

Each scenario is independent of the previous ones and independent of successive ones.

In each scenario, I initially consume 50 beers and 50 wines. My initial income is $200. The initial price of beer is $2 and the initial price of wine is $2.

1. The price of wine falls to $1. The price of beer rises to $3. I increase my consumption of wine to 80 wines and decrease my consumption of beer to 40 beers.
2. The price of beer and the price of wine both increase to $4. My income rises to $400. I continue to consume 50 beers and 50 wines.
3. The price of beer and the price of wine both rise to $4. I decrease my consumption to 25 beers and 25 wines.
4. The price of wine falls to $1. I increase my consumption of wine to 150 wines and decrease my consumption of beer to 25 beers.

Demand Curves for Beer and Wine

For the scenario above where my consumption of beer and wine is affected by a combined income and substitution effect:

5. Draw the initial budget constraint and indifference curve. Place beer on the vertical axis and wine on the horizontal axis.
6. Draw the substitution effect. Be sure to LABEL that substitution effect.
7. Draw the income effect. Be sure to LABEL that income effect.
8. Is wine a gross substitute for beer? OR is wine a gross complement to beer?
9. Draw the demand curve for beer and the demand curve for wine. If a demand curve shifts, then illustrate that shift. If there is movement along a demand curve, then illustrate that movement.

◆◆◆

Short answer questions on profit-maximization and elasticity

A firm in perfect competition faces a perfectly elastic demand for its product (i.e. they are price takers). In reality however, firms usually have some ability to set the price of their product.

- What assumptions lead to the result that firms face perfectly elastic demand?
- How might real life firm make the demand for its product less elastic?
- How would a firm benefit from having a less elastic demand curve?

(continued on the next page)
Short answer questions on perfect competition

- Economists often talk about firms that operate in a perfectly competitive industry. Describe the three characteristics of a perfectly competitive industry.
- Discuss the three reasons why economists assume that firms in a perfectly competitive industry make zero profit in the long run.

Short answer questions on profit-maximization

You are given the following data on a profit-maximizing firm’s production in the short-run (its capital stock is fixed).

The quantity of output that it produces ($Q$) depends on the amount of capital ($K$) and labor ($L$) it employs.

The firm must pay rent on its capital stock at a rate of about $2 per unit, so its total rent bill is $1000.

It also must pay a wage rate of $5 per unit of labor.

Calculate the marginal product of labor for each output level. (NB: the marginal product of labor is the change in output per unit change in labor).

<table>
<thead>
<tr>
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<th>K</th>
<th>L</th>
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</thead>
<tbody>
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<td>73</td>
<td>--</td>
</tr>
<tr>
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</tr>
<tr>
<td>318.20</td>
<td>500</td>
<td>82</td>
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</table>

Up until what point does a profit-maximizing firm hire labor? (Hint: the answer is in Lecture 6)

If the firm can sell all of its output on a perfectly competitive market at a price of $5 per unit, then how much should it produce? How many units of labor should it employ?

Using your answers to the previous two questions, what will the firm’s profit be?

Could it maintain this profit in the long-run? Why or why not?