Lecture Notes
on
Economic Growth and Economic Fluctuations

Eric Doviak

3rd edition, July 2011

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To learn more about me, please visit: wdowiak.me If you would like to contact me, you may email me at: economics@doviak.net

Thanks again for downloading my notes. I hope you enjoy them.

Sincerely,
- Eryk Wdowiak

Preface

In 1961, Pres. John F. Kennedy proposed substantial cuts in personal and corporate taxes. When a reporter asked why, he replied:

> "To stimulate the economy. Don't you remember your Economics 101?"

Unfortunately, that quote has become a poison pill and Americans swallowed it hard.

Most economists believe that tax cuts can temporarily provide a temporary stimulus to the economy in times of recession, but they also know that the living standards of future generations depend heavily on the rate at which an economy saves for investment in future productive capacity.

Empirical evidence has shown that certain forms of taxation discourage saving, but incessant tax cutting has created large budget deficits that have reduced gross saving from 21 percent of gross national income in 1961 (when Kennedy made his remark) to 14 percent in 2004.

Kennedy is not responsible for this sad state of affairs however. Economists are.

In contrast to the modeling process that students learn when they study the business cycle, most introductory textbooks relegate economic growth to two dozen pages of text. A person who only took the introductory course, simply does not remember those vague pages on the determinants of output per worker as well as he remembers the results that he had to derive algebraically.

It's no surprise therefore that many Americans believe that tax cuts are the cure for every economic ailment and it's no surprise that politicians unfurl the tax cut banner at the first opportunity. After all, tax cuts are the faith that economists have taught them.

Our fear of rigorously teaching growth theory has left a generation of students with a fundamentally flawed understanding of macroeconomic policy. As a result, the average American fails to see how incessant tax-cutting reduces the national saving rate and deprives his children of a better standard of living.

These *Lecture Notes* represent a first attempt to repair the damage. In writing these *Lecture Notes*, I have placed an extraordinary emphasis on long-run economic growth, so that students complete the course with a firm understanding of the politically unappealing choice that policymakers face between stimulating the economy in the short-run and laying the foundation for long-run growth through increased saving.

In writing these *Lecture Notes*, I have followed the framework of N. Gregory Mankiw by first describing the goods, money and labor markets in the long run and then discussing how these markets may deviate from long run equilibrium over short periods of time when prices are not completely flexible.

A wonderful feature of Mankiw's framework is how it has enabled me to provide microeconomic foundation to the models of the economy in the short run.

Finally, I have not shied away from using mathematics in developing the models, but where I use math, I also provide intuitive explanations of the mathematical assumptions and results. I hope this will enable students to see how powerful a tool mathematics can be.

Knowledge of mathematics will become increasingly more important as computer technology makes increasingly more data available to us in the coming years. Students may dislike studying math today, but in the long run, they'll be better off … and that's the theme of this whole course.

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Lecture 1

Introduction and Math Review

Eric Doviak

Economic Growth and Economic Fluctuations

Helpful hints

- **Economics doesn't have to be difficult**
- **BUT... some people make**
- **it difficult for themselves.**
- **I did.**
- **If a model is unclear, don't try to think of an example from the $15 trillion US economy.**
- **Instead, apply the model to a small rural village.**
- **Most important part of any economic model are the:**

ASSUMPTIONS

  - **If you understand the assumptions of the model, you will understand the conclusions.**
  - **You will NOT understand the conclusions, if you don't understand the assumptions.**
  - **WHEN READING, DON'T SKIP CHAPTERS!**

Scope & Method of Economics

Why should I study economics?

- **To learn a way of thinking!** Hopefully, you'll learn to use three key concepts in your daily lives:
  - **efficient markets**
  - **marginalism** and
  - **opportunity cost**

Efficient markets

- Profit opportunities are rare because everyone is looking for them.
- **Efficient markets** eliminate profit opportunities immediately.
- Ex. You'll never find a good parking space, because if there was a good one, it would already be taken before you got there.

Marginalism

Average cost – total cost divided by quantity

- If I spend 300 hours preparing 30 lessons for you:
- You had better study!
- My average cost per lesson is 10 hours.

Sunk cost – costs that can no longer be avoided because they have already been "sunk"

- If I teach this class again next semester, I will have already sunk 300 hours into preparation.

Marginal cost – cost of producing one more unit

- Next semester I can recycle my notes, so my marginal cost per lesson will equal 75 minutes.
- Compare that with my current 10 hours!

Opportunity Cost

- We all face choices. **Resources are "scarce."**
- We can't spend more time or money than we have, so we have to give up one opportunity to take advantage of another.
- If I have a choice between earning $1000 per month by teaching this course OR earning $500 per month by working at McDonald's, then:
  - It takes me one month to ***produce*** $1000 worth of teaching.
  - It takes me one month to ***produce*** $500 worth of burger flipping.
- **Q: What's my opportunity cost of teaching?**
- **A: Half a burger flipping per unit of teaching.**

$$\frac{\text{one month per \$1000 of teaching}}{\text{one month per \$500 of burger flipping}} = \frac{\frac{\text{one month}}{\text{\$1000 of teaching}}}{\frac{\text{one month}}{\text{\$500 of burger flipping}}}$$

$$= \frac{\text{\$500 of burger flipping}}{\text{\$1000 of teaching}} = \frac{1}{2}\,\frac{\text{burger flippings}}{\text{teaching}}$$

Math – tool of econ. analysis

Point plotting (X,Y):

- **the first point in a pair lies on the X axis** (horizontal axis)
- **the second point in a pair lies on the Y axis** (vertical axis)

Let's graph the following equation in red (square points):

$$y = -5x + 20$$

Connect points:
(0,20), (1,15), (2,10), (3,5) & (4,0)

y-intercept:

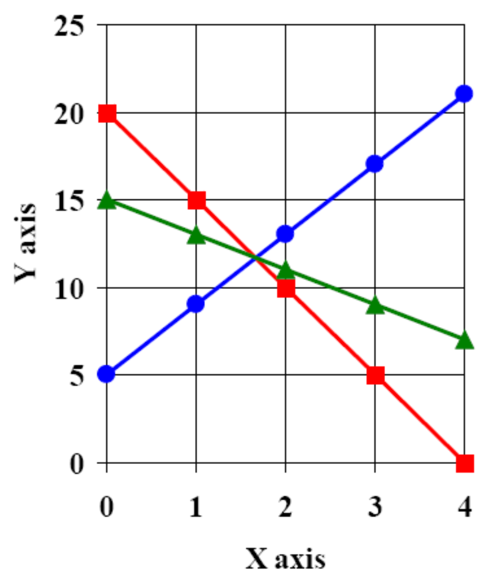
- **the value of y, when x = 0**
- here it's **20**, because: **20 = (–5*0) + 20**

slope: (we'll get back to that)

More examples:

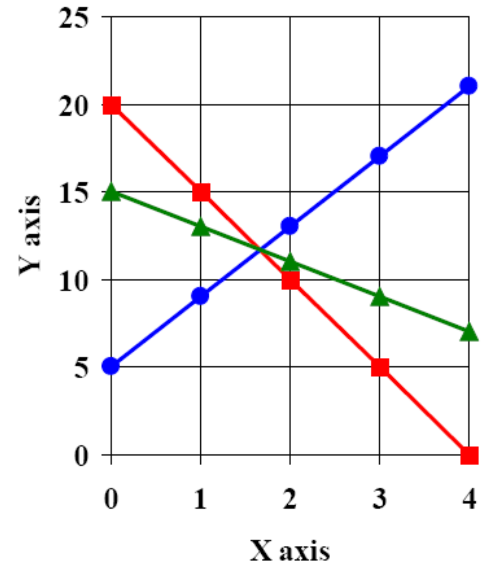
$y = 4x + 5$ (blue, round points)

$y = -2x + 15$ (green, triangle points)



equation:	slope:	y-int:
$y = -5x + 20$	-5	20
$y = 4x + 5$	4	5
$y = -2x + 15$	-2	15

Math – tool of econ. analysis

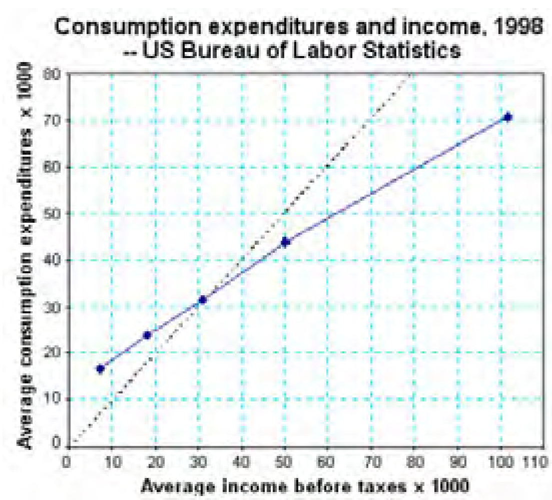


What is SLOPE?

- **the change in y divided by the change in x**
  - $y = -5x + 20$
  - x increases from 1 to 2
  - y decreases from 15 to 10
  - slope: $\frac{10-15}{2-1} = \frac{-5}{1} = -5$
- **positive slope:** x and y increase and decrease together
- **negative slope:** x and y increase and decrease inversely (when one rises the other falls)

NB: in linear functions (such as the ones here) the slope equals the value of the parameter by the variable X.

Analyzing Graphs



The graph illustrates relationship between average household income and average consumption expenditure. Along the 45 degree line, income equals expenditure.

- **Why does curve slope up?**
- **When is avg. consumption greater than avg. income? How is this possible?**

- **Suppose that the relationship between avg. income and avg. consumption is:**

  $c = 0.60*y + 14,000$

  where: c = avg. consumption and y = avg. income

- **What's the significance of the intercept** ($14,000)?
- **What's the significance of the parameter next to the variable "y"** (0.60)?

c = 0.60*y + 14,000
marginal propensity to consume

- **If your boss increased your income from $37,000 to $38,000, how much more would you consume?**
  - **On average, you would consume an extra $600 worth of goods.**
  - **Put differently, if you were an average person, your expenditure on consumption goods would rise from $37,200 to $37,800.**
- **Every $1000 increase in income raises consumption by $600. Why?**
- **marginal propensity to consume = 0.60** (NB: that's the slope of the line!)

- **What if you got fired? How much would you consume?**
- **Your income would fall to zero, but you'd still consume $14,000 worth of goods. After all, you've got to eat!**

- **When your income is less than $37,500 your expenditures on consumption goods exceed your income.** (You run down your savings)**.**
- **When your income is more than $37,500 your income exceeds your expenditures on consumption goods.** (You save some of your income)**.**

A few more definitions

c = 0.60*y + 14,000

- **Model** – the formal statement of a theory, often presented using mathematical equations
- **Variable** – a measure that can change such as consumption or income
  - **Dependent variable**
  - **Independent variable**
  - In the example above, consumption **depends** on income.
- **Parameters** – values which remain constant in an equation (here: 0.60 and 14,000)

Y = C + I + G + (X–M)

- **Ceteris paribus – "all else equal"**
- How does an increase in investment, **I**, affect national income, **Y**?
- To answer this question we must hold all other variables constant, while we determine the effect of investment alone.

Micro vs. Macro

MICROeconomics

- Study of the decision-making of individuals, households and firms
- Study of distribution of wealth

MACROeconomics

- Study of aggregates
- What factors affect:
  - Gross Domestic Product?
  - the price level?
  - the unemployment rate?

Positive vs. Normative Economics

Positive

- No judgements
- Just asking how the economy operates

Normative

- Makes judgements
- Evaluates the outcomes of economic behavior
- Policy recommendations

Economic policy

- **Positive** – economic policy starts with positive theories and models to develop an understanding of how the economy works
- Then economic policy evaluates **(normative)** on the basis of:
  - **Efficiency** – Is the economy producing what people want at the least possible cost? (quantifiable)
  - **Equity** – Is the distribution of wealth ***fair***? Are landlords treating low-income tenants ***fairly***? (non-quantifiable)
  - **Growth** – Increase in total output of the economy. Note: efficiency gains lead to growth (quantifiable)
  - **Stability** – steady growth, low inflation and full employment of resources – capital and labor (quantifiable)
- And recommends **(normative)** courses of action to policy-makers (presidents, congressmen, etc.)

Homework #1A

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

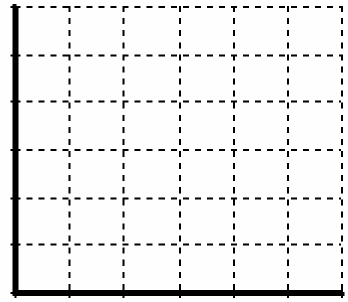
Homework #1B
More Math Review Problems

1. Graph these equations (placing Y on the vertical axis and X on the horizontal axis):

- Y = 2X + 2
- Y = 4X + 2

Comparing the two equations, which is different: the slope or the Y-intercept? How is it different? Are the lines parallel or do they intersect?

X	Y(1st)	Y(2nd)

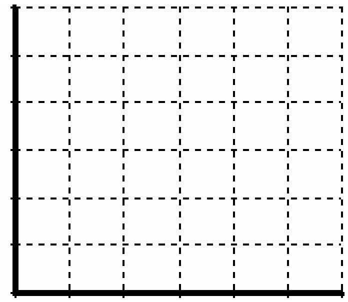


2. Graph these equations (placing Y on the vertical axis and X on the horizontal axis):

- Y = 2 + 2X
- Y = 2 – 2X

Comparing the two equations, which is different: the slope or the Y-intercept? How is it different? Are the lines parallel or do they intersect?

X	Y(1st)	Y(2nd)

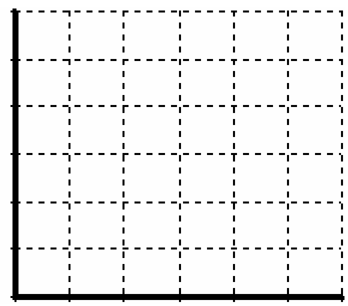


3. Graph these equations (placing Q on the vertical axis and P on the horizontal axis):

- Q = 4 + 2P
- Q = 2 + 2P

Comparing the two equations, which is different: the slope or the Q-intercept? How is it different? Are the lines parallel or do they intersect?

P	Q(1st)	Q(2nd)

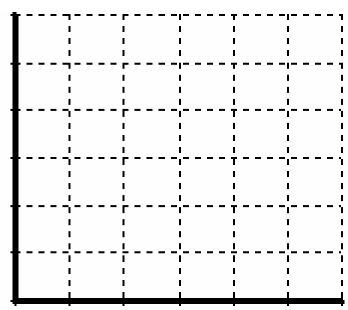


4. Graph these equations (placing Q on the vertical axis and P on the horizontal axis):

- Q = 4 – 2P
- Q = 2 + 2P

These two equations have different slopes and different Q-intercepts. Do the lines intersect? If so, can you find the value of P and Q at which they intersect?

P	Q(1st)	Q(2nd)



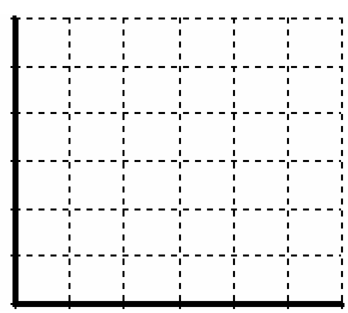
If demand curves slope down and supply curves slope up, then which of these two equations resembles a demand curve? Which resembles a supply curve?

5. *Solve these two equations for P.* Then graph the new equations by placing P on the vertical axis and Q on the horizontal axis:

- Q = 4 – 2P
- Q = 2 + 2P

Do the lines intersect? If so, can you find the value of P and Q at which they intersect?

Q	P(1st)	P(2nd)



6. The Law of Demand says that consumers purchase more of a good when its price is lower and they purchase less of a good when its price is higher. Can you give that statement a mathematical interpretation? (***Hint:*** Does price depend on quantity purchased? or does quantity purchased depend on price?)

Is price an independent variable or a dependent variable? Is quantity purchased an independent variable or a dependent variable? What is the difference between a dependent variable and an independent variable?

On which axis (the vertical or horizontal) do mathematicians usually place the independent variable? On which axis do mathematicians usually place the dependent variable?

When economists draw supply and demand diagrams, they usually place price on the vertical axis and quantity purchased on the horizontal axis. Why is that "wrong"?

7. (A question about percentages) 0.750 = ______%

8. (A question about fractions) $\frac{2}{3}$ = ______%

Eric Doviak
Economic Growth and
Economic Fluctuations

What's the difference between Marginal Cost and Average Cost?

"Marginal cost is *not* the cost of producing the "last" unit of output. The cost of producing the last unit of output is the same as the cost of producing the first or any other unit of output and is, in fact, the *average* cost of output. Marginal cost (in the finite sense) is the increase (or decrease) in cost resulting from the production of an extra increment of output, which is not the same thing as the "cost of the last unit." The decision to produce additional output entails the greater utilization of factor inputs. In most cases … this greater utilization will involve losses (or possibly gains) in input efficiency. When factor proportions and intensities are changed, the marginal productivities of the factors change because of the law of diminishing returns, therefore affecting the cost per unit of output."

– Eugene Silberberg, *The Structure of Economics* (1990)

Let's break Silberberg's definition of marginal cost into its component pieces. First, he ascribes changes in marginal cost to changes in marginal productivities of factor inputs. (By factor inputs, he means factors of production, like labor and capital).

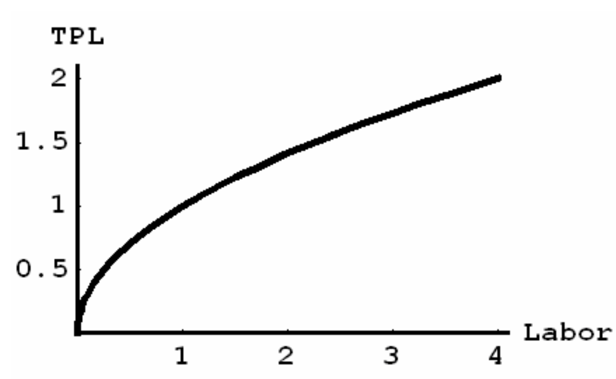
So what is the marginal product of labor and how is it affected by the law of diminishing returns?

Imagine a coal miner traveling deep underground to swing his pick at the coal face. The longer he swings his pick, the more coal he will produce, but it's exhausting work, so if his boss were to require him to work a double shift, the miner wouldn't double the amount of coal that he produces.

To be more specific, let's assume that the miner produces an amount of coal equal to the square root of the number of hours he works. The tonnage of coal that he produces is his "total product of labor (TPL)." So if he puts in zero hours, he produces zero tons of coal. If he puts in one hour he produces one ton. If he puts in two hours, he produces $\sqrt{2} = 1.41$ tons of coal, etc.

$$\text{tons of coal} = \sqrt{\text{hours}}$$

hours	coal	$\Delta\text{coal}/\Delta\text{hours}$
0	0	–
1	1	1
2	1.41	0.41
3	1.73	0.32
4	2	0.27



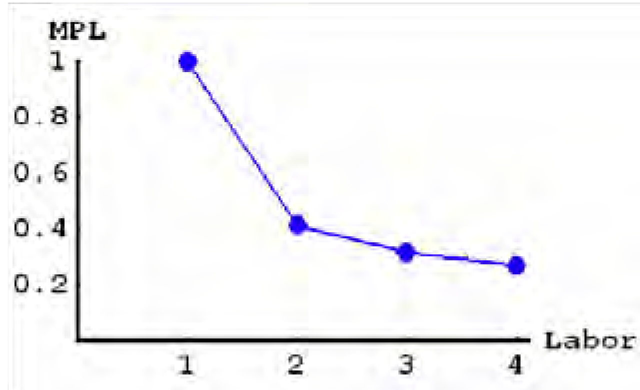
If the miner increases the number of hours that he spends mining from one hour to two hours, his output of coal will increase by 0.41 tons. Increasing the miner's hours from two to three hours only increases his output of coal by 0.32 tons however. Notice that **the additional coal he produces per additional hour that he works diminishes. This is the law of diminishing marginal returns.**

Notice also that the table lists **the ratio of the change in coal output to the change in the amount of hours worked.** That's the **slope of the total product function**, or the **"marginal product of labor."**

Plotting the miner's marginal product of labor against the amount of hours that he works shows the rate of output change at each amount of working hours.

$$\text{tons of coal} = \sqrt{\text{hours}}$$

hours	coal	$\Delta\text{coal}/\Delta\text{hours}$
0	0	–
1	1	1
2	1.41	0.41
3	1.73	0.32
4	2	0.27



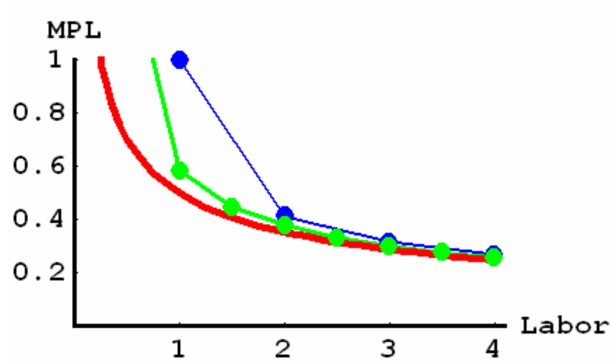
We don't need to measure the changes in the miner's coal output in one hour increments however. In fact, economists are usually more interested in a continuous rate of change. Those of you who have taken a course in calculus should know that the continuous rate of change in output is simply the first derivative of the total product function with respect to the number of hours worked.

For those of you who have not taken a course in calculus, imagine that we can measure the miner's total output at each second in time. If we look at how much the miner's output increases from one second to the next and divide that change by one second, we'll have a good approximation of the first derivative.

For example, one second is $1/360$ of an hour or 0.002778 hours. If the miner works for exactly two hours, then he'll produce 1.414214 tons of coal. If he works for exactly two hours and one second, then he'll produce 1.415195 tons of coal. In other words, adding one second to a two hour workday increases his output of coal by 0.000982 tons. The marginal product of labor evaluated at two hours and one second is:

$$\frac{\sqrt{2.002778}-\sqrt{2}}{0.002778} = \frac{1.415195-1.414214}{0.002778} = \frac{0.000982}{0.002778} = 0.353431 \text{ tons per hour}$$

To see how successively smaller changes in units of time (by which output changes are measured) lead to closer and closer approximations to the first derivative, consider this graph of true marginal product (red), the change in output per half-hour change in work hours (green) and the change in output per one hour change in work hours (blue).



A table of the data points in the graph is given on the next page.

Now that you know what the "marginal product of labor" is, what do you think "marginal cost" is? It's the change in total cost per unit change in output, calculated for an infinitesimally small change in output.

Just as the marginal product of labor measures the slope of the total product of labor function, marginal cost measures the slope of the total cost function.

hours	coal	true MPL	$\Delta\text{coal}/\Delta\text{hrs.}$ $\Delta\text{hrs.} = 0.5$	$\Delta\text{coal}/\Delta\text{hrs.}$ $\Delta\text{hrs.} = 1$
0	0	infinite	–	–
0.5	0.71	0.71	1.41	–
1.0	1.00	0.50	0.59	1
1.5	1.22	0.41	0.45	–
2.0	1.41	0.35	0.38	0.41
2.5	1.58	0.32	0.33	–
3.0	1.73	0.29	0.30	0.32
3.5	1.87	0.27	0.28	–
4.0	2.00	0.25	0.26	0.27

Notice also what happens when we sum the last two columns of the table above (i.e. the columns of $\Delta\text{coal}/\Delta\text{hrs.}$). The column listing a time interval of one hour sums to 2, which is exactly how many tons of coal are produced. The area under the marginal product curve equals total product because increasing the number of hours from zero to one yields one additional ton of coal per hour, increasing the number of hours from one to two yields 0.41 additional tons of coal per hour, etc.

The column listing a time interval of half an hour sums to 4, which when multiplied by 0.5 hours also equals 2 (tons of coal), so once again the area under the marginal product curve equals total product.

The column listing the true marginal product of labor sums to 3.1 plus infinity, which at first glance seems to contradict the results above, but keep in mind that the true marginal product is calculated using infinitesimally small intervals of time. So we'd have to multiply infinity plus 3.1 by the infinitesimally small intervals of time that we used to obtain the true marginal product to obtain 2 tons of coal. (In mathematical terms: we could integrate the true marginal product of labor from zero hours to four hours with respect to the number of hours the miner works to obtain 2 tons of coal).

♦ ♦ ♦

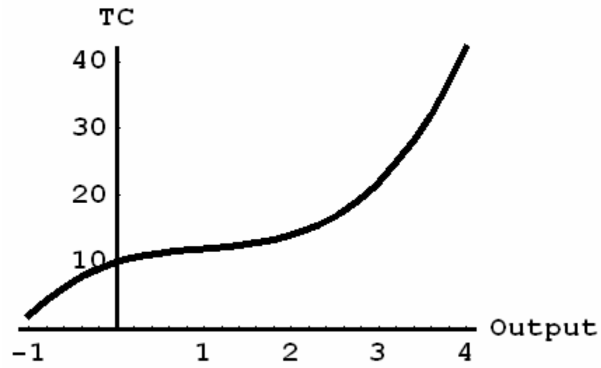
Now that we understand the law of diminishing returns and the concept of marginalism, let's reexamine Silberberg's quote. He says that to produce more output, a firm must hire more factors of production (like labor or capital) and/or use them more intensively, but such increased utilization reduces the efficiency of those factors of production (due to the law of diminishing returns) and raises the marginal cost of output.

For example, if the mining company only employs one miner and pays him a wage of $1 and that one miner is the only factor of production, then producing one ton of coal requires one hour of labor from the miner and costs a total of $1, but producing two tons requires four hours of labor and costs a total of $4. In this case, the marginal cost of increasing output from zero tons to one ton is $1 and the marginal cost of increasing output from one ton to two is: $4 – $1 = $3.

So let's examine a hypothetical firm's total, average and marginal costs by assuming that it faces a fixed cost of $10 and its variable cost is given by: $VC = X^3 - 3X^2 + 4X$, where X is the amount of output that it produces. Total cost is equal to fixed cost plus variable cost, so: $TC = X^3 - 3X^2 + 4X + 10$.

In the specification above, the firm's variable costs increase as the firm produces more output (and decrease as it produces less), therefore marginal cost reflects changes in variable cost. By definition, the firm's fixed costs do not change when it increases or decreases the amount of output it produces, therefore marginal cost does not reflect changes in fixed cost – because there are no changes in fixed cost.

In the graph at right, I have drawn a total cost curve running from negative one units of output to four units of output.



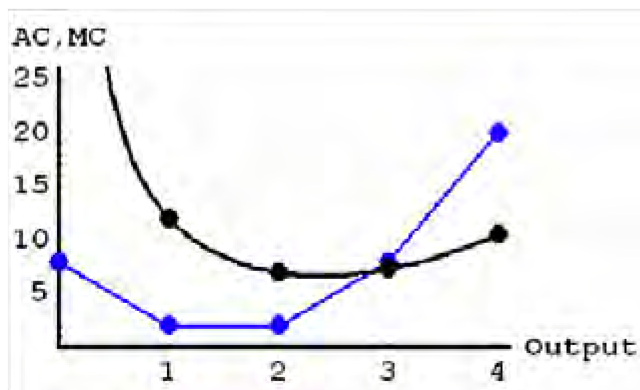
Now it should be obvious to you that a firm would not produce a negative output.

I drew the total cost function from negative one to better show the shape of the curve and because I'll use negative one to approximate the marginal cost at zero units of output.

The total cost curve depicted is everywhere increasing as output increases (i.e. is everywhere positively sloped), but it is not increasing at a constant rate. Initially total cost rises at a fairly rapid rate, but then the rate of increase slows, yielding a somewhat flat section. Finally, the rate of increase accelerates again. Since marginal cost is the rate of change in total cost (the slope of the total cost curve), the marginal cost curve will be U-shaped.

$$TC = X^3 - 3X^2 + 4X + 10$$

X	TC	VC	AC	$\Delta TC/\Delta X$ $\Delta X = 1$
–1	2	–8	–	–
0	10	0	infinite	8
1	12	2	12	2
2	14	4	7	2
3	22	12	$7\frac{1}{3}$	8
4	42	32	$10\frac{1}{2}$	20



If the firm faces a U-shaped marginal cost curve, then at low levels of output, it can increase the marginal productivity of its inputs by using them more intensively (a possibility I ruled out in the miner example), but at higher outputs, the firm confronts the law of diminishing returns and faces rising marginal cost.

The firm also faces a U-shaped average cost curve. The firm's average costs fall when it increases its production from zero to a moderate amount of output because its fixed cost is spread over a larger amount of output and (to a much lesser extent) because its average variable cost falls as inputs are used more efficiently (i.e. they yield a higher marginal product).

At high levels of output, the firm's average fixed cost approaches zero, but its variable costs rise rapidly due to the law of diminishing returns. At high levels of output, marginal cost exceeds both average variable cost and average cost, because the **averages spread the rising variable cost over the total amount of output, whereas marginal cost reflects changes in variable cost over small intervals.**

Finally, notice that the sum of the entries in the column containing the firm's marginal cost equals \$32 – the variable cost. (Ignore the marginal cost of \$8 that occurs when the firm produces zero units of output because it was calculated by increasing output from negative one to zero). It equals \$32 because marginal cost examines changes in variable cost, so the sum of the marginal costs must equal variable cost at that level of output.

Eric Doviak

Calculus Tricks #1

Calculus is <u>not</u> a pre-requisite for this course. However, the foundations of economics are based on calculus, so what we'll be discussing over the course of the semester is the intuition behind models constructed using calculus.

It's not surprising therefore that the students who do better in economics courses are the ones who have a better understanding of calculus – **even when calculus is not a required part of the course**. So if you want to do well in this course, you should learn a little calculus.

♦ ♦ ♦

Many times throughout the course, we'll be discussing marginalism – e.g. marginal cost, marginal revenue, marginal product of labor, marginal product of capital, marginal propensity to consume, marginal propensity to save, etc.

Whenever you see "marginal …" it means "the derivative of …"

A derivative is just a slope. So, for example, let's say **labor is used to produce output**

- if TP stands for Total Production (quantity produced),
- if L stands for Labor input and
- if Δ denotes a change,

then if I write: $\dfrac{\Delta TP}{\Delta L}$ that's the change in Total Production divided by the change in Labor.

- It's the slope of the total production function.
- It's the derivative of the production function with respect to labor input.
- It's the marginal product of labor (MPL).

So if you understand derivatives, you'll understand the course material much better.

♦ ♦ ♦

a few preliminaries – exponents

You should recall from your high school algebra classes that when you see an exponent, it simply means multiply the number by itself the number of times indicated by the exponent.

$$x^3 = x \cdot x \cdot x$$

Now if you divide both sides of the above equation by x:

$$\frac{x^3}{x} = \frac{x \cdot x \cdot x}{x} = x^2$$

But what if you see the something like: x^0? Well, that's simply equal to:

$$x^0 = \frac{x^1}{x} = \frac{x}{x} = 1$$

$$\begin{aligned}
2^3 &= 2\cdot2\cdot2 = 8 = \frac{16}{2} = \frac{2^4}{2} \\
2^2 &= 2\cdot2 = 4 = \frac{8}{2} = \frac{2^3}{2} \\
2^1 &= 2 = 2 = \frac{4}{2} = \frac{2^2}{2} \\
2^0 &= 1 = 1 = \frac{2}{2} = \frac{2^1}{2} \\
2^{-1} &= \frac{1}{2} = \frac{1}{2} = \frac{1}{2} = \frac{2^0}{2} \\
2^{-2} &= \frac{1}{2\cdot2} = \frac{1}{4} = \frac{1/2}{2} = \frac{2^{-1}}{2} \\
2^{-3} &= \frac{1}{2\cdot2\cdot2} = \frac{1}{8} = \frac{1/4}{2} = \frac{2^{-2}}{2}
\end{aligned}$$

Similarly, $x^{-1} = \frac{x^0}{x} = \frac{1}{x}$ and $x^{-2} = \frac{x^{-1}}{x} = \frac{1/x}{x} = \frac{1}{x \cdot x} = \frac{1}{x^2}$

But what about $x^{0.5}$? That's the square root of x: $x^{0.5} = \sqrt{x}$. Ex. $16^{0.5} = \sqrt{16} = 4$

By the same logic as before: $x^{-0.5} = \frac{1}{\sqrt{x}}$. Ex. $9^{-0.5} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

♦ ♦ ♦

a few preliminaries – functions

You may have seen something like this in your high school algebra classes: $f(x)$. This notation means that there is a function named "f" whose value depends on the value of the variable called "x."

Some examples of functions in economics include:

- The quantity of output that a firm produces depends on the amount of labor that it employs. In such a case, we can define a function called "TP" (which stands for Total Production) whose value depends on a variable called "L" (which stands for Labor). So we would write: $TP(L)$.
- A firm's total cost of producing output depends on the amount of output that it produces. In such a case, we can define a function called "TC" (which stands for Total Cost) whose value depends on a variable called "Q" (which stands for Quantity). So we would write: $TC(Q)$.
- A firm's total revenue from selling output depends on the amount of output that it produces. In such a case, we can define a function called "TR" (which stands for Total Revenue) whose value depends on a variable called "Q" (which stands for Quantity). So we would write: $TR(Q)$.

♦ ♦ ♦

derivatives

Now let's return to the original purpose of these notes – to show you how to take a derivative.

A derivative is the slope of a function. For those of you who saw $f(x)$ in your high school algebra classes, you may recall taking a derivative called "f-prime of x," $f'(x)$.

What you were doing was you were finding the slope of the function $f(x)$. You were finding how much the value of the function $f(x)$ changes as x changes.

So let's define the function: $f(x) = 3x^2$ and let's look at how the value of $f(x)$ changes as we increase x by one unit increments. Once again, let Δ denote a change.

x	$f(x)$	$\frac{\Delta f(x)}{\Delta x}$	true $f'(x)$
0	0	$--$	0
1	3	3	6
2	12	9	12
3	27	15	18

The third column is our rough measure of the slope. The fourth column – entitled true $f'(x)$ – is the true measure of the slope of $f(x)$ evaluated at each value of x. The values differ greatly between the two columns because we are looking at "large" changes in x (in the third column) as opposed to the infinitesimally small changes described in the notes entitled: “What’s the Difference between Marginal Cost and Average Cost?” (The infinitesimally small changes are listed in the fourth column).

Why does it make a difference whether we look at small or large changes? Consider the following derivation of the slope of $f(x)$:

$$f'(x) = \frac{\Delta f(x)}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{3(x+\Delta x)^2 - 3x^2}{\Delta x} = \frac{3(x+\Delta x)\cdot(x+\Delta x) - 3x^2}{\Delta x} = \frac{3\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 3x^2}{\Delta x}$$

$$= \frac{3x^2 + 6x\Delta x + 3(\Delta x)^2 - 3x^2}{\Delta x} = \frac{6x\Delta x + 3(\Delta x)^2}{\Delta x} = \frac{6x\Delta x}{\Delta x} + \frac{3(\Delta x)^2}{\Delta x}$$

$$f'(x) = 6x + 3\Delta x$$

If we look at one unit changes in the value of x – i.e. $\Delta x = 1$ – then the slope of $f(x)$ evaluated at each value of x is equal to $6x + 3\Delta x$ which equals $6x + 3$ since $\Delta x = 1$.

If we look at changes in x that are so small that the changes are approximately zero – i.e.: $\Delta x \approx 0$ – then the slope of $f(x)$ evaluated at each value of x is approximately equal to $6x$ and gets closer and closer to $6x$ as the change in x goes to zero.

So if $f(x) = 3x^2$, then $f'(x) = 6x$.

Since we’ll be looking at infinitesimally small changes in x, we’ll stop using the symbol Δ to denote a change and start using the letter d to denote an infinitesimally small change.

♦ ♦ ♦

calculus tricks – an easy way to find derivatives

For the purposes of this course, there are only a handful of calculus rules you’ll need to know:

1. the constant-function rule
2. the power-function rule,
3. the sum-difference rule,
4. the product-quotient rule and
5. the chain rule.

We’ll focus on the first three of these rules now. We’ll discuss the last two after we have a firm grasp on the first three.

the constant-function rule

If $f(x) = 3$, then the value of $f(x)$ doesn’t change x as changes – i.e. $f(x)$ is constant and equal to 3.

So what’s the slope? Zero. Why? Because a change in the value of x doesn’t change the value of $f(x)$. In other words, the change the value of $f(x)$ is zero. So if $f(x) = 3$, then $\frac{d\,f(x)}{d\,x} = f'(x) = 0$.

the power-function rule

Now if the value of x in the function $f(x)$ is raised to a power (i.e. it has an exponent), then all we have to do to find the derivative is "roll the exponent over."

To roll the exponent over, multiply the original function by the original exponent and subtract one from the original exponent. For example:

$$f(x) = 5x^3$$

$$\frac{d\,f(x)}{d\,x} = f'(x) = 15x^2$$

$$5x^3 \;\rightarrow\; 3 \cdot 5x^{3-1} \;=\; 15x^2$$

$$g(x) = 4x^{1/2} = 4\sqrt{x}$$

$$\frac{d\,g(x)}{d\,x} = g'(x) = 2x^{-1/2} = \frac{2}{\sqrt{x}}$$

$$4x^{1/2} \;\rightarrow\; \frac{1}{2} \cdot 4x^{\frac{1}{2}-1} \;=\; 2x^{-1/2}$$

the sum-difference rule

Now, say the function you are considering contains the variable x in two or more terms.

$$k(x) = 2x^2 - 3x + 5$$

if we define:

$$f(x) = 2x^2 \qquad g(x) = -3x^1 = -3x \qquad h(x) = 5$$

then:

$$\begin{aligned} k(x) &= f(x) + g(x) + h(x) \\ &= 2x^2 - 3x + 5 \end{aligned}$$

Now we can just take the derivatives of $f(x)$, $g(x)$ and $h(x)$ and then add up the individual derivatives to find $k'(x)$. **After all, the change in a sum is equal to the sum of the changes.**

$$\frac{d\,k(x)}{d\,x} = \frac{d\,f(x)}{d\,x} + \frac{d\,g(x)}{d\,x} + \frac{d\,h(x)}{d\,x}$$

$$k'(x) = f'(x) + g'(x) + h'(x)$$

$$k'(x) = 2 \cdot 2x^{2-1} - 1 \cdot 3x^{1-1} + 0 = 4x - 3$$

Example #1 – Total Revenue and Marginal Revenue

Total Revenue, denoted TR, is a function of the quantity of output that a firm produces, denoted Q, and the price at which the firm sells its output, denoted p. Specifically, Total Revenue is equal to the amount of output that a firm sells times the price. For example, if the firm sells 20 widgets at a price of $5 each, then its Total Revenue is $100.

If a firm is in a perfectly competitive market, then the firm cannot sell its output at a price higher than the one that prevails in the market (otherwise everyone would buy the products of competitor firms). So we can assume that the price is **constant**.

So what is a firm's Marginal Revenue? It's Marginal Revenue, denoted MR, is the derivative of Total Revenue with respect to a change in the quantity of output that the firm produces.

$$TR(Q) = p \cdot Q \quad \rightarrow \quad MR = \frac{d\,TR(Q)}{d\,Q} = p$$

Example #2 – Total Product and Marginal Product of Labor

If a firm produces output using "capital" – a fancy word for machinery – and labor, then the quantity of output that it produces – i.e. its Total Product, denoted by TP – is a function of two variables: capital, denoted by K, and labor, denoted by L.

$$TP(K,L) = K^{0.3} \cdot L^{0.7}$$

So what is the Marginal Product of Labor, denoted MPL? Marginal Product of Labor is the change in Total Product caused by an increase in Labor input. Marginal Product of Labor is the derivative of Total Product with respect to Labor.

Notice that we're looking solely at the change in Total Product that occurs when we vary the Labor input. We're not changing the capital stock, so when we take the derivative of Total Product with respect to Labor, we'll hold the firm's capital stock is fixed – i.e. we'll hold it **constant**.

$$TP(K,L) = K^{0.3} \cdot L^{0.7} \quad \rightarrow \quad MPL = \frac{d\,TP(K,L)}{d\,L} = 0.7 \cdot K^{0.3} \cdot L^{-0.3} = 0.7 \cdot \left(\frac{K}{L}\right)^{0.3}$$

Homework #1C

1. Find the derivative of each of the following functions:

a. $g(x) = 7x^6$

b. $k(y) = 3y^{-1}$

c. $m(q) = \frac{3}{2}q^{-2/3}$

d. $h(w) = -aw^2 + bw + \frac{c}{w}$

e. $u(z) = 5$

f. $y(x) = mx + b$

2. The Total Product of a firm, denoted by TP, depends on the amount of capital and labor that it employs. Denote capital by K and denote labor by L.

The Total Product function is given by: $TP(K, L) = K^{0.5} \cdot L^{0.5}$.

Throughout this problem, assume that the firm's capital stock is fixed at one unit.

a. Plot the Total Product function from zero units of Labor to four units of Labor. (Hint: Use graph paper if you have it).

b. Now find the Marginal Product of Labor by taking the derivative of the Total Product function with respect to Labor.

c. Plot the Marginal Product of Labor from zero units of Labor to four units of Labor.

3. Plot each of the following functions. Then find the derivative of each function and plot the derivative directly underneath your plot of the original function.

a. $f(x) = x^{1.5}$

b. $g(x) = x^{-0.5}$

If you plot the functions correctly, you will notice that the height of the plotted derivative is higher when the slope of the original function is steeper. Conversely, the height of the plotted derivative is lower when the slope of the original function is shallower.

4. The Total Cost function of a firm depends on the quantity of output that it produces, denoted by Q.

The Total Cost function is given by: $TC(Q) = Q^3 - 6Q^2 + 18Q + 6$.

a. Plot the Total Cost function from zero units of output to five units of output. (Hint: Use graph paper if you have it).

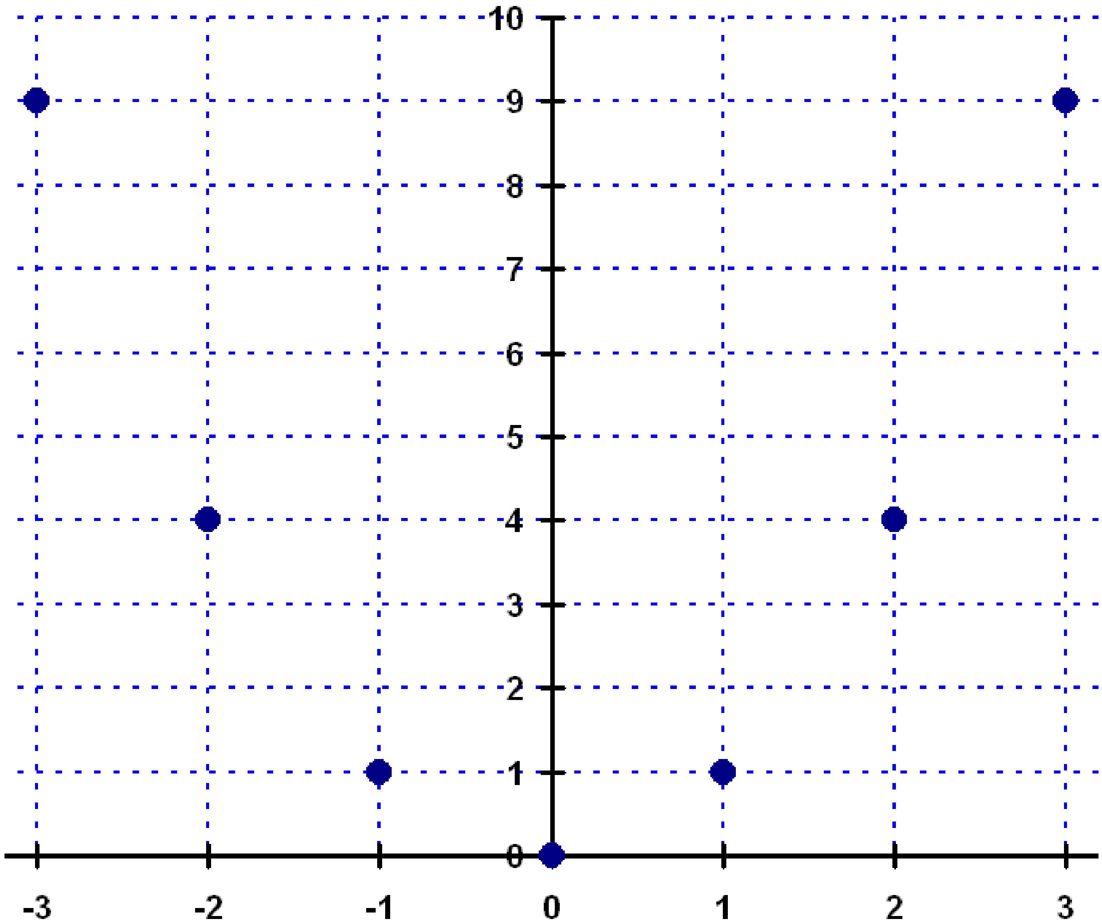
b. Does the Total Cost function ever slope downward? Or is it everywhere increasing?

c. Now find the Marginal Cost function by taking the derivative of the Total Cost function with respect to the quantity of output that the firm produces.

d. Plot the Marginal Cost function from zero units of output to five units.

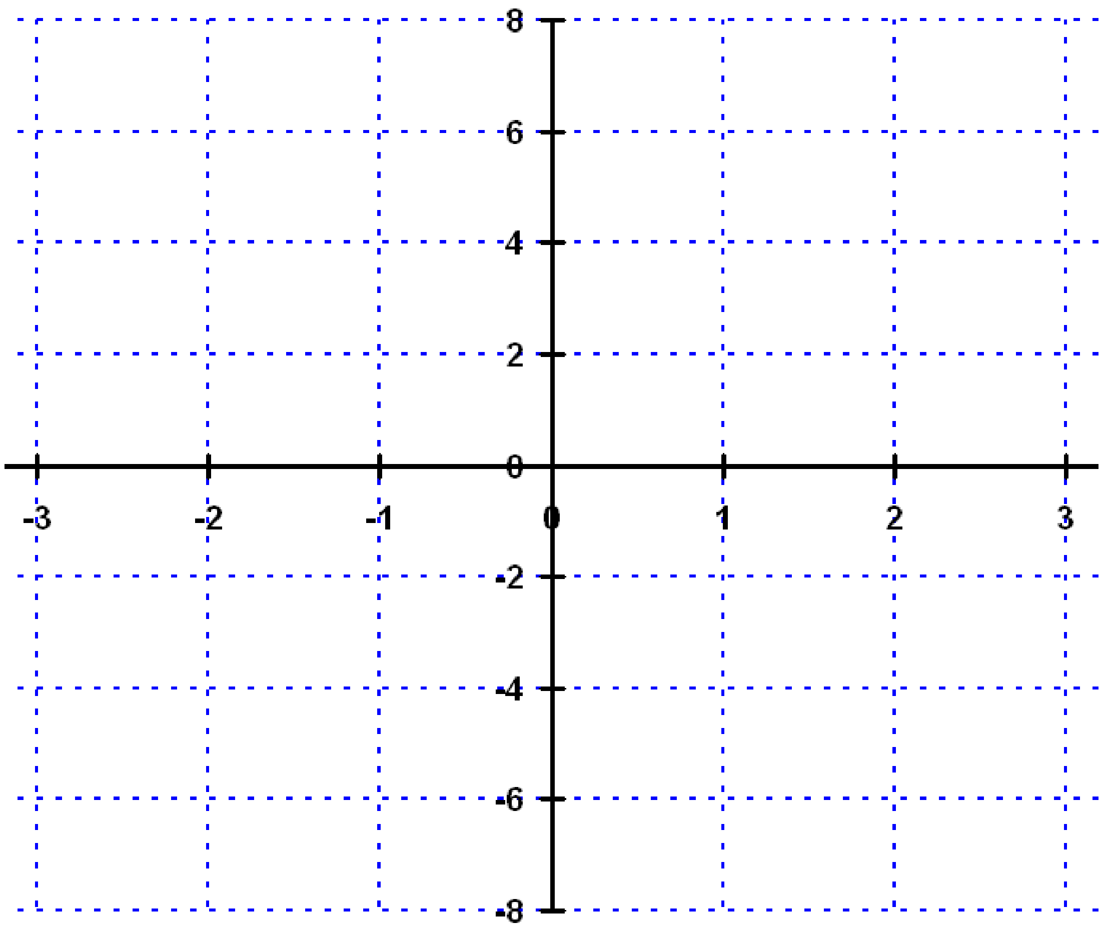
e. Does the Marginal Cost function ever slope downward? Or is it everywhere increasing?

f. If the Total Cost function never slopes downward, then why does the Marginal Cost function slope downward over some ranges of output?



$f(x) = x^2$

x	$f(x)$	$\frac{\Delta f(x)}{\Delta x}$
-3.0	9	
-2.5		
-2.0	4	
-1.5		
-1.0	1	
-0.5		
0.0	0	
0.5		
1.0	1	
1.5		
2.0	4	
2.5		
3.0	9	



$f'(x) = 2x$

x	$\frac{\Delta f(x)}{\Delta x}$	$f'(x)$
-3.0		
-2.5		
-2.0		
-1.5		
-1.0		
-0.5		
0.0		
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		

Calculus Tricks #2

This set of calculus tricks explains the chain rule and the product-quotient rule. For the purposes of this course, our only need for these rules will be to show that:

- The percentage change in a product of two variables is equal to the sum of the percentage changes in each of the two variables.
- The percentage change in the ratio of two variables is equal to the percentage change in the numerator minus the percentage change in the denominator.

For example, if we're interested in the percentage change in Total Revenue, i.e. $TR = p \cdot Q$, then:

$$\frac{\Delta TR}{TR} = \frac{\Delta(p \cdot Q)}{(p \cdot Q)} = \frac{\Delta p}{p} + \frac{\Delta Q}{Q}$$

To take another example, if we're interested in the percentage change in GDP per capita, i.e. GDP/N (where N denotes population), then:

$$\frac{\Delta \text{GDP per capita}}{\text{GDP per capita}} = \frac{\Delta(GDP/N)}{(GDP/N)} = \frac{\Delta GDP}{GDP} - \frac{\Delta N}{N}$$

♦ ♦ ♦

the chain rule

Say you are considering a function that is a function of a function. That is:

$$h(x) = f(g(x))$$

In other words, the value of $h(x)$ changes as the function named "f" changes and the function named "f" changes as the function $g(x)$ changes.

To analyze this change, we can analyze a chain of causality that runs from x to $h(x)$.

$$x \rightarrow g(x) \rightarrow f(g(x)) = h(x)$$

So the derivative of $h(x)$ with respect to x is:

$$\frac{d\,h(x)}{d\,x} = \frac{d\,f(x)}{d\,g(x)} \cdot \frac{d\,g(x)}{d\,x}$$

which looks like the chain of causality flipped around:

$$h(x) = f(g(x)) \leftarrow g(x) \leftarrow x$$

So for example, if $g(x) = 3x + 1$ and if $f(g(x)) = (g(x))^2$, then $h(x) = (3x+1)^2$.

So there are two ways to take the derivative of $h(x)$ with respect to x. Using the methods you already learned, you could expand the terms in the function $h(x)$:

$$h(x) = (3x+1)^2 = 9x^2 + 6x + 1$$

and then take the derivative of $h(x)$ with respect to x, so that:

$$h'(x) = \frac{d\,h(x)}{d\,x} = 18x + 6$$

Expanding the terms of $(3x+1)^2$ can be rather tedious when you're working with a complicated function. Fortunately, the chain rule enables us to arrive at the same result, but in a somewhat quicker fashion:

$$\left.\begin{array}{lcl} f(g(x)) = (g(x))^2 & \Rightarrow & f'(g(x)) = 2 \cdot g(x) \\ & & \\ g(x) = 3x+1 & \Rightarrow & g'(x) = 3 \end{array}\right\} \rightarrow \begin{aligned} h'(x) &= f'(g(x)) \cdot g'(x) \\ &= 2 \cdot g(x) \cdot 3 \\ &= 6 \cdot (3x+1) \\ &= 18x + 6 \end{aligned}$$

which yields exactly the same result as the one above.

♦♦♦

the product-quotient rule

Say you are considering a function that is the product of two functions, each of which is a function of the variable x. That is:

$$h(x) = f(x) \cdot g(x)$$

If we knew the explicit functional forms of $f(x)$ and $g(x)$, then we could multiply $f(x)$ by $g(x)$ and take the derivative of $h(x)$ with respect to x using the rules you already know. For example,

$$\text{if } f(x) = 3x \text{ and } g(x) = x^2, \text{ then } \begin{aligned} h(x) &= f(x) \cdot g(x) \\ &= 3x \cdot x^2 \\ &= 3x^3 \end{aligned} \quad \text{and} \quad \frac{d\,h(x)}{d\,x} = h'(x) = \mathbf{9x^2}$$

But we can also consider the change in $h(x)$ as $f(x)$ changes holding $g(x)$ constant and the change in $h(x)$ as $g(x)$ changes holding $f(x)$ constant.

In other words: $$\frac{d\,h(x)}{d\,x} = \frac{d\,f(x)}{d\,x} \cdot g(x) + \frac{d\,g(x)}{d\,x} \cdot f(x) \quad \text{or} \quad h'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Using the previous case where $f(x) = 3x$ and $g(x) = x^2$, we can write:

$$\begin{aligned} h'(x) &= f'(x) \cdot g(x) + g'(x) \cdot f(x) \\ &= 3 \cdot x^2 + 2x \cdot 3x \\ &= 3x^2 + 6x^2 \\ &= 9x^2 \end{aligned}$$

which yields exactly the same result as the one above.

Now let's say you are considering a function that is a ratio of two functions, each of which is a function of the variable x. That is:

$$h(x)=\frac{f(x)}{g(x)} \quad \text{which can be rewritten as:} \quad h(x)=f(x)\cdot(g(x))^{-1}$$

To find the derivative of $h(x)$ with respect to x, we can perform the exact same analysis as we did in the previous example, but with the twist that we also have to use the chain rule on the term $(g(x))^{-1}$.

If we define a function $k(x)$ which is identically equal to $(g(x))^{-1}$, i.e. $k(x)\equiv(g(x))^{-1}$, then we can rewrite the function $h(x)$ as:

$$h(x)=f(x)\cdot k(x)$$

The derivative of $h(x)$ with respect to x is:

$$h'(x)=f'(x)\cdot k(x)+k'(x)\cdot f(x)$$

And the derivative of $k(x)$ with respect to x is:

$$\frac{d\,k(x)}{d\,x}=\frac{d\,(g(x))^{-1}}{d\,g(x)}\cdot\frac{d\,g(x)}{d\,x}$$

$$k'(x)=-1\cdot(g(x))^{-2}\cdot g'(x)=-\frac{g'(x)}{(g(x))^{2}}$$

Plugging that into the derivative of $h(x)$ with respect to x:

$$h'(x)=f'(x)\cdot(g(x))^{-1}-\frac{g'(x)}{(g(x))^{2}}\cdot f(x)$$

$$h'(x)=\frac{f'(x)}{g(x)}-\frac{g'(x)\cdot f(x)}{(g(x))^{2}}$$

So let's consider: $h(x)=\frac{f(x)}{g(x)}$, where $f(x)=6x^4+2x^2$ and $g(x)=2x$. In such a case, $h(x)=3x^3+x$ and $h'(x)=\mathbf{9x^2+1}$. To illustrate the rule we just derived, let's use the rule to obtain the same result:

$$h'(x)=\frac{f'(x)}{g(x)}-\frac{g'(x)\cdot f(x)}{(g(x))^{2}}$$

$$\left.\begin{array}{lll} f(x)=6x\quad+2x^2 & \Rightarrow & f'(x)=24x^3+4x \\ g(x)=2x & \Rightarrow & g'(x)=2 \end{array}\right\} \Rightarrow \quad h'(x)=\frac{24x^3+4x}{2x}-\frac{2\cdot\left(6x^4+2x^2\right)}{(2x)^2}$$

$$h'(x)=12x^2+2\quad-3x^2-1 \qquad =\mathbf{9x^2+1}$$

Now, let's return to the original purpose of this set of Calculus Tricks, i.e. to show that:

- The percentage change in a product of two variables is equal to the sum of the percentage changes in each of the two variables.
- The percentage change in the ratio of two variables is equal to the percentage change in the numerator minus the percentage change in the denominator.

♦ ♦ ♦

Example #1 – a percentage change in Total Revenue

Once again Total Revenue is given by $TR = p \cdot Q$. Let's assume now that the price of output and the quantity of output produced evolve over time, so that $p = p(t)$ and $Q = Q(t)$, where "t" represents time. In such a case Total Revenue would also evolve over time $TR = TR(t)$.

So what's the percentage change in Total Revenue over time? First, we need to find the changes:

$$\frac{d\,TR(t)}{d\,t} = \frac{d\,p(t) \cdot Q(t)}{d\,t} = \frac{d\,p(t)}{d\,t} \cdot Q(t) + p(t) \cdot \frac{d\,Q(t)}{d\,t}$$

$$= TR'(t) \qquad = p'(t) \cdot Q(t) + p(t) \cdot Q'(t)$$

Since we're interested in a percentage change, we need to divide both sides by Total Revenue to get the percentage change in Total Revenue:

$$\frac{TR'(t)}{TR(t)} = \frac{p'(t) \cdot Q(t)}{p(t) \cdot Q(t)} + \frac{p(t) \cdot Q'(t)}{p(t) \cdot Q(t)}$$

$$\begin{matrix}\%\Delta \\ TR\end{matrix} = \frac{p'(t)}{p(t)} + \frac{Q'(t)}{Q(t)} = \begin{matrix}\%\Delta \\ \text{price}\end{matrix} + \begin{matrix}\%\Delta \\ \text{quantity}\end{matrix}$$

♦ ♦ ♦

a note on time derivatives

When working with dynamic changes – that is: a change over time – economists usually denote a time derivative by placing a dot over the variable. I will frequently use this notation.

So for example, the derivative of price with respect to time would be denoted by $\dot{p}$

$$\frac{d\,p(t)}{d\,t} = p'(t) = \dot{p}$$

and the derivative of quantity with respect to time would be denoted by $\dot{Q}$

$$\frac{d\,Q(t)}{d\,t} = Q'(t) = \dot{Q}$$

(continued on the next page)

Example #2 – a percentage change in the Capital-Labor ratio

The Capital-Labor ratio – denoted: k – is defined as: $k \equiv \frac{K}{L}$, where K and L denotes capital and labor respectively.

Suppose that these two variables evolve over time so that: $K = K(t)$ and $L = L(t)$. This implies that the Capital-Labor ratio also evolves over time, so $k = k(t)$.

To avoid clutter, I'll drop the "t" from the functional notations.

So how does the Capital-Labor ratio evolve over time?

$$\begin{aligned}
\frac{dk}{dt} &= \frac{d}{dt}\cdot\left(K\cdot L^{-1}\right) \\
\dot{k} &= L^{-1}\cdot\frac{dK}{dt} + K\cdot\frac{dL^{-1}}{dL}\cdot\frac{dL}{dt} \\
&= \frac{\dot{K}}{L} - K\cdot\frac{\dot{L}}{L^2} = \frac{K}{L}\cdot\left(\frac{\dot{K}}{K}-\frac{\dot{L}}{L}\right)
\end{aligned}$$

Since $k \equiv \frac{K}{L}$, the derivation above implies that:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

The percentage change in the Capital-Labor ratio over time is equal to the percentage change in Capital over time minus the percentage change in Labor over time.

Some students have told me that they understand the product-quotient rule better when I explain the rules using difference equations.

Example #1 revisited – a percentage change in Total Revenue

Since Total Revenue is given by: $TR = p \cdot Q$, the percentage change in Total Revenue is:

$$\frac{\Delta TR}{TR} = \frac{\Delta(p \cdot Q)}{p \cdot Q} = \frac{p_2 \cdot Q_2 - p_1 \cdot Q_1}{p_1 \cdot Q_1}$$

where: p_1 is the inital price, Q_1 is the inital quantity, p_2 is the new price, Q_2 is the new quantity

Next, we're going to add a zero to the equation above. Adding zero leaves the value of the percentage change in Total Revenue unchanged.

We're going to add that zero in an unusual manner. The zero that we're going to add is:

$$0 = \frac{p_1 \cdot Q_2 - p_1 \cdot Q_2}{p_1 \cdot Q_1}$$

Adding our "unusual zero" yields:

$$\frac{\Delta TR}{TR} = \frac{p_2 \cdot Q_2 - p_1 \cdot Q_1}{p_1 \cdot Q_1} + \frac{p_1 \cdot Q_2 - p_1 \cdot Q_2}{p_1 \cdot Q_1}$$

Rearranging terms, we get:

$$\frac{\Delta TR}{TR} = \frac{(p_2 - p_1) \cdot Q_2}{p_1 \cdot Q_1} + \frac{p_1 \cdot (Q_2 - Q_1)}{p_1 \cdot Q_1}$$

Now notice that: $\Delta p = (p_2 - p_1)$ and $\Delta Q = (Q_2 - Q_1)$, therefore:

$$\frac{\Delta TR}{TR} = \frac{\Delta p}{p_1} \cdot \frac{Q_2}{Q_1} + \frac{\Delta Q}{Q_1}$$

Since we're considering very small changes: $\Delta Q \approx 0$, which implies that: $Q_2 \approx Q_1$ and $\frac{Q_2}{Q_1} \approx 1$.

Therefore we can write:

$$\frac{\Delta TR}{TR} = \frac{\Delta p}{p} + \frac{\Delta Q}{Q}$$

Example #2 revisited – a percentage change in the Capital-Labor ratio

Once again, define k as the Capital-Labor ratio, i.e.: $k \equiv \frac{K}{L}$, where K denote capital and L denotes labor. The percentage change in the Capital-Labor ratio is:

$$\frac{\Delta k}{k} = \frac{\Delta(K/L)}{K/L} = \frac{\frac{K_2}{L_2} - \frac{K_1}{L_1}}{K_1/L_1}$$

where: K_1 is the inital capital stock, L_1 is the inital labor force, K_2 is the new capital stock, L_2 is the new labor force

Once again, we're going to add an "unusual zero."

$$0 = \frac{\frac{K_1}{L_2} - \frac{K_1}{L_2}}{K_1/L_1}$$

Adding our "unusual zero" yields:

$$\frac{\Delta(K/L)}{K/L} = \frac{\frac{K_2}{L_2} - \frac{K_1}{L_1}}{K_1/L_1} + \frac{\frac{K_1}{L_2} - \frac{K_1}{L_2}}{K_1/L_1}$$

Rearranging terms, we get:

$$\begin{aligned}
\frac{\Delta(K/L)}{K/L} &= \frac{\frac{K_2}{L_2} - \frac{K_1}{L_2}}{K_1/L_1} + \frac{\frac{K_1}{L_2} - \frac{K_1}{L_1}}{K_1/L_1} \\
&= \left(\frac{K_2}{L_2} - \frac{K_1}{L_2}\right) \cdot \frac{L_1}{K_1} + \left(\frac{K_1}{L_2} - \frac{K_1}{L_1}\right) \cdot \frac{L_1}{K_1} \\
&= \left(\frac{K_2 - K_1}{K_1}\right) \cdot \frac{L_1}{L_2} + \left(\frac{L_1}{L_2} - \frac{L_1}{L_1}\right) \cdot \frac{K_1}{K_1} \\
&= \frac{\Delta K}{K_1} \cdot \frac{L_1}{L_2} + \left(\frac{L_1}{L_2} - 1\right) \qquad \leftarrow \text{note that} : 1 = \frac{L_2}{L_2} \\
&= \frac{\Delta K}{K_1} \cdot \frac{L_1}{L_2} + \left(\frac{L_1 - L_2}{L_2}\right) \\
&= \frac{\Delta K}{K_1} \cdot \frac{L_1}{L_2} - \frac{\Delta L}{L_2}
\end{aligned}$$

The derivation above uses the definitions: $\Delta K = K_2 - K_1$ and $\Delta L = L_2 - L_1$.

Since we're considering very small changes: $\Delta L \approx 0$, which implies that: $L_2 \approx L_1$ and $\frac{L_1}{L_2} \approx 1$.

Therefore we can write:

$$\frac{\Delta(K/L)}{K/L} = \frac{\Delta K}{K} - \frac{\Delta L}{L}$$

Homework #1D

1. Let $Y(t)$ denote output as a function of time and let $L(t)$ denote the labor force as a function of time.

a. What is the ratio of output per worker?

b. How does it evolve over time?

2. Let $Y(t)$ denote output as a function of time, let $L(t)$ denote the labor force as a function of time and let $A(t)$ denote a level labor efficiency, so that $A(t)\cdot L(t)$ is the "effective labor force."

a. What is the ratio of output per unit of effective labor?

b. How does it evolve over time?

3. Let $K(t)$ denote the capital stock as a function of time, let $L(t)$ denote the labor force as a function of time and let $A(t)$ denote a level labor efficiency, so that $A(t)\cdot L(t)$ is the "effective labor force." Let $\tilde{k}(t)$ denote the ratio of capital to effective labor.

a. What is the ratio of capital per unit of effective labor?

b. How does it evolve over time?

c. Find the derivative: $\dfrac{d\,\tilde{k}(t)^{\alpha}}{d\,t}$. Hint: Use the chain rule. It makes life a lot easier.

Eric Doviak
Economic Growth and
Economic Fluctuations

Notes on Logarithms

When I initially designed this course, I did not plan to teach you how to use logarithms. Van den Berg's textbook however assumes that you understand logarithms, so I've written these notes to enable you to better understand the equations in his text.

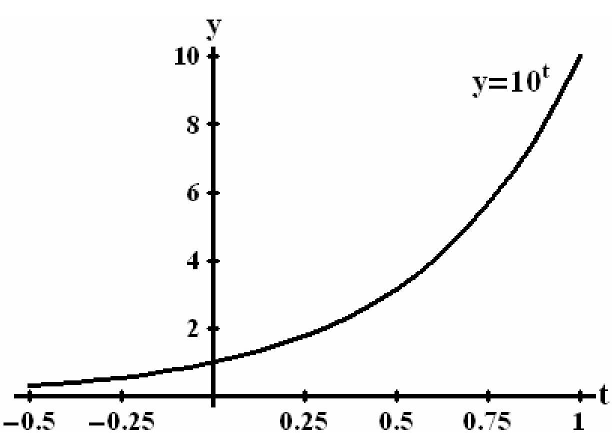
Logarithms start with a given base number. The base number can be any real number. The simplest base to use is 10, but the preferred base is the irrational number: $e = 2.71828...$. These notes explain the basic idea of logarithms using the base number 10. Then once you've grasped the basic idea behind logarithms, these notes will introduce the preferred base.

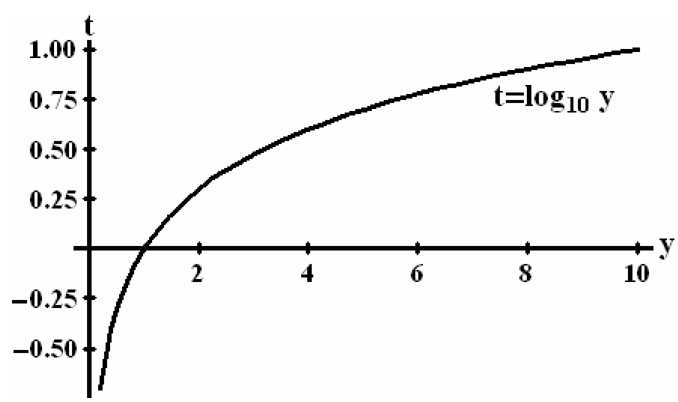
Now that we've temporarily chosen a base of 10, let's pick another number, say: 1000. The basic idea of logarithms is to answer the question: "10 raised to what power will equals 1000?" The answer of course is: "10 raised to the third power equals 1000." That is: $10^3 = 1000$. Mathematically, we say: "The logarithm of 1000 to the base of 10 equals 3." That is: $\log_{10} 1000 = 3$.

$10^3 = 1000$	$\log_{10} 1000 = 3$
$10^2 = 100$	$\log_{10} 100 = 2$
$10^1 = 10$	$\log_{10} 10 = 1$
$10^0 = 1$	$\log_{10} 1 = 0$
$10^{-1} = 0.1$	$\log_{10} 0.1 = -1$
$10^{-2} = 0.01$	$\log_{10} 0.01 = -2$
$10^{-3} = 0.001$	$\log_{10} 0.001 = -3$

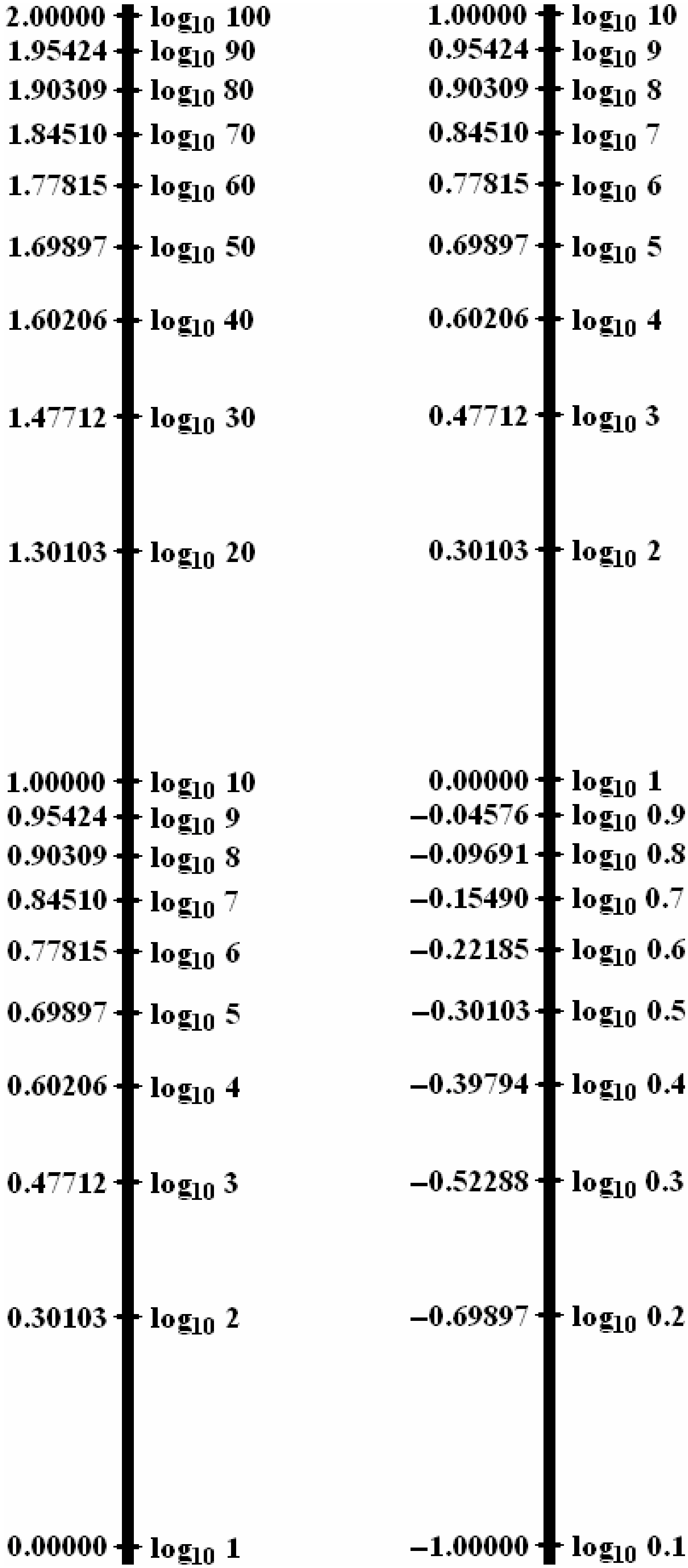
Now let's pick another number, say: 0.01 and once again ask: "10 raised to what power will equals 0.01?" The answer this time is: "10 raised to the power –2 equals 0.01." That is: $10^{-2} = 0.01$. Mathematically, we say: "The logarithm of 0.01 to the base of 10 equals –2." That is: $\log_{10} 0.01 = -2$.

This relationship is summarized in the table above and is depicted in the graphs below.





It should also be intuitively clear that if we had chosen a different base number, say: 4, then we could ask the question: "4 raised to what power equals 16?" The answer this time is: "4 raised to the second power equals 16." That is: $4^2 = 16$. Mathematically, we say: "The logarithm of 16 to the base of 4 equals 2." That is: $\log_4 16 = 2$.



Logarithms are useful because they allow us to perform the mathematical operations of multiplication and division using the simpler operations of addition and subtraction.

For example, you already know that: $2 \times 4 = 8$, so look at the logarithmic scales at left and observe that:

$$\begin{array}{rr} \log_{10} 2 & 0.30103 \\ + \log_{10} 4 & + 0.60206 \\ \hline \log_{10} 8 & 0.90309 \end{array}$$

Similarly, you know that: $\frac{40}{8} = 5$.
Looking again at the logarithmic scales, you can see that:

$$\begin{array}{rr} \log_{10} 40 & 1.60206 \\ - \log_{10} 8 & - 0.90309 \\ \hline \log_{10} 5 & 0.69897 \end{array}$$

In fact, before technology enabled us all to carry a calculator our pocket, people performed multiplication and division using slide rules that had base 10 logarithmic scales.

So why does this “trick” work? To answer this question, first recall that:

$$10^2 \cdot 10^3 = 10^5$$
$$100 \cdot 1000 = 100{,}000$$

$$10^2 \cdot 10^{-3} = 10^{-1}$$
$$\frac{100}{1000} = 0.1$$

So the “trick” works because the numerical value of a logarithm is an exponent and because you can add (or subtract) exponents in a multiplication problem (or division problem) so long as the exponents are the powers of a common base number.

On the previous page, we established two rules of logarithms:

$$\text{Rule I}: \quad \log_{10}(a \cdot b) = \log_{10} a + \log_{10} b$$
$$\text{Rule II}: \quad \log_{10}\left(\frac{a}{b}\right) = \log_{10} a - \log_{10} b$$

We can use Rule I to establish yet another rule:

$$\text{Rule III}: \quad \log_{10}\left(a^{c}\right) = c \cdot \log_{10} a$$

For example: $4^3 = 4 \cdot 4 \cdot 4$, therefore:

$$\begin{aligned}\log_{10}\left(4^3\right) &= \log_{10}(4 \cdot 4 \cdot 4) \\ &= \log_{10} 4 + \log_{10} 4 + \log_{10} 4 \\ &= 3 \cdot \log_{10} 4\end{aligned}$$

Of course, **the rules above apply to logarithms to all bases.** After all, the numerical value of a logarithm is just an exponent and an exponent can be attached to any base number.

We've been working with logarithms to the base of 10, but in analytical work the preferred base is the irrational number: $e = 2.71828...$. Logarithms to the base of e are called **natural logarithms** (abbreviated "ln"): $\log_e a \equiv \ln a$. The rules of natural logarithms are the same as the ones derived above:

$$\text{Rule I}: \quad \ln(a \cdot b) = \ln a + \ln b$$
$$\text{Rule II}: \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$
$$\text{Rule III}: \quad \ln\left(a^{c}\right) = c \cdot \ln a$$

♦ ♦ ♦

pitfalls to avoid

Finally, there are two pitfalls to avoid.

First, observe from Rule I that $\ln(a + b)$ is NOT equal to $\ln a + \ln b$. Similarly, Rule II tells us that $\ln(a - b)$ is NOT equal to $\ln a - \ln b$.

Second, logarithms of non-positive numbers are undefined. For example, in the graphs on the first page, we used the equation $y = 10^t$ to obtain the relationship $t = \log_{10} y$. Therefore if $y = 0$, then the value of t must be negative infinity.

So what would the value of t be if y were a negative number? … That's a trick question. If y were a negative number, then t could not possibly be a real number. For this reason, logarithms of negative numbers are undefined.

Lecture 2

The Production Process

Eric Doviak

Economic Growth and Economic Fluctuations

Purpose of this Lecture

This lecture is NOT intended to give you a comprehensive understanding of firm behavior.

If you're interested in how economists model firm behavior, then take my course in Microeconomics.

The purpose is to provide you with a basic understanding of:

- marginal product of labor and marginal product of capital
  - why firms hire labor until the wage rate is equal to price times the marginal product of labor, i.e. $w = p \cdot MPL$
  - why firms hire capital until the rental rate on capital is equal to price times the marginal product of capital, i.e. $r = p \cdot MPK$
- assumption that firms make zero economic profit in the long-run if they face **constant returns to scale**

Intro to Firm Behavior

- **<u>production</u> – process by which inputs are combined, transformed, and turned into outputs**
- **<u>firm</u> – person or a group of people that produce a good or service to meet a perceived demand**
- **we'll assume that firms' goal is to <u>maximize</u> <u>profit</u>**

Perfect Competition

- **many firms, each small relative to overall size of the industry, producing homogenous** (virtually identical) **products**
- **no firm is large enough to have any control over price**
- **new competitors can freely enter and exit the market**

Competitive Firms are Price Takers

- **firms have no control over price**
- **price is determined by the market**

Firms' Basic Decisions

1. **How much of each input to demand**
2. **Which production technology to use**
3. **How much supply**

Short-Run vs. Long-Run

In the <u>short-run</u>, two conditions hold:

1. **firm is operating under a fixed scale of production – i.e. at least one input is held fixed** (ex. it may be optimal for a firm to buy new machinery, but it can't do so overnight)
2. **firms can neither enter nor exit an industry**

In the <u>long-run</u>:

- **there are no fixed factors of production, so firms can freely increase or decrease scale operation**
- **new firms can enter and existing firms can exit the industry**

Profit-Maximization

(economic) **profit = total revenue – total** (economic) **cost**

total revenue – **amount received from the sale of the product** (price times number of goods sold)

total (economic) **cost** – **the total of:**

1. **out of pocket costs** (ex. prices paid to each input)
2. **opportunity costs:**
   a. **normal rate of return on capital and**
   b. **opportunity cost of each factor of production** – ex. if an employee in my firm could earn $30,000 if he/she worked for another firm, then I'd have to pay him/her at least $30,000, otherwise he/she would leave. (In reality, you might work in your parents' firm and not be paid. In such a case, their firm's accounting profit would be higher than their firm's economic profit).

normal rate of return on capital – **rate of return that is just sufficient to keep investors satisfied** (ex. real interest rate on corporate bonds)

- **nearly the same as the real interest rate on risk-free government bonds for relatively risk-free firms**
- **higher for relatively more risky firms**

Production Process

optimal method of production minimizes cost

production technology – **relationship betw/n inputs & outputs**

- **labor-intensive technology relies heavily on labor instead of capital**
- **capital-intensive technology relies heavily on capital instead of labor**

production function – **units of total product as func. of units of inputs**

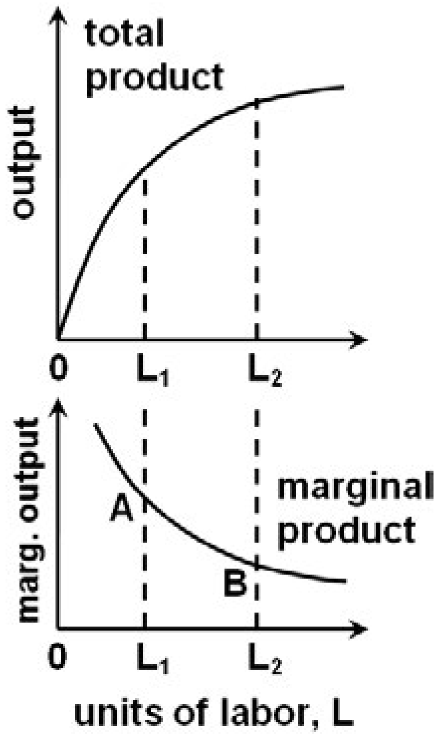
average product – **average amount produced by each unit of a variable factor of production (input)**

$$\text{avg. product of labor} = \frac{\text{total product}}{\text{total units of labor used}}$$

marginal product – **additional output produced by adding one more unit of a variable factor of production (input), ceteris paribus**

$$\text{marg. product of labor} = \frac{\Delta\ \text{total product}}{\Delta\ \text{units of labor used}}$$

Total and Marginal Product



diminishing marginal returns – **when additional units of a variable input are added to fixed inputs, the marginal product of the variable input declines**

marginal product is the slope of the total product function

at labor input L_1, the slope of the total product function is relatively higher than it is at L_2, so the marginal product of labor is higher at point A than it is at point B

in the case depicted, the marginal product of labor diminishes over entire range of labor input

Production with Two Inputs

Inputs often work together and are complementary.

- **Ex. cooks** (Labor) **and grills** (Capital)
- **If you hire more cooks, but don't add any more grills, the marginal product of labor falls** (too many cooks in the kitchen)
- **If you hire (rent) more grills, but don't add any more cooks, the marginal product of capital falls** (grills sit idle)**.**

Given the technologies available, a profit-maximizing firm:

- **hires labor up to the point where the wage equals the price times the marginal product of labor (MPL)**
- **hires capital up to the point where the rental rate on capital equals the price times the marginal product of capital (MPK)**

$$\text{Profits } (\Pi) = \text{Total Revenue} - \text{Total Cost}$$
$$\Pi = p_X X(K,L) - wL - rK$$

at profit maximum:

$$w = p_X MPL$$

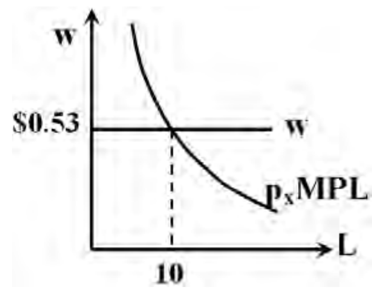
$$r = p_X MPK$$

Profit Maximization with Two Inputs

- **In a perfectly competitive industry:**
  - **firms are price takers in both input output markets,**
  - **firms cannot affect the price of their product, nor can they affect the price of inputs** (wages, rental rate on capital)**.**

- **Assume that a firm produces X with capital and labor and that its production function is given by:**

$$X = K^{2/3} * L^{1/3}$$



- **Assume also that:**
  - **price of the firm's output is $1**
  - **wage rate and rental rate are both $0.53**
- **So its profits are given by:**

$$\Pi = p_x X - r*K - w*L$$
$$= \$1* K^{2/3}*L^{1/3} - \$0.53*K - \$0.53*L$$

- **and firm should hire labor until** $w = p_X MPL$

Profit Maximization with Two Inputs

- **If the firm has 20 units of capital on hand** (this is the short-run)**:**
  - **how much labor should it hire?**
  - **how much should it produce?**
  - **how much profit will it make?**

Output of X	Capital	Labor	MPL
14.74	20	8	0.61
15.33	20	9	0.57
15.87	**20**	**10**	**0.53**
16.39	20	11	0.50
16.87	20	12	0.47

- **In the case depicted, the firm:**
  - **would hire 10 labor units**
  - **would produce 15.87 units of X**
  - **would make zero profit** … (I'm foreshadowing a little here)

$$\Pi^* = \$1*\mathbf{15.87} - \$0.53*20 - \$0.53*\mathbf{10} = \$0$$

- **Hiring more labor or less labor would lower profit:**

$$\Pi_8 = \$1*\mathbf{14.74} - \$0.53*20 - \$0.53*\mathbf{8} = -\$0.14$$
$$\Pi_9 = \$1*\mathbf{15.33} - \$0.53*20 - \$0.53*\mathbf{9} = -\$0.07$$

$$\Pi_{11} = \$1*\mathbf{16.39} - \$0.53*20 - \$0.53*\mathbf{11} = -\$0.03$$
$$\Pi_{12} = \$1*\mathbf{16.87} - \$0.53*20 - \$0.53*\mathbf{12} = -\$0.06$$

Costs in the Short Run

Fixed cost:

- **any cost that does not depend on the firm's level of output.** (The firm incurs these costs even if it doesn't produce any output).
- **firms have no control over fixed costs in the short run.** (For this reason, fixed costs are sometimes called sunk costs)**.**
    - **obvious examples:** property taxes, loan payments, etc.
    - **not-so-obvious example:** firm must pay "rent" to hired capital. If that level of capital cannot be adjusted immediately ("fixed factor"), then rental payments are a fixed cost in the short-run

Variable cost:

- **depends on the level of production**
- **derived from production requirements and input prices**
    - **variable cost rises as output rises because firm has to hire more inputs** (capital and labor) **to produce larger quantities of output**
    - (in the Long Run all costs are variable)

Marginal Cost

Marginal cost:

- **increase in total cost from producing one more unit of output** (the additional cost of inputs required to produce each successive unit of output)
- **only reflects changes in variable costs**
    - fixed cost does not increase as output increases
    - marginal cost is the slope of both total cost and variable cost

Shape of the Marginal Cost Curve

In the short run, the firm is constrained by a fixed input, therefore:

1. **the firm faces diminishing returns to variable inputs and**
2. **the firm has limited capacity to produce output**

As the firm approaches that capacity it becomes increasingly costly to produce successively higher levels of output. Marginal costs ultimately increase with output in the short run.

Short-Run Average and Marginal Cost

- **If a firm's capital stock is fixed in the short-run, then the rental payments that the firm makes on its capital stock is a fixed cost.**
- **We can use that assumption to derive short-run average and marginal cost curves.**
- **So start by assuming that a firm's production function is given by:**

$$X=K^{2/3}L^{1/3}$$

- **Since the firm's capital stock is fixed** (by assumption) **we can solve the production function for labor to find the amount of labor needed to produce various levels of output:**

$$L=\frac{X^3}{K^2}$$

- **Its total costs are given by:**

$$TC=rK+wL$$

$$TC=rK+w\frac{X^3}{K^2}$$

Short-Run Average and Marginal Cost

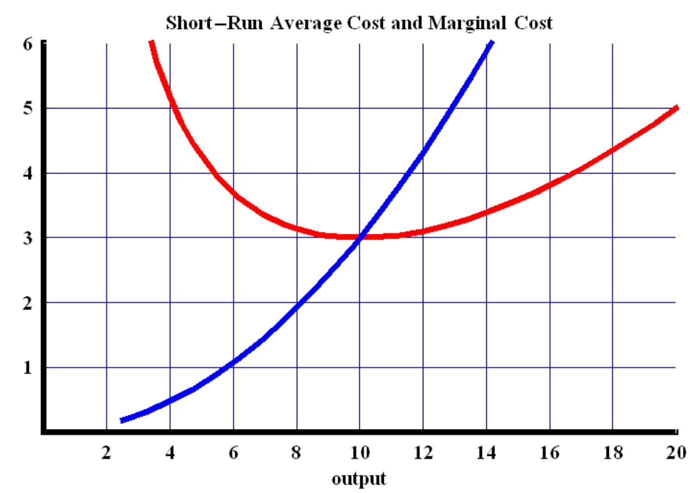
To find Short-Run Average Cost simply divide by Total Cost by X:

$$AC=\frac{TC}{X} \quad \Rightarrow \quad AC=\frac{rK}{X}+w\frac{X^2}{K^2}$$

To find Short-Run Marginal Cost take the derivative of Total Cost with respect to X:

$$MC\equiv\frac{d\,TC}{d\,X} \quad \Rightarrow \quad MC=3w\frac{X^2}{K^2}$$

So if the wage rate is $1 per unit of labor, i.e. $w=\$1$, **and the rental rate is $2 per unit of capital, i.e.** $r=\$2$, **and the firm has a capital stock of 10 units, then:**

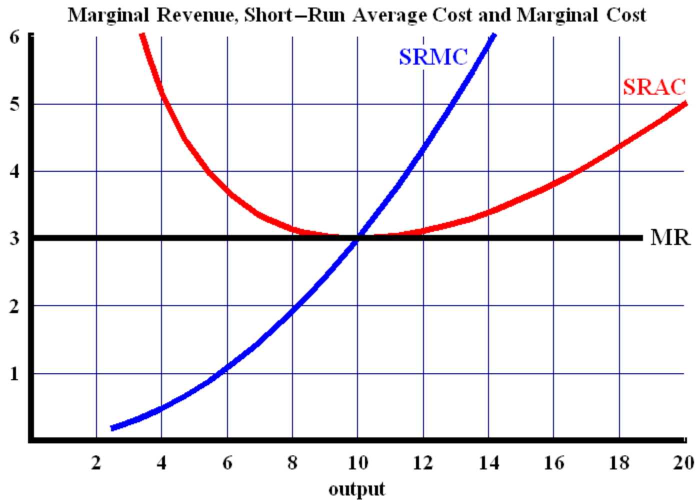


Revenue, Costs and Profit-Maximization

- **Total Revenue – amount that firm receives from sale of its output**
- **Marginal Revenue – additional revenue that a firm takes in when it increases output by one additional unit.**
- For example, if a firm is operating in a competitive industry – so that it has no ability to influence the market price – and if the market price is $3, then:

Comp. Firm's MR = p*

p	Q	TR	MR
3	0	0	3
3	1	3	3
3	2	6	3
3	3	9	3
3	4	12	3
3	5	15	3
3	6	18	3



- **Profit-maximizing level of output occurs where a firm's MR = MC**
- **Since MR = p*, it will produce up to the point where p* = MC**

But wait …

- **In the example above, the firm is making zero** (economic) **profit**
- **If it produces up to the point where p* = MC, then:**
  - **it will produce 10 units of output and**
  - **sell it at a price of $3, for total revenue of $30**
  - **but it's total costs will also be $30,**
    - **since at an output of 10 units, its average cost** (per unit) **is $3**
    - **and average cost times output equals total cost**
  - **so its total revenue equals its total cost**

Could a firm ever make positive (economic) **profit? Yes.**

- **If the price were higher** (say: $4)**, then it would make positive** (economic) **in the SHORT RUN, but**
- **in the LONG RUN new firms will enter the market and push down the market price until the firm's profit is zero**
- **It could also make positive** (economic) **profit if it's a monopoly**

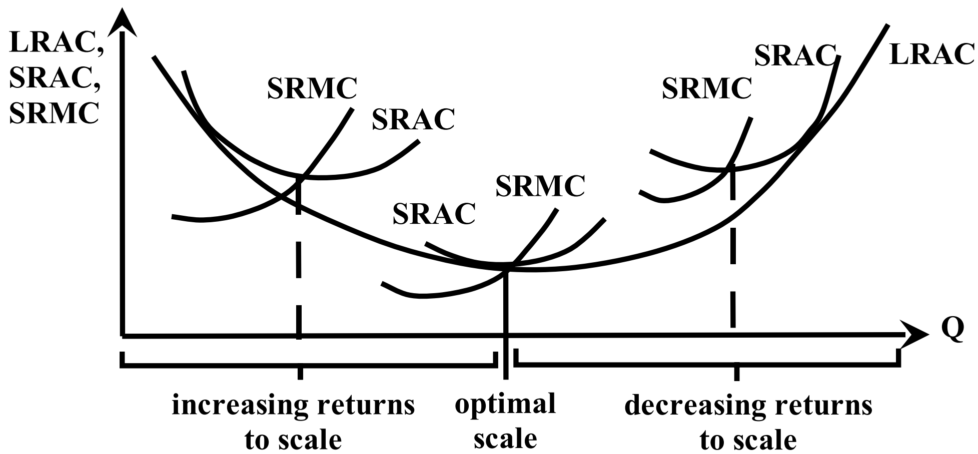
Long-Run Costs: Returns to Scale

- **In the SHORT RUN, firms have to decide how much to produce in the current scale of plant** (factory size is fixed)**.**
- **In the LONG RUN firms, have to choose among many potential scales of plant** (they can expand the factory)**.**

- **Increasing returns to scale** (or economies of scale)**, refers to an increase in a firm's scale of production, which leads to lower average costs per unit produced.**
- **Constant returns to scale refers to an increase in a firm's scale of production, which has no effect on average costs per unit produced.**
- **Decreasing returns to scale** (or diseconomies of scale) **refers to an increase in a firm's scale of production, which leads to higher average costs per unit produced.**

Long-Run Average Cost Curve

- **The Long-Run Average Cost (LRAC) curve shows the different scales on which a firm can operate in the long-run. Each scale of operation defines a different short-run.**
- **The Long-Run Average Cost curve of a firm:**
  - **is downward-sloping when the firm exhibits increasing returns to scale.**
  - **is upward sloping when the firm exhibits decreasing returns to scale.**
- **The optimal scale of plant is the scale that minimizes long-run average cost.**

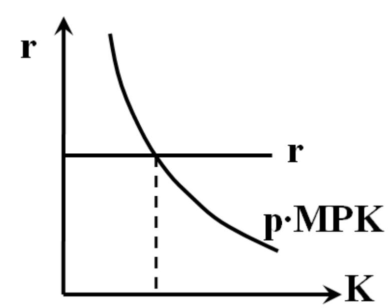


Returns to Scale

- **As mentioned previously, the only two cases where a firm can make positive profit is:**
  - o if the firm is a monopoly
  - o if there is not enough competition to drive the price down to the point where competitive firms make zero profit
- **Monopoly arises when the firm faces increasing returns to scale over the whole range of output**
  - o So if the monopolist were to double his/her inputs of capital and labor, then his/her output would more than double
  - o So his/her costs would double, but he/she would be able to sell more than two times the amount of output
- **The production process of a competitive firm faces exhibits constant returns to scale**
  - o So if the firm were to double its inputs of capital and labor, then its output would exactly than double
  - o So its costs would double and it would be able to sell exactly two times the amount of output

Factor Demand in the Long Run

- **When we derived the firm's short-run marginal and average cost functions, we assumed that the firm was operating under a fixed scale of production.**
- **Specifically, we assumed that the firm's capital stock was held fixed.**
- **So when we derived the condition for profit-maximization, we focused on the condition that:**
  - o the firm hires labor up to the point where the wage equals the price times the marginal product of labor (MPL): $\mathbf{w} = \mathbf{p} \cdot \mathbf{MPL}$
- **In the long run, the other condition must also hold:**
  - o firm hires capital up to the point where the rental rate on capital equals the price times the marginal product of capital (MPK): $\mathbf{r} = \mathbf{p} \cdot \mathbf{MPK}$



Zero Profit in the Long Run

Now let's bring it all together.

- **Firm's profit is given by:** $\Pi = p \cdot K^{2/3} \cdot L^{1/3} - r \cdot K - w \cdot L$
- **Firm's factor demands are given by:** $r = p \cdot MPK$ **and** $w = p \cdot MPL$
- **Using the calculus tricks that you learned:**
  - $MPK = \frac{2}{3} \cdot K^{-1/3} \cdot L^{1/3}$ **and** $MPL = \frac{1}{3} \cdot K^{2/3} \cdot L^{-2/3}$
- **Plugging the factor demands into the profit function:**

$$\Pi = p \cdot K^{2/3} \cdot L^{1/3} - \left(p \cdot \frac{2}{3} \cdot K^{-1/3} \cdot L^{1/3} \right) \cdot K - \left(p \cdot \frac{1}{3} \cdot K^{2/3} \cdot L^{-2/3} \right) \cdot L$$

$$= p \cdot \left(K^{2/3} \cdot L^{1/3} \right) - \frac{2}{3} \cdot p \cdot \left(K^{2/3} \cdot L^{1/3} \right) - \frac{1}{3} \cdot p \cdot \left(K^{2/3} \cdot L^{1/3} \right) = 0$$

Zero Profit in the Long Run

So what does this mean?

- **If the firm operates in a competitive industry**
  - **with a large number of firms**
  - **so that it cannot affect market prices**
- **if the firm's production process exhibits constant returns to scale, and**
- **if the firm varies capital and labor optimally, in order to maximize** (economic) **profit, then**

- **its <u>maximum</u>** (economic) **profit will be zero in the long run**
- **even though it is producing at the lowest possible cost per unit of output** (lowest possible average cost)

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Economic Fluctuations

Why does a Firm Maximize its Profit where Marginal Revenue equals Marginal Cost?

If a firm is operating in a competitive industry, then its total revenue is simply equal to the market price times the quantity it produces, so we can depict Total Revenue as a linear function of output (a straight line) in the graph on the next page (i.e. $TR = pQ$).

In the graph, I've assumed that the firm's Total Cost is increasing at an increasing rate (due to diminishing marginal product of labor).

Notice that if the firm produces a very low level of output (quantity produced), it will not be profitable. If it produces too much, its costs will once again exceed its revenues and it will not be profitable.

Over the range of output where the firm's total revenue exceeds its total cost, the firm is making positive profit (in the short-run anyway).

The firm maximizes its profit in the middle of that range, but at what point specifically?

In the range of output where the slope of the Total Revenue curve is greater than the slope of the Total Cost curve, the firm could increase its profit by producing more output.

In the range of output where the slope of the Total Revenue curve is less than the slope of the Total Cost curve, the firm could increase its profit by producing less output.

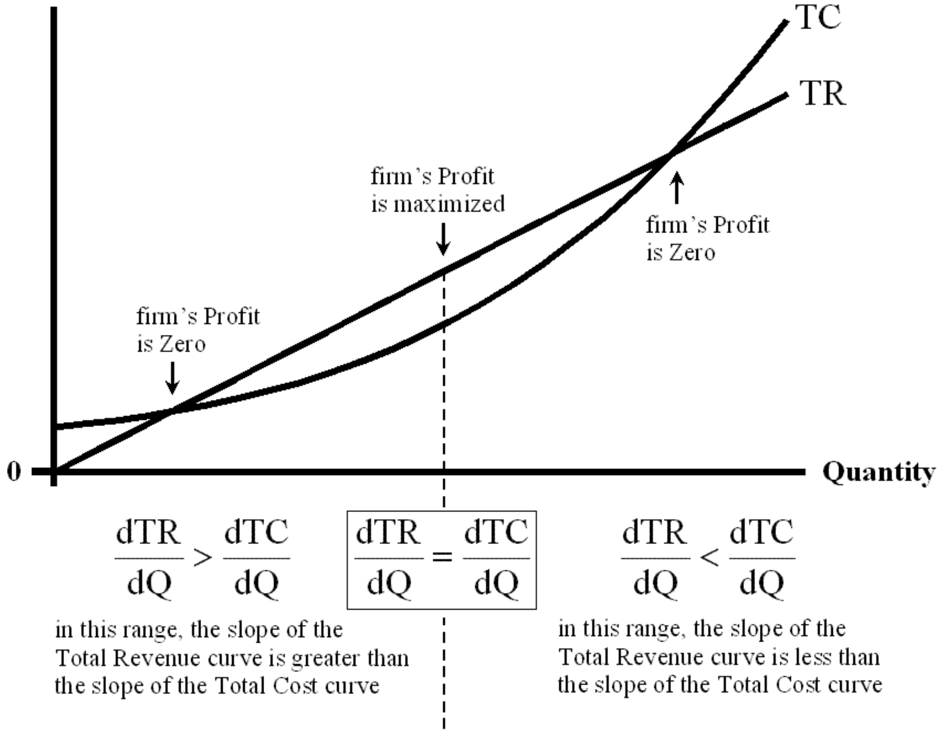
When the slope of the Total Revenue curve is equal to the slope of the Total Cost curve, the firm's profit is maximized.

Since Marginal Revenue is the slope of the Total Revenue curve and since Marginal Cost is the slope of the Total Cost curve, the point at which the firm maximizes its profit corresponds to the point where Marginal Revenue equals Marginal Cost.

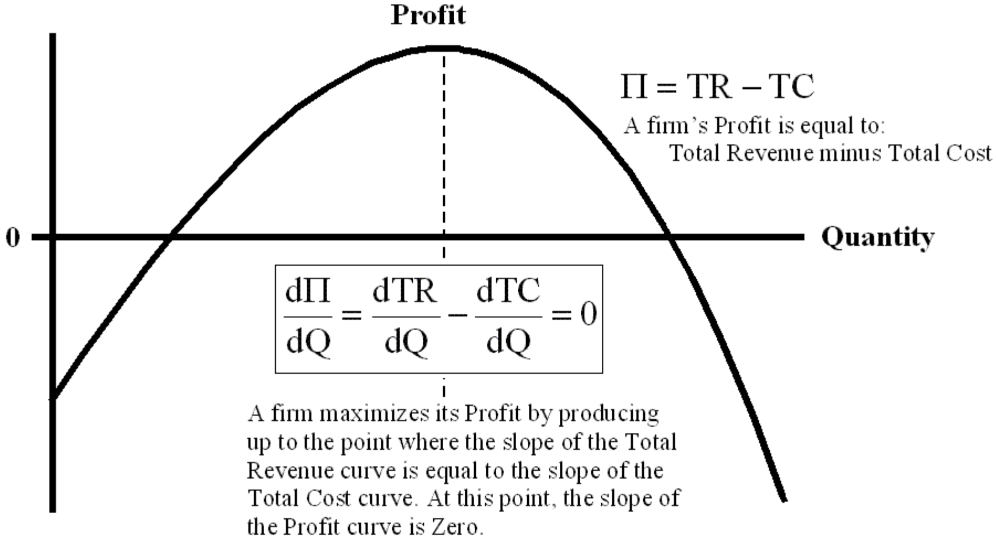
Since we've assumed that the firm is operating in a competitive industry, the firm's Marginal Revenue is simply equal to the market price over all ranges of output because it faces an infinitely elastic (horizontal) demand curve.

Because the firm produces up to the point where Marginal Revenue equals Marginal Cost (in order to maximize its profit), the Marginal Cost curve is the firm's Supply curve.

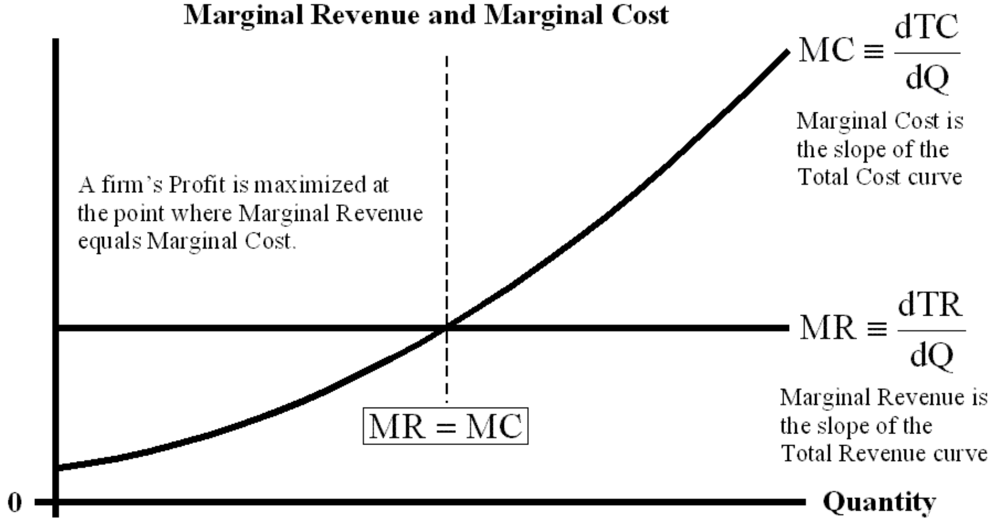
Total Revenue and Total Cost



Profit



Marginal Revenue and Marginal Cost



Notes on Profit Maximization

Eric Doviak

January 30, 2011

Brooklyn College, Microeconomics

These notes are a supplement to a lecture that I will deliver in class. They contain the tables that I will use to provide a simple, numerical example of profit-maximization. They do not contain much explanation, so please do not treat them as a substitute for the lecture.

1 A Firm's Profit

A firm's profit, Π, is equal to the difference between its total revenue and its total costs:

$$\Pi = TR - TC \tag{1}$$

For the purposes of this lecture, we will assume that the firm is in perfect competition and, therefore, cannot affect the market equilibrium price of the good that it produces. It simply takes the price as given. We will also assume that it can sell any quantity that it desires at the market price. These assumptions imply that total revenue is equal to price times quantity sold:

$$TR = p \cdot Q \tag{2}$$

We will also make the simplifying assumption that the only input into the production of the good that it sells is labor, so that its total costs are equal to the wage rate times the amount of labor that it employs:

$$TC = w \cdot L \tag{3}$$

2 Production Assumptions

Because labor is the only input into the production process, the quantity that the firm produces depends only on the amount of labor that it employs. We will assume that the firm faces **diminishing marginal returns** to the labor that it employs. In other words, "as you add more and more cooks to the kitchen, the additional amount of food produced by each additional cook falls."

To formalize this concept, we will assume that the quantity that the firm produces is proportional to the square root of the amount of labor that it employs:

$$Q = A \cdot \sqrt{L} \tag{4}$$

Its **marginal product of labor** is the additional quantity that it produces when it employs an additional unit of labor:

$$MPL \equiv \frac{\Delta Q}{\Delta L} \tag{5}$$

Those of you who have taken a course in calculus may notice that the marginal product of labor is the derivative of quantity produced with respect to labor:

$$\frac{dQ}{dL} = \frac{A}{2} \cdot \frac{1}{\sqrt{L}} \tag{6}$$

Finally, if the firm wished to produce a particular quantity of output, that choice would determine how much labor it employs:

$$L = \left(\frac{Q}{A}\right)^2 \tag{7}$$

3 Profit Maximization

Now that we have defined a firm's profit and production assumptions, we will use a simple, numerical example to find the quantity of output that maximizes its profit and the amount of labor it would employ to maximize its profit.

Specifically, we'll assume that the market price of the firm's output is $4 per unit, the wage rate is $10 per unit of labor and the scale factor in its production is 10. In other words:

- $p = 4$
- $w = 10$
- $A = 10$

3.1 Brute-Force Method

One way to find the profit-maximizing levels of output and labor is to compute the firm's profit at each level of output:

Table 1: Brute-Force Method

Q	L	TR	TC	Π
10	1	40	10	30
20	**4**	80	40	**40**
30	9	120	90	30
40	16	160	160	0

This method shows that the firm would maximize profit by employing 4 units of labor to produce 20 units of output.

3.2 Cost-Benefit Analysis

Another way to find the profit-maximizing levels of output and labor is to compare the benefits associated with producing an additional unit of output (in terms of increased revenue) with the cost of producing an additional unit of output.

If producing an additional unit of output would bring in additional revenue that exceeds the additional cost associated with producing another unit, then the firm would increase its profit by producing that additional unit. By contrast, if the additional revenue were less than the additional cost, then the firm's profit would fall if it produced that additional unit.

A firm's **marginal revenue** is the additional revenue that it receives when it sells an additional unit of output:

$$MR \equiv \frac{\Delta TR}{\Delta Q} \tag{8}$$

Its **marginal cost** is the additional cost that it incurs by producing an additional unit of output:

$$MC \equiv \frac{\Delta TC}{\Delta Q} \tag{9}$$

The firm maximizes profit by producing output up to the point at which marginal revenue equals marginal cost.

In Table 1, we saw that the firm maximizes profit by producing 20 units of output. Table 2 shows that the firm's marginal revenue exceeds marginal cost when it produces fewer than 20 units. Table 2 also shows that the firm's marginal revenue is less than marginal cost when it produces more than 20 units.

Table 2: MR = MC Method

Q	L	TR	MR	MC	TC
10	1	40			10
			$40/10 = 4$	$3 = 30/10$	
20	4	80			40
			$40/10 = 4$	$5 = 50/10$	
30	9	120			90
			$40/10 = 4$	$7 = 70/10$	
40	16	160			160

In other words, it would not make sense for the firm to produce 15 units. Producing an additional unit would bring in additional revenue that exceeds the additional cost associated with producing it, so it should produce more than 15 units.

Similarly, it would not make sense for the firm to produce 25 units. Producing an additional unit would bring in additional revenue that falls short of the additional cost associated with producing it. Notice also that if it reduced production by one unit, the subtracted revenue would be less than the subtracted cost, so it should produce fewer than 25 units. (Ideally, it should produce 20 units).

3.3 Cost-Benefit Analysis, One More Time

Another form of cost-benefit analysis compares the cost of hiring an additional unit of labor to the additional benefit (in term of increased revenue) that hiring an additional unit of labor brings to the firm.

In this case, the additional cost of employing an additional unit of labor (i.e. the marginal cost) is simply the wage rate, w. The additional revenue is the price of the firm's output times the additional output that is produced when an additional unit of labor is hired:

$$p \cdot \frac{\Delta Q}{\Delta L} \equiv p \cdot MPL \tag{10}$$

The term $p \cdot MPL$ is usually called the **"marginal revenue product of labor,"** but some economists call it the "marginal value product of labor."

If employing an additional unit of labor would bring in additional revenue that exceeds the wage rate (i.e. the additional cost associated with another unit of labor), then the firm would increase its profit by hiring that additional unit of labor. By contrast, if the additional revenue were less than the wage rate, then the firm's profit would fall if it hired that additional unit.

To maximize profit, the firm will hire labor up to the point at which the marginal revenue product of labor is equal to the wage rate:

$$w = p \cdot MPL \tag{11}$$

Table 3: w = p MPL Method

Q	L	TR	$p \cdot MPL$	w
10	1	40		
			$40/3 \approx 13.3$	10
20	4	80		
			$40/5 = 8.00$	10
30	9	120		
			$40/7 \approx 5.71$	10
40	16	160		

In Table 1, we saw that the firm maximizes profit by employing 4 units of labor. Table 3 shows that the marginal revenue product of labor is greater than the wage rate when it employs fewer than 4 units of labor. Table 3 also shows that the marginal revenue product of labor is less than the wage rate when it employs more than 4 units.

In other words, it would not make sense for the firm to employ 2 units of labor. Hiring an additional unit would bring in additional revenue that exceeds the wage rate (i.e. the additional cost associated with hiring another unit of labor), so it should employ more than 2 units.

Similarly, it would not make sense for the firm to employ 6 units of labor. Hiring an additional unit would bring in additional revenue that falls short of the wage rate. Notice also that if it reduced the amount of labor that it employs by one unit, the subtracted cost would be less than the subtracted revenue, so it should employ fewer than 6 units. (Ideally, it should employ 4 units).

Eric Doviak
Economic Growth and
Economic Fluctuations

Notes on the Zero-Profit Result

In Lecture 3, I gave examples of firms operating in a competitive industry that make zero profit in the short-run, but I didn't fully explain when such a situation would arise.

Firms in competitive industries may make positive profits in the short-run, but – if there is free entry into the industry and firms face constant returns to scale over some range of output – then in the long-run, the firms' profits will be driven to zero.

one reason why profits go to zero in the long run

Suppose that there is free entry into the industry in which my firm operates, that the technology I use is widely available and that my firm is making positive profits in the short run. Since I'm making positive profits, another person (call him John) will use the same technology that I am using to produce output.

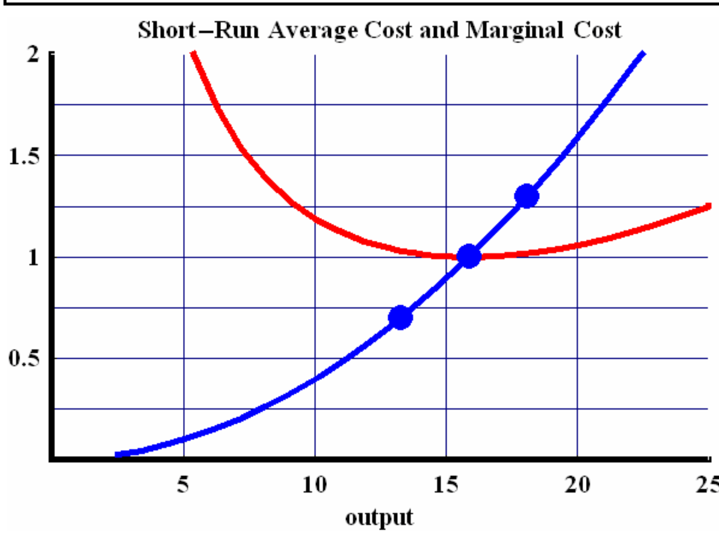
John will enter the industry, increase the market supply and lower the market price. This will reduce my profit, but if John and I are still making a positive profit, then yet another person (call her Jane) will enter the industry and use the same technology to produce output. Jane's output will further increase market supply and lower the market price. This process will continue until there are no more profits to be made.

Eventually, John, Jane and I will all be producing output at the minimum point along our long-run average cost curves – this corresponds to the point where our firms all face constant returns to scale.

The table shows that the firm maximizes profit by employing labor up to the point where the wage equals the price times the marginal product of labor. (Here, capital is held fixed since this is the short-run)**.**

The graph corresponds to the table and shows that the firm maximizes its profit by producing up to the point where price (marginal revenue) **equals marginal cost.**

case where p = $1.30 and where r = w = $0.53					
X	**K**	**L**	**MPL**	**p*MPL**	**profit**
17.32	20	13	0.44	0.58	5.06
17.76	20	14	0.42	0.55	5.09
18.17	**20**	**15**	**0.40**	**0.52**	**5.10**
18.57	20	16	0.39	0.50	5.09
18.95	20	17	0.37	0.48	5.05
case where p = $1.00 and where r = w = $0.53					
X	**K**	**L**	**MPL**	**p*MPL**	**profit**
14.74	20	8	0.61	0.61	-0.08
15.33	20	9	0.57	0.57	-0.02
15.87	**20**	**10**	**0.53**	**0.53**	**0.00**
16.39	20	11	0.50	0.50	-0.02
16.87	20	12	0.47	0.47	-0.06
case where p = $0.70 and where r = w = $0.53					
X	**K**	**L**	**MPL**	**p*MPL**	**profit**
11.70	20	4	0.97	0.68	-4.51
12.60	20	5	0.84	0.59	-4.41
13.39	**20**	**6**	**0.74**	**0.52**	**-4.39**
14.09	20	7	0.67	0.47	-4.42
14.74	20	8	0.61	0.43	-4.50



returns to scale

As we saw in Lecture 8, it's assumed that firms have a U-shaped long-run average cost curve. At low output levels, firms face increasing returns to scale. At high output levels, they face decreasing returns to scale. At the minimum point on the long-run average cost curve, they face constant returns to scale.

If a firm facing **constant returns to scale** doubles all of its inputs, then its output will exactly double. For example, if a firm's production function is given by:

$$X = K^{2/3}\cdot L^{1/3}$$

then if it doubles its inputs of capital and labor, its output doubles.

$$\begin{aligned} 2X &= (2K)^{2/3}\cdot (2L)^{1/3} \\ &= 2^{2/3}\cdot K^{2/3}\cdot 2^{1/3}\cdot L^{1/3} \\ 2X &= 2\cdot K^{2/3}\cdot L^{1/3} \end{aligned}$$

Similarly, you can easily see that when a firm doubles all of its inputs, its output:

- **more than doubles** when it faces **increasing returns to scale**
- **less than doubles** when it faces **decreasing returns to scale**.

increasing returns to scale:

$$\begin{aligned} X &= K^{2/3}\cdot L^{2/3} \\ 2X &< (2K)^{2/3}\cdot (2L)^{2/3} \\ &< 2^{2/3}\cdot K^{2/3}\cdot 2^{2/3}\cdot L^{2/3} \\ 2X &< 2^{4/3}\cdot K^{2/3}\cdot L^{2/3} \end{aligned}$$

decreasing returns to scale:

$$\begin{aligned} X &= K^{1/3}*L^{1/3} \\ 2X &> (2K)^{1/3}\cdot (2L)^{1/3} \\ &> 2^{1/3}\cdot K^{1/3}\cdot 2^{1/3}\cdot L^{1/3} \\ 2X &> 2^{2/3}\cdot K^{1/3}\cdot L^{1/3} \end{aligned}$$

two more reasons why profits go to zero in the long run

Since a firm facing constant returns to scale can double its output by doubling each of its inputs, then if it doubled its inputs and output, its profits would also double.

$$\begin{aligned} \Pi &= p\cdot X \quad - w\cdot L \quad - r\cdot K \\ 2\Pi &= p\cdot(2X) - w\cdot(2L) - r\cdot(2K) \end{aligned}$$

But if the firm can double its profit by doubling its inputs and output, then it could also quadruple its profit by quadrupling its inputs and output. So when exactly would a firm ever maximize its profit? The only reasonable assumption to make therefore is that the firm's profits go to zero in the long run. (Two times zero is zero).

There are three reasons why the profits of firms operating in competitive industries go to zero in the long run. The first reason was discussed above – as firms enter they drive down the market price to the point where no firm profits.

Another reason is because at very high levels of output, a firm may encounter logistical difficulties. For example, it may have difficulty coordinating the activities of all of its plants. Coordination difficulties at high levels of output are simply an example of decreasing returns to scale.

Finally, if the firm were lucky enough to face increasing returns to scale over all ranges of output, then it could always make ever higher profits by growing ever larger. Such a firm would eventually dominate its industry and become a monopolist – thus, the firm no longer operates in a competitive industry.

Homework #2

The Americans with Disabilities Act (ADA) requires that institutions like Brooklyn College accommodate individuals with disabilities in such a manor that everyone will have equal access to all facilities. Issues of accessibility include physical entrances to buildings and classrooms as well as the technology that students use (e.g. computers must be fitted with audio output for students with visual impairments).

Some interpret the ADA to mean that the disabled have a civil right to full parity in access to resources. Others complain about the costs.

degree of parity	total benefit	total cost
0 %	0	0
10 %	36	39
20 %	64	67
30 %	85	86
40 %	100	98
50 %	110	105
60 %	116	109
70 %	119	112
80 %	120	116
90 %	120	123
100 %	120	135

1. Does the total benefit of parity exhibit diminishing marginal returns? Explain your answer.
2. When does the benefit of parity exceed the cost of parity?
3. To find Brooklyn College's optimal degree of parity, what condition should you look for?
4. What is Brooklyn College's optimal degree of parity?
5. In the context of this model, why shouldn't Brooklyn College strive for full parity?

♦ ♦ ♦

You are given the following data on a profit-maximizing firm's production in the short-run (its capital stock is fixed).

The quantity of output that it produces (Q) depends on the amount of capital (K) and labor (L) it employs.

The firm must pay rent on its capital stock at a rate of about \$2 per unit, so its total rent bill is \$1000.

It also must pay a wage rate of \$5 per unit of labor.

Calculate the marginal product of labor for each output level. (NB: the marginal product of labor is the change in output per unit change in labor).

Q	K	L	MPL
309.02	500	73	--
310.08	500	74	
311.13	500	75	
312.17	500	76	
313.20	500	77	
314.22	500	78	
315.23	500	79	
316.23	500	80	
317.22	500	81	
318.20	500	82	

Up until what point does a profit-maximizing firm hire labor?

If the firm can sell all of its output on a perfectly competitive market at a price of \$5 per unit, then how much should it produce? How many units of labor should it employ?

Using your answers to the previous two questions, what will the firm's profit be?

Could it maintain this profit in the long-run? Why or why not?

Lecture 3

Distribution and Allocation of National Income

Eric Doviak

Economic Growth and Economic Fluctuations

Gross Domestic Product

- **Gross Domestic Product (GDP) is measures the total market value of country's output. It's the:**
  - market value of all final goods and services
  - produced within a given period of time
  - by factors of production located within country

What's In?

- **value of all final goods** (end products)
- **sale of a new car**
- Value of intermediate goods and services are implicitly present in the market value of the final good or service.

What's Out?

- **value of intermediate goods** (ex. steel used to build a car)
- **sale of a used car** (but the value of labor used to repair a used car is in GDP)
- **sales of stocks, bonds, etc.** (but the value of the broker's services is in GDP)

Gross Domestic Product

- **In the previous lecture, we discussed a firm's profit:**

$$\Pi = p \cdot K^{\alpha} \cdot L^{1-\alpha} - r \cdot K - w \cdot L$$

$$\text{where: } 0 < \alpha < 1$$

- **But since we've assumed that economic profit is zero – i.e.:** $\Pi = 0$

$$p \cdot K^{\alpha} \cdot L^{1-\alpha} = r \cdot K + w \cdot L$$

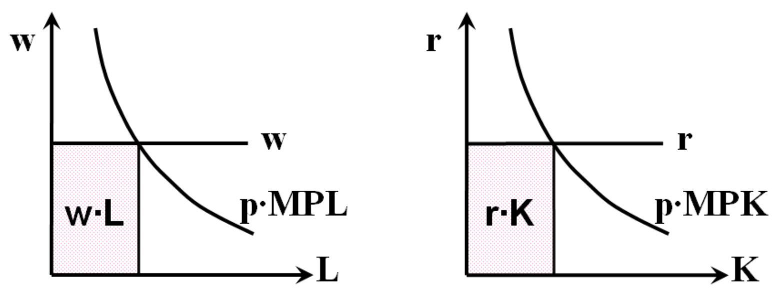
- **So the value of a firm's output is equal to the value of its capital and labor inputs.**
  - GDP is the aggregate sum of the market value of the output of all firms in the economy
  - GDP is also the aggregate sum of the value of all capital and labor inputs in the economy
- **By definition, these two measures of GDP should be equal**

Distributing National Income to the Factors of Production

- **So one approach to calculating GDP – the income approach – is based primarily on national income:**

$$\text{national income} \approx r \cdot K + w \cdot L$$

- $w \cdot L$ **is the total payments to human labor, where:** $w = p \cdot MPL$
- $r \cdot K$ **is the total payments to physical capital, where:** $r = p \cdot MPK$



- **but National Income only makes up about 80 percent of GDP**
- **so the income approach consists of more than just summing the incomes accruing to all factors of production**

the Income Approach to GDP

- **In practice, calculating $r \cdot K$ is impossible because:**
  - most firms own rather than rent the capital they use
  - firms can at times earn economic profit
- **When discussing the concept of National Income however, we're interested in how much everyone in the economy has earned**
  - If **economic profit** is approximately zero, then $r \cdot K$ is approximately equal to the **accounting profit** of the economy's firms (after depreciation).

$$\text{accounting profit} = \text{economic profit} + r \cdot K - \text{depreciation}$$

- **The difference between national income and GDP consists of:**
  - **depreciation** – decrease in capital's value which occurs as capital wears out or becomes obsolete (which is subtracted from accounting profits)
  - **indirect taxes minus subsidies** – accounts for the difference in price between that which the consumer pays and that which producer receives
  - **net factor payments to the rest of the world** – foreigners own factors of production in America (such as factories), but their income is not counted in U.S. national income since they are not U.S. nationals
- **Each of these are ADDED to national income to arrive at GDP in order to balance our measure of GDP on the income and expenditure side**

Allocating National Income

- **Now, it should be obvious to you that capital and labor won't be paid very much if no output is sold.**
- **So we also need to examine the purchases of aggregate output – the expenditure approach to calculating GDP:**
  - **Consumption –** denoted C
  - **Investment –** denoted I
  - **Government purchases –** denoted G
  - **Net Exports –** denoted NX
    - **net exports is simply total export minus total imports, so**
    - **we can write: $(X - M)$, where X is exports and M is imports**

- **Since we'll frequently use Y to denote GDP, we can write:**

$$Y = C + I + G + (X - M)$$

Consumption

- **Consumption – refers to the aggregate personal consumption expenditures of households in the economy and consists of:**
  - **Durable goods** – cars, refrigerators, furniture (things that will last a few years)
  - **Non-durable goods** – food, gasoline, paper (things that don't last very long)
  - **Services** – legal, medical (things that we value, but that do not represent the production of a tangible item)

Investment

- **Investment – consists of goods that firms and households purchase for future use – as opposed to present use**
  - **Non-residential fixed investment** – purchases of <u>new</u> plants and equipment by firms
  - **Residential fixed investment** – purchases of <u>new</u> housing by households and landlords
  - **Inventory investment** – investment to meet future demand, this component is negative when firms run down their inventories
    - **GDP is the market value of total p<u>roduction</u> in a period, not sales**
    - GDP = final sales + inventory investment

Government purchases

- **Government purchases – consist of the goods and services bought by federal, state and local governments**
  - military equipment
  - highway construction
  - services of government employees
- **Government purchases do NOT include transfer payments, such as Social Security payments, AFDC, etc.**
  - such payments reallocate income from some households to others
  - they do not represent the production of a good or service

Net Exports

- **Since U.S. expenditures include purchases of goods produced abroad, we must subtract those purchases from U.S. expenditures to find the value of goods and services produced in the U.S.**

- **Similarly, since foreign consumption includes purchases of goods produced in the U.S., we must add those purchases to U.S. expenditures to find the value of goods and services produced in the U.S.**

Lecture 4

Economic Growth: the Solow Model

Eric Doviak

Economic Growth and Economic Fluctuations

Why Study Economic Growth?

Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.

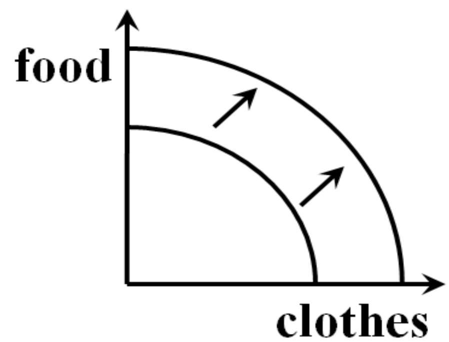
– Robert E. Lucas, Jr.

What is Economic Growth?

- **Before the Industrial Revolution in Great Britain, every society in the world was agrarian.**
- **Then technical change and capital accumulation increased British society's ability to produce textiles and agricultural products**
- **and a rapid and sustained increase in real output per capita began**
- **As a result, more could be produced with fewer resources**
    - new products
    - more output and
    - wider choice

- **Economic growth shifts the society's production possibility frontier up and to the right**

- **Economic growth allows each member of society to produce and consume more of all goods**



Plan of this Lecture

- **In the previous lecture, we learned some basic measures of how national income is distributed among factors of production and how national income is allocated among the goods produced.**
- **In this lecture, we'll examine the model of economic growth developed by Robert M. Solow in the 1950s.**

- **The Solow Model was one of the first attempts to describe how:**
    - **saving,**
    - **population growth and**
    - **technological progress**
- **affect the growth of output per worker over time – i.e. we're looking at LONG RUN economic growth.**

- **We'll use Solow's model to examine:**
    - **why the standards of living vary so widely among countries and**
    - **how economic policy can be used to influence standards of living**
    - **How much of an economy's output should be consumed today and how much should be saved for the future?**

Demand-Side Assumptions

- **To simplify the discussion, we'll examine a closed economy without a government. In other words:**
  - **there is no international trade** (so net exports equal zero) **and**
  - **government purchases equal zero**
- **so output is divided among consumption and investment:**

$$Y = C + I$$

- **Ultimately, we want to examine living standards, so we want to focus on per capita output, consumption and investment.**
- **It's easier to examine per worker variables in this model however.**

- **Nonetheless, per worker variables will yield fairly good approximations of living standards, so if we denote the labor force by L, we can define:**
  - **output per worker** as: $y \equiv Y/L$
  - **consumption per worker** as: $c \equiv C/L$ and **so:** $y = c + i$
  - **investment per worker** as: $i \equiv I/L$

Demand-Side Assumptions

- **Consumption per worker is the amount of output that is not invested**

$$c = y - i$$

- **The Solow Model assumes that consumption and investment** (per worker) **are proportional to income:**

$$c = (1 - s) \cdot y \qquad \textbf{where: } i = s \cdot y$$

- **So that the saving rate – denoted by the letter s – is constant.**

- **In other words, every year a fraction $(1 - s)$ of income is consumed and a fraction s of income is saved.**

Supply-Side Assumptions

- **We'll also assume that:**
  - **output is produced using capital, K, and labor, L**
  - **there is no technological progress** (we'll drop this assumption later)
  - **the production function exhibits constant returns to scale (CRS)**

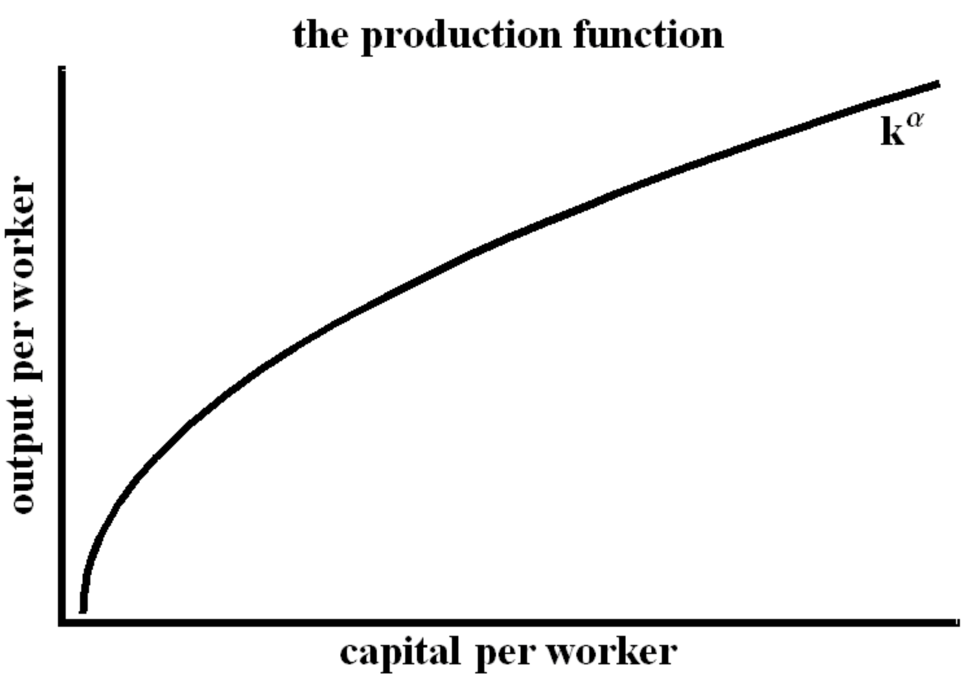
$$Y = K^{\alpha} \cdot L^{1-\alpha} \qquad \text{where: } 0 < \alpha < 1$$

- **Once again, we want to focus on per worker variables, so define:**
  - **output per worker** as: $y \equiv Y/L$ and
  - **capital per worker** as: $k \equiv K/L$

- **One convenient feature of the assumption of CRS is that we can define output per worker entirely in terms of capital per worker:**

$$\left.\begin{aligned} \frac{Y}{L} &= K^{\alpha} \cdot \frac{L^{1-\alpha}}{L} \\ &= K^{\alpha} \cdot L^{-\alpha} \end{aligned}\right| \quad \Rightarrow \quad y = k^{\alpha}$$

Production per Worker



Supply-Side Assumptions

- **Another convenient feature of the assumption of CRS concerns the Marginal Product of Capital (MPK)** – the derivative of output per worker with respect to capital**:**

$$MPK \equiv \frac{dY}{dK}$$

- **the Marginal Product of Capital equals the derivative of output per worker with respect to capital per worker:**

$$\frac{dY}{dK} = \alpha \cdot K^{\alpha-1} \cdot L^{1-\alpha} \qquad y = k^{\alpha}$$

$$\frac{dY}{dK} = \alpha \cdot \frac{K^{\alpha-1}}{L^{\alpha-1}} = \alpha \cdot k^{\alpha-1} \qquad \frac{dy}{dk} = \alpha \cdot k^{\alpha-1}$$

- **What this tells us is that increases in the capital stock per worker increase output per worker, but each successive increase in the capital stock yield ever smaller increases in output per worker – because output per worker exhibits diminishing marginal returns.**

Accumulation of Capital

- **The underlying theory behind the Solow Model:**
  - **countries with higher levels of capital per worker**
  - **have higher levels of output per worker.**

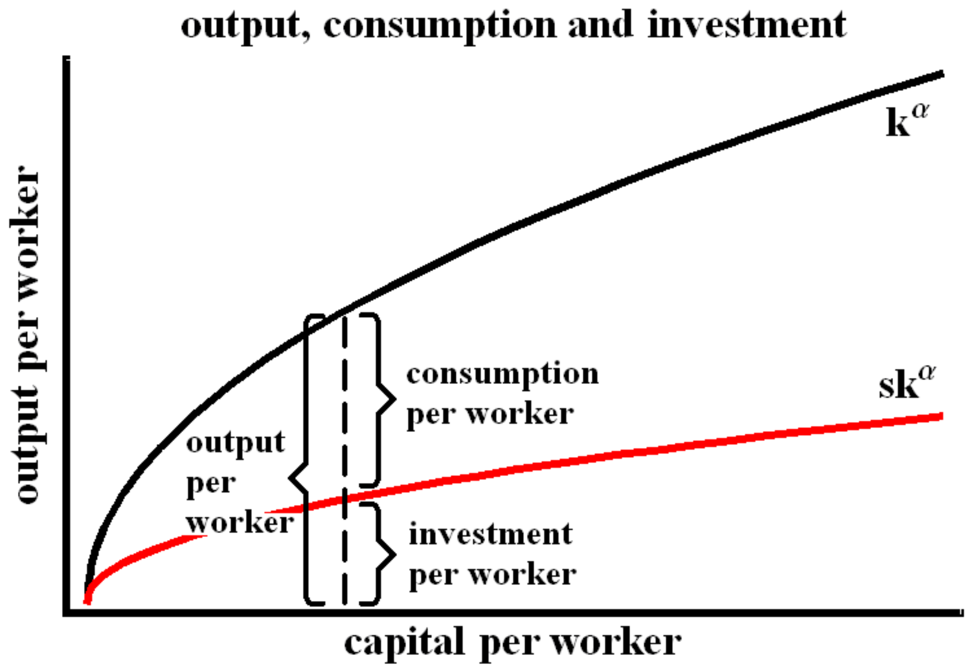
- **Think about that a second.**
- **If Solow's theory is correct, then all we have to do to increase output per worker – and lift billions of people out of poverty – is increase the amount of capital that they have to work with.**

- **So what determines the level of capital per worker in a country?**
- **The Solow Model assumes that:**
  - **investment increases the capital stock, but**
  - **a constant fraction of the capital stock depreciates each year**

- **Our definition of investment,** $i = s \cdot y$ **implies that annual investment in capital is a fraction,** s**, of the total output per year, i.e.** $I = s \cdot Y$
- **Let** δ **denote the fraction of the capital stock that depreciates in a year.**

- **Therefore:** $\dot{K} = sY - \delta K$ **where:** $\dot{K} = \frac{dK}{dt}$

Tradeoff between Consumption and Investment



- **The saving rate s determines the allocation of output per worker between consumption and investment.**

Evolution of Capital per Worker

- **Now that we now how the total capital stock evolves from year to year, finding out how the capital stock per worker evolves from one year to the next is straightforward.**
- Recall from the Calculus Tricks that the percentage change in a ratio is equal to the percentage change in the numerator minus the percentage change in the denominator.
- **So we can find the evolution capital per worker over time:**

$$\frac{\dot{k}}{k} = \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{k} = \frac{K}{L} \cdot \left(\frac{sY - \delta K}{K} - \frac{\dot{L}}{L}\right)$$

$$= \frac{sY - \delta K}{L} - k \cdot \frac{\dot{L}}{L}$$

$$\text{define: } n \equiv \frac{\dot{L}}{L} \qquad \dot{\mathbf{k}} = \mathbf{s}\mathbf{k}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{n}) \cdot \mathbf{k}$$

- Note that: n is the constant **exogenous** annual growth rate of the labor force
- the model assumes that there are no cyclical fluctuations in employment

Key Equation of the Solow Model

$$\dot{\mathbf{k}} = \mathbf{sk}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{n}) \cdot \mathbf{k}$$

- **Growth of the capital stock per worker over time,** $\dot{k}$
  - **is an increasing function of investment per worker, i.e.** sk^{α}
  - **a decreasing function of the depreciation rate** and
  - **a decreasing function of the growth rate of the labor force**

- **Although we've used math to obtain this result, the result should also be intuitive:**
  - **The capital stock per worker increases at higher saving rates because at higher saving rates more output is being devoted to accumulating capital.**
  - **By definition, depreciation decreases the capital stock, so faster rates of depreciation reduce the capital stock per worker.**
  - **Faster rates of growth of the labor force will also lead to lower levels of capital per worker, because the total capital stock must be spread over a larger labor force.**

Evolution of Capital per Worker

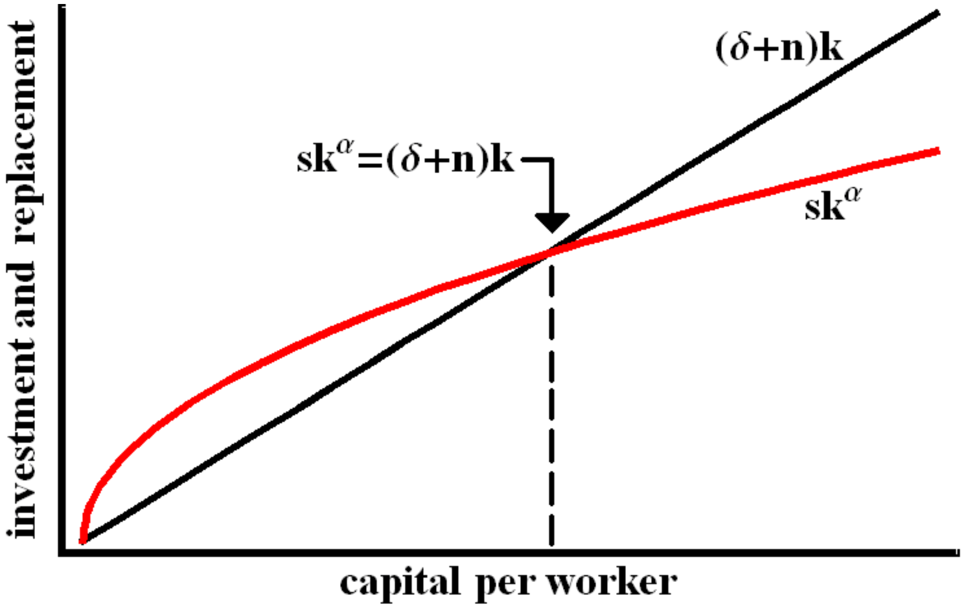
Whether capital per worker is growing, falling or remaining constant over time, depends on whether investment in new capital per worker exceeds, falls short of or is equal to the replacement requirement: $(n + \delta) \cdot k$.

- **if** $sk^{\alpha} > (n + \delta) \cdot k$**, then capital per worker increases over time**
  - in this case, **investment in new capital per worker exceeds the replacement requirement** and
  - **output per worker is growing over time**

- **if** $sk^{\alpha} < (n + \delta) \cdot k$**, then capital per worker decreases over time**
  - in this case, **investment in new capital per worker falls short of the replacement requirement** and
  - **output per worker is falling over time**

- **if** $sk^{\alpha} = (n + \delta) \cdot k$**, then capital per worker remains constant over time**
  - in this case, **investment in new capital per worker equals the replacement requirement** and
  - **output per worker is constant over time**
  - **this is called the steady state** (since the level of capital per worker is "steady")

Steady State



- **The capital per worker must converge to the steady state**
- **Once capital per worker converges to the steady state level it remains at that level,** unless the saving rate, depreciation rate or growth rate of the labor force changes.

Convergence to the Steady State

- **As an example of convergence consider an economy that initially starts at a level of capital per worker that is below the steady state level.**
- **If initial** $k=1$ **and if** $\alpha=0.5$ $y=k^{0.5}$ $s=0.08$ $\delta=0.02$ and $n=0.02$

year	k	y	$c=(1-s)y$	$i=s\cdot k^{\alpha}$	$(\delta+n)\cdot k$	Δk
0	1.000	1.000	0.920	0.080	0.040	0.0400
1	1.040	1.020	0.938	0.082	0.042	0.0400
2	1.080	1.039	0.956	0.083	0.043	0.0399
3	1.120	1.058	0.974	0.085	0.045	0.0399
4	1.160	1.077	0.991	0.086	0.046	0.0398
5	1.200	1.095	1.008	0.088	0.048	0.0396
…						
10	1.396	1.182	1.087	0.095	0.056	0.0387
…						
25	1.945	1.394	1.283	0.112	0.078	0.0338
…						
100	3.484	1.867	1.717	0.149	0.139	0.0100
…						
∞	4.000	2.000	1.840	0.160	0.160	0

- After 35 years, this economy's level of output per worker will have converged halfway to its steady state level – i.e. $y=1.5$ at 35 years.

Steady State

- **Once the economy has converged to its steady state, the level of capital per worker stops growing** (or falling as the case may be)**, i.e.** $\dot{k} = 0$
- **At the steady state:** $sk^{\alpha} = (\delta + n) \cdot k$**. If we solve this equation for** k**, we find the steady state level of capital per worker:**

$$k_{SS} = \left(\frac{s}{n + \delta} \right)^{\frac{1}{1-\alpha}}$$

- **which implies that the steady state level of output per worker is:**

$$y_{SS} = \left(\frac{s}{n + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- **In the long-run, the steady-state level of output per worker is constant and depends only on:**
    - **the saving rate**
    - **the labor force growth rate and**
    - **the rate at which capital depreciates**

Economic Growth

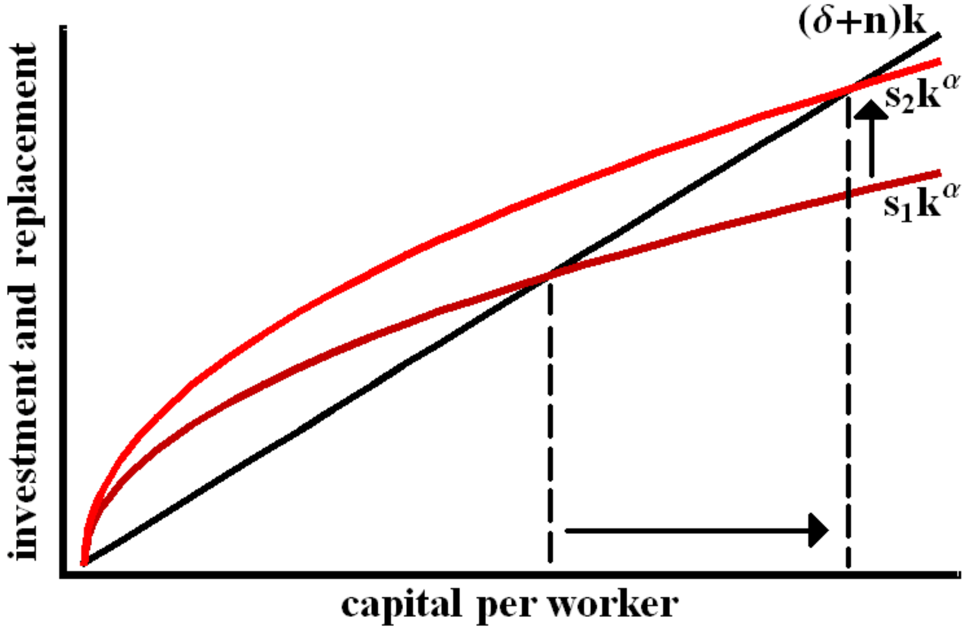
- **The interesting thing about this model of economic growth is that there is no growth once the economy reaches steady state.**
- **But what do politicians say?**
    - Politicians say tax cuts will be great for economic growth
    - Politicians say protecting open space will be great for economic growth
    - Politicians say building a new stadium will be great for economic growth
- **Are they lying?**

- **Public policy cannot affect the steady state growth rate.**
- **But public policy can affect the steady state level of output per worker, which will affect living standards.**

- **If policymakers found a way to increase the saving rate the economy will converge to a higher steady state level of output per worker.**

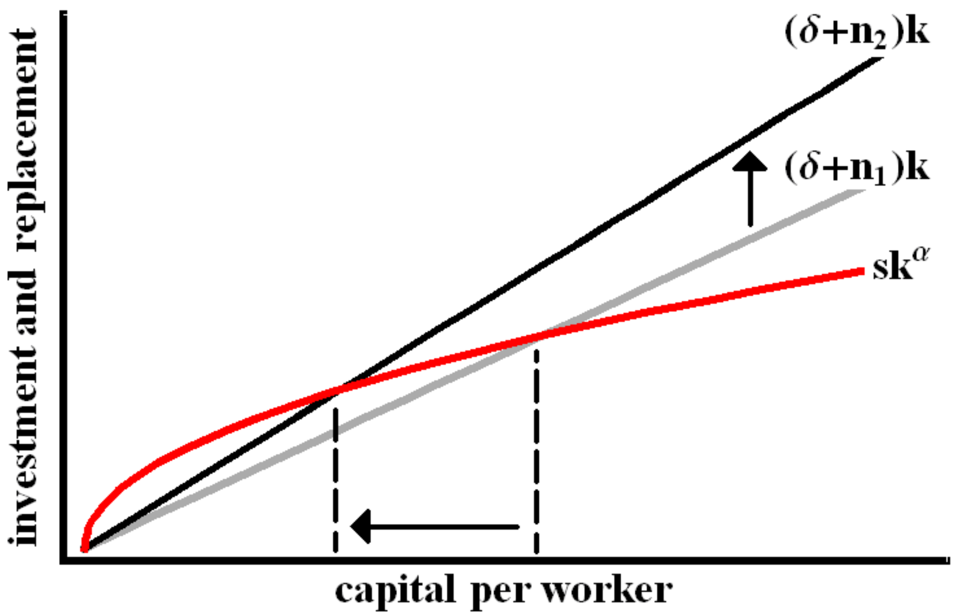
- **Conversely, if policymakers pursued a policy that increased the labor force growth rate, then the economy would converge to a lower steady state level of output per worker.**

Increasing the Saving Rate



- **Many economists favor low corporate tax rates as a way to encourage saving, in the hope that lower rates will stimulate savings/investment.**
- **At a higher saving rate, the economy will converge to a higher steady state level of output per worker.**

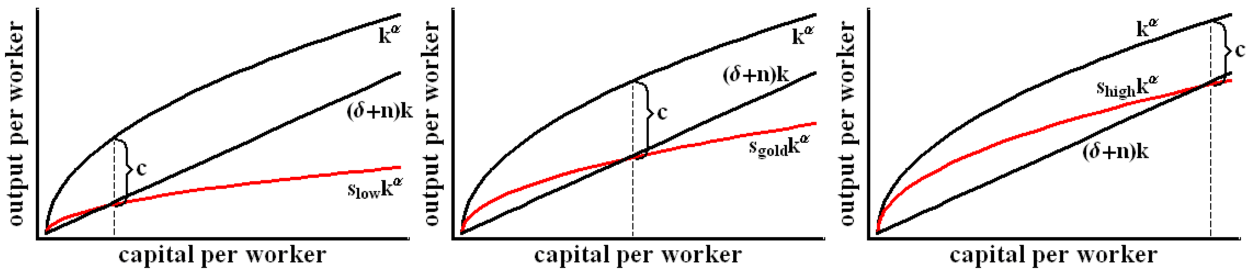
Increasing the Labor Force Growth Rate



- **One reason living standards are low in some countries is because they have high rates of population growth** (high rates of labor force growth)**.**
- **At a higher labor force growth rate, the economy will converge to a lower steady state level of output per worker.**

the Golden Rule

- **So far, it would appear that the goal of public policy should be to reach a higher steady state level of output per worker**
- **In practice, that should be the goal – our saving rate is too low – but …**
- **If a benevolent policymaker could choose the saving rate – which would enable him/her to choose the steady state level of output per worker, then which steady steady state should he/she choose?**
    - Extreme example #1: You wouldn't want a saving rate of 1%
    - Extreme example #2: You wouldn't want a saving rate of 99%



- **If the policymaker followed the Golden Rule of "Do unto others …" then he/she would want to choose the steady state with the highest level of consumption per worker.** This case is depicted in the middle panel.

the Golden Rule

- Using our Calculus Tricks, we can find the saving rate which maximizes consumption across steady states in the same way that we found a firm's maximum profit:
    - take the derivative of consumption with respect to the saving rate
    - and set it equal to zero
- This time it's easier because we can use the chain rule:

$$\max_{s} \quad c_{SS}(s) = k_{SS}(s)^{\alpha} - (n+\delta)\cdot k_{SS}(s)$$

$$c'_{SS}(s) = \left(\alpha \cdot k_{SS}^{\alpha-1} - (n+\delta)\right)\cdot k'_{SS}(s) = 0 \quad \Rightarrow \quad \alpha \cdot k_{GOLD}^{\alpha-1} = (n+\delta)$$

- In other words, the Golden Rule steady state level of capital per worker corresponds to level of capital per worker which equates the Marginal Product of Capital to $(n+\delta)$.

- **The Golden Rule level is a CHOICE.**
- **The economy does NOT converge to the Golden Rule level on its own!**

Technological Progress

- **You may have noticed that if the steady state level of output per worker is constant, then:**
  - **as the economy approaches steady state**
  - **growth of output per worker is zero and therefore**
  - **growth of income per worker is zero**

- **Is this realistic? No.**

- **We can introduce more realism into the model if we introduce technological progress into the model.**
- Technological progress occurs when firms find ways to produce more from the same amount of resources.
- **If we define a variable A to denote the efficiency of labor**
  - **which reflects society's knowledge about production methods or**
  - **which reflects improvements in the health, education of skills of the labor force**
- **then as the available technology improves, the efficiency of labor rises.**

Technological Progress

- **So redefine the production function as:**

$$Y = K^{\alpha} \cdot (AL)^{1-\alpha} \qquad \text{where: } 0 < \alpha < 1$$

- **Once again, we want to focus on per worker variables, but now we have to focus on labor in efficiency units, so define:**
  - **output per unit of effective labor** as: $\tilde{y} \equiv Y/AL$ and
  - **capital per unit of effective labor** as: $\tilde{k} \equiv K/AL$

- **Since the production function still assumes constant returns to scale, so we can define output per unit of effective labor in terms of capital per unit of effective labor:**

$$\left.\begin{aligned} \frac{Y}{AL} &= K^{\alpha} \cdot \frac{(AL)^{1-\alpha}}{AL} \\ &= K^{\alpha} \cdot (AL)^{-\alpha} \end{aligned}\right\} \Rightarrow \quad \tilde{y} = \tilde{k}^{\alpha}$$

Evolution of Capital per unit of Effective Labor

- **Using the Calculus Tricks once again, we can find the evolution capital per unit of effective labor over time:**

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \left(\frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{\tilde{k}} = \frac{K}{AL} \cdot \left(\frac{sY - \delta K}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right)$$

$$\text{define: } g \equiv \frac{\dot{A}}{A} \qquad = \frac{sY - \delta K}{AL} - \tilde{k} \cdot \left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right)$$

$$\text{define: } n \equiv \frac{\dot{L}}{L} \qquad \dot{\tilde{\mathbf{k}}} = \mathbf{s}\tilde{\mathbf{k}}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{g} + \mathbf{n}) \cdot \tilde{\mathbf{k}}$$

- Note that: g is the **exogenous** annual growth rate of technological progress
  n is the **exogenous** annual growth rate of the labor force

- We're assuming that technology grows at a constant rate and that there are no cyclical fluctuations in the level of technology.
- We're still assuming that the labor force grows at a constant rate and that there are no cyclical fluctuations in employment.

Key Equation of the Solow Model with Technological Progress

$$\dot{\tilde{\mathbf{k}}} = \mathbf{s}\tilde{\mathbf{k}}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{g} + \mathbf{n}) \cdot \tilde{\mathbf{k}}$$

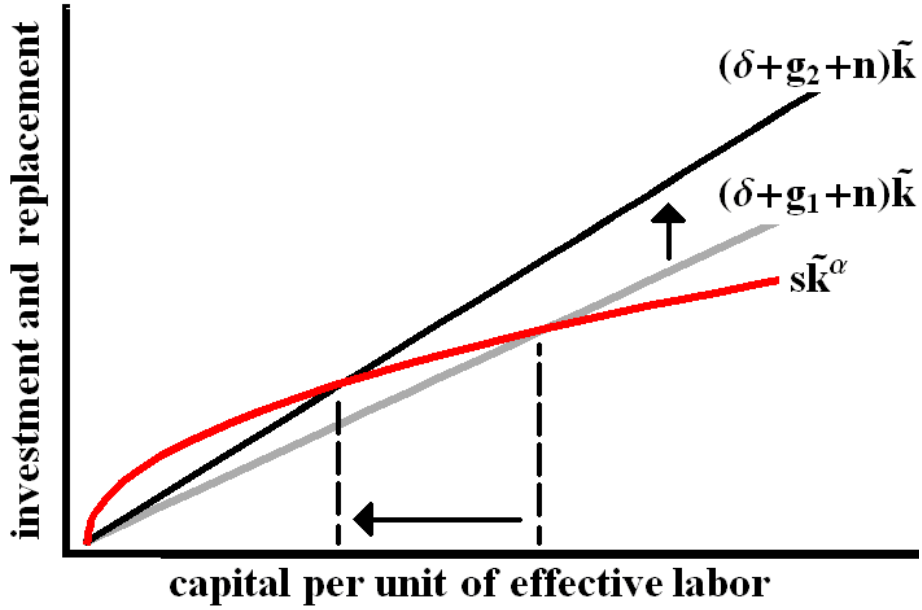
- **This "Key Equation" has the same interpretation as the previous one but this time:**
  - **growth of the capital stock per unit of effective labor over time, $\dot{\tilde{k}}$,**
  - **is a decreasing function of the growth rate of technological progress, g.**
- **And this time the steady state level of the capital stock per unit of effective labor, will occur when:**

$$\dot{\tilde{k}} = 0 \Rightarrow s\tilde{k}^{\alpha} = (\delta + g + n) \cdot \tilde{k}$$

- **Solve this equation for $\tilde{k}$, we can find obtain the steady state levels of capital and output per unit of effective labor:**

$$\tilde{k}_{SS} = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \quad \textbf{and} \quad \tilde{y}_{SS} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Increasing the Rate of Technological Progress



- **If policymakers were able to find a way to increase the rate of technological progress, the steady state level of capital per unit of effective labor would fall, but …**
- **… this would be a GOOD THING**

the Rate of Technological Progress

- **A faster rate of technological progress would lower the steady state level of capital per unit of effective labor, but …**
- **… this would be a GOOD THING**

- **The rate of growth of output per unit of effective labor is:**

$$\frac{\dot{\tilde{y}}}{\tilde{y}} = \frac{\dot{Y}}{Y} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$
$$= \frac{\dot{Y}}{Y} - g - n$$

- **The rate of growth of output per worker is:**

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$
$$= \frac{\dot{Y}}{Y} - n$$

- **In steady state, the growth rate of output per unit of effective labor is zero, but this implies that the rate of growth of output per worker is equal to the rate of growth of technological progress.**

$$\frac{\dot{\tilde{y}}}{\tilde{y}} = 0 \;\Rightarrow\; \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} \quad \text{or more simply}: \; \frac{\dot{y}}{y} = g$$

- **So a faster rate of growth of technological progress implies a rapidly rising standard of living for the residents of that economy**

Homework #4

1. In its introduction to the Solow Model without technological progress, Lecture 5 contains a derivation of the marginal product of capital.

a. If the production function is given by: $Y = K^{\alpha} L^{1-\alpha}$, what is the marginal product of <u>labor</u>?

b. Assuming that $\alpha = 0.5$ and that $K = 1$, calculate the marginal product of labor from one unit of labor input to five units. Hint: Use a calculator!

c. On a graph, plot the marginal product of labor using the values you just calculated.

d. Assuming that $\alpha = 0.5$ and that $K = 2$, calculate the marginal product of labor from one unit of labor input to five units.

e. On the <u>same</u> graph, plot the marginal product of labor using the values you just calculated.

f. What happens to the marginal product of labor when the economy's stock of capital increases?

2. In Lecture 2, you learned that a firm hires labor up to the point where the wage equals the price times the marginal product of labor (MPL), i.e. $w = p \cdot MPL$, where labor is supplied at wage rate, w, and the labor demand is given by $p \cdot MPL$. Since we're now discussing economy-wide aggregates, it's convenient to normalize the price level to $p = 1$.

a. If we assume that the wage rate, w, is constant at a given point in time, then how will the quantity of labor that the economy demands respond to a sudden increase in the capital stock?

b. If we assume that the quantity of labor supplied, L, is constant at a given point in time, then how will the wage rate respond to a sudden increase in the capital stock?

c. Which assumption does the Solow Model make?

3. In 2003, Pres. George W. Bush convinced Congress to reduce the maximum tax rate that shareholders pay on dividends from 38.6 percent to 15 percent. In lobbying for this measure, he argued that cutting the tax would encourage people to invest more – i.e. increase the economy's saving rate.

Opponents of the policy argued that cutting the tax on dividends was a giveaway to Pres. Bush's rich friends and that it would not benefit workers.

Answer the following questions using the Solow Model without technological progress. Throughout the problem, assume that the U.S. economy was in steady state when Pres. Bush announced his dividend tax plan. Until part **e.**, assume that Pres. Bush's tax policy would increase the saving rate.

a. Under what condition would Pres. Bush's tax policy increase steady-state consumption per worker? Under what condition would it decrease steady-state consumption per worker?

b. How would the marginal product of labor differ between the initial steady state and the one to which the economy will converge to after reduction of the tax on dividends?

c. How would Pres. Bush's tax policy affect wages, w? **Hint:** remember that: $w = p \cdot MPL$

d. Given your answers to the previous three questions, was Pres. Bush's tax policy a giveaway to the rich without any benefit for workers?

e. Now assume that Pres. Bush's tax policy would <u>not</u> increase the saving rate. Under this assumption, was the tax policy a giveaway to the rich without any benefit for workers?

Lecture 5

Economic Growth: Transition Dynamics

Eric Doviak

Economic Growth and Economic Fluctuations

Saving

- **In our discussion of the Solow model, we assumed that:**
  - **annual physical capital investment is a fraction, s, of the total output per year, i.e.** $I = s \cdot Y$ and
  - **a fraction, δ, of the capital stock depreciates each year**
- **So once again:** $\dot{K} = sY - \delta K$
- **In practice, the saving rate depends on:**
  - **the decisions of individuals within the economy**
  - **government decisions about how much to collect in tax revenue and how much to spend**
- **Government Saving is the difference between Tax Revenues, T, and Government Purchases, G, so we can define the government saving <u>rate</u> as:** $s_G \equiv (T - G)/Y$
- **If the government is running a budget deficit, then s_G is negative**
- **If we define s_P as the "private" saving rate, then the economy's saving rate is:** $s \equiv s_G + s_P$

Saving

- **Now consider the table below** (sources: BEA and BLS).
  - "Gross Saving Rate" is a measure of s
  - "Government Saving Rate" is a measure of s_G
  - "Net Nonresid. Invest. Rate" is a measure of $\dot{K}/Y$
  - the last row gives the growth rate of the civilian noninstitutional population aged 20 to 64 – a measure of n , the labor force growth rate

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Gross Saving Rate*	18.2	18.6	18.1	17.8	16.2	14.6	13.9	14.4	14.9	15.9
Government Saving Rate*	-0.3	0.9	1.5	2.2	0.2	-2.9	-3.7	-3.2	-2.0	-1.1
Net Nonresid. Invest. Rate*+	3.7	4.1	4.2	4.4	3.3	2.0	1.7	2.0	2.3	2.7
%Δ Civ. Nonist. Pop. 20-64	1.2	1.0	1.2	–	1.4	1.4	1.8	1.0	1.2	1.1

* as a percentage of Gross National Income
+ "Net Private Domestic Nonresidential Fixed Investment Rate"

- **Government saving fell dramatically – a result of:**
  - fluctuations – the recession of the early 2000s reduced tax revenues
  - the wars in Iraq and Afghanistan and
  - **TAX CUTS**
- **Due to the fall in saving, net capital investment had already fallen prior to the financial crisis of the late 2000s.**
- **All else equal this reduces steady state output per worker.**

Steady State Income per Worker

- **I don't know how much each of the aforementioned factors contributed to the growth of the federal budget deficit**
- **but the focus of this lecture will be on why we might prefer low saving rates, even though low saving rates lead to low steady state levels of consumption per worker**
- **To illustrate this preference, I'll:**
  - **use the Solow Model without technology,** i.e.: $A = 1$, $g = 0$ and
  - **focus on the effect of tax cuts on the saving rate**

DISCLAIMER

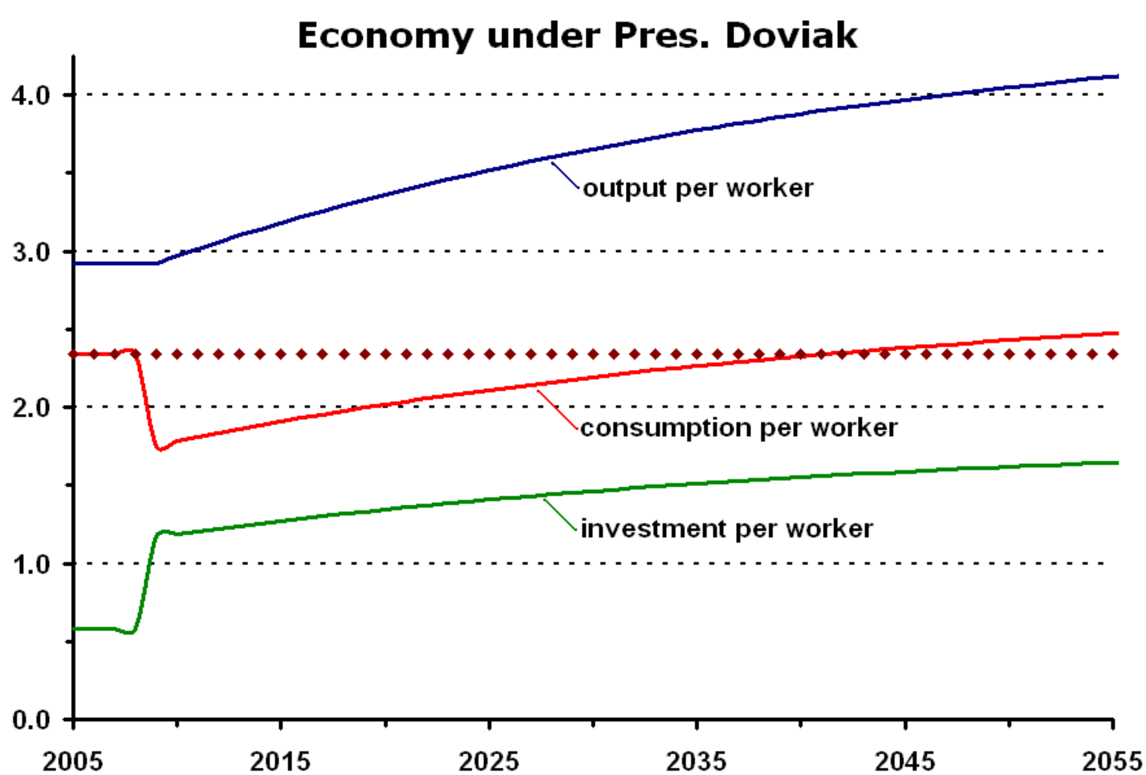
- Before diving into the discussion, I want to emphasize that empirical evidence suggests that:
  - high personal and corporate income tax rates may discourage net capital investment and thus lower steady state output per worker
  - although potentially beneficial, major tax reforms designed to increase steady state output per worker will be <u>not</u> be self-financing and
  - well-designed government spending can also increase steady state output per worker

Doviak for President

Investing for the Future

- **Currently, our nation saves a mere 20 percent of its output and finds itself at a steady state level of consumption per worker that is 17 percent below the Golden Rule level**
- **We could now be enjoying a much higher standard of living had budget deficits not crowded out capital investment all these years**
- **To reach the Golden Rule level however, we need to double our saving rate by repealing Pres. Bush's tax cuts**
- **The tax increases I propose will immediately reduce your consumption 29 percent, but don't worry …**
- **the capital investments resulting from the higher saving rate will increase output per worker over time and by the year 2042 your consumption will have returned to its current level**
- **And it will keep growing over time enabling your great-grandchildren to enjoy the highest possible level of consumption per worker, given the rate at which capital depreciates and our rate of labor force growth**

Note: all numbers cited in this mock campaign speech are fictional.

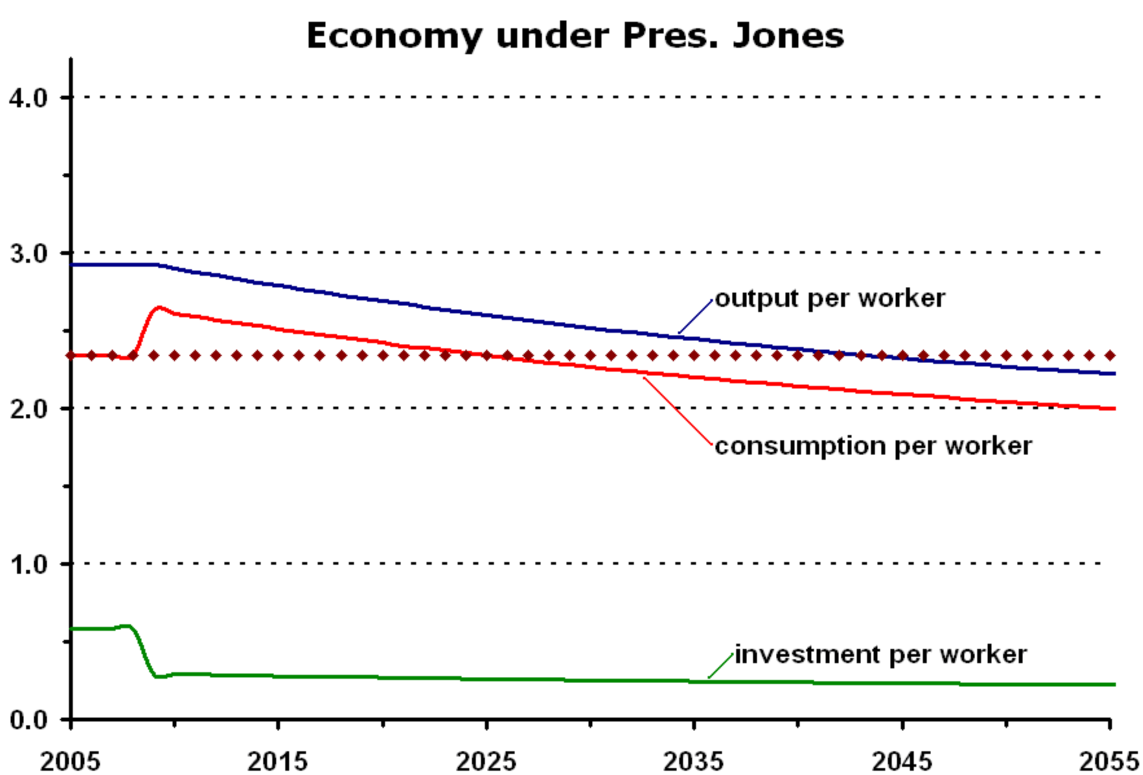


- **The increase in the saving rate causes consumption to drop immediately.**
- **The fall in consumption is matched by an increase in investment.**
- **Over time output, consumption and investment all increase together.**

Jones for President

More Lunches in Every Pail

- **Saving Schmaving!**
- **What's all this gobble-de-gook about a Golden Rule?**
- **There's only one Golden Rule – the American people need more gold**
- **Don't listen to this Ivory Tower Elitist! He's outta touch with reality.**
- **Real people need more consumption now, not 40 years from now!**
- **Let him scratch Greek letters on a college blackboard, but don't let him run your economy!**
- **As your next president, my tax cuts will be so deep that the national saving rate will fall by half!**
- **And when I cut the saving rate in half, you'll consume more than you did before!**



- **The decrease in the saving rate causes consumption to rise immediately.**
- **The increase in consumption is matched by a fall in investment.**
- **Over time output, consumption and investment all decrease together.**

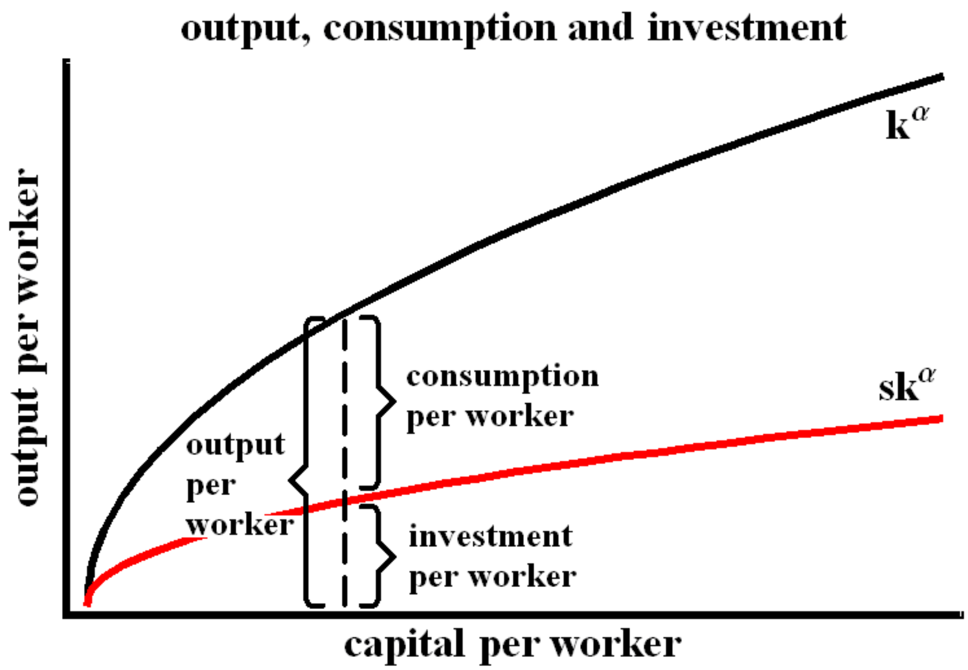
So who would you vote for?

- **You'd vote for Jones.**
- **He immediately increases your consumption 12 percent and your consumption doesn't slip below its original level until 2026.**

- **So how does he get away with it?**
- **Remember from Lecture 5 that there's a tradeoff between consumption and investment.**
- **Each level of capital per worker corresponds to a unique level of output per worker and the saving rate s determines the allocation of output between consumption and investment**

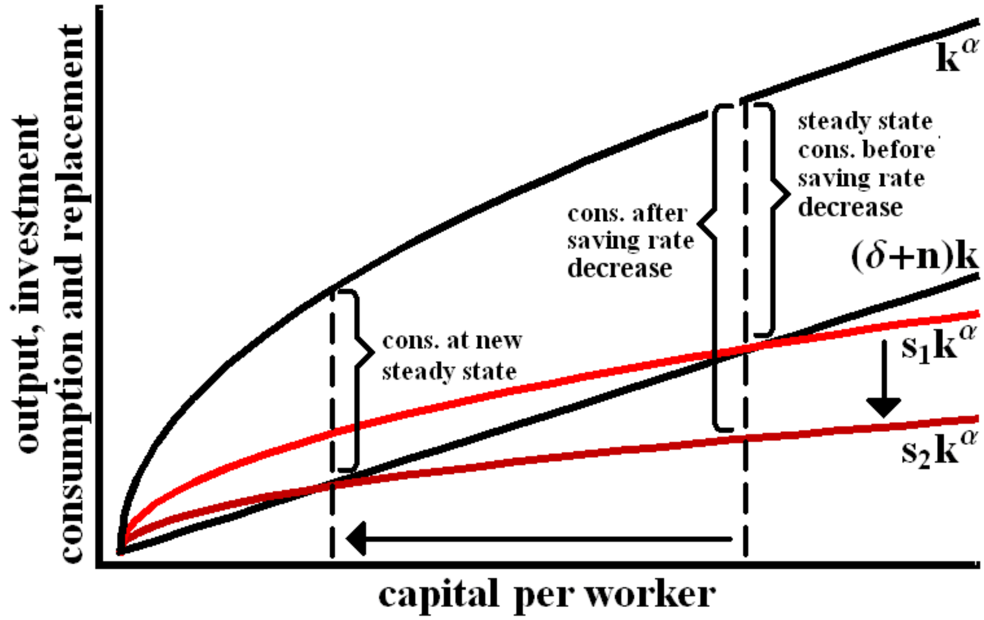
- **The level of consumption is free to vary but capital must be accumulated** (or depleted) **<u>over</u> <u>time</u> therefore:**
  - **on any given day, you can decide to consume more or less than you did the day before, but**
  - **to consume more, you save less – which will decrease the rate of capital accumulation**
  - **to consume less, you save more – which will increase the rate of capital accumulation**

Tradeoff between Consumption and Investment



- **the saving rate s determines the allocation of output between consumption and investment**

Effect of Jones' Policy



- **The decrease in the saving rate causes consumption to rise immediately.**
- **The increase in consumption is matched by a fall in investment.**
- **Over time output, consumption and investment all decrease together.**

Appendix

- **At the beginning of the lecture, I wrote that the economy's saving rate,** s**, is the sum of the private saving rate,** s_P**, and the government saving rate,** s_G**, so that:** $s \equiv s_G + s_P$
- **Recalling from Lecture 4 that when Net Exports are zero, then:**

$$I = Y - C - G$$

$$I = (Y - T - C) + (T - G)$$

$$I = \text{private saving} + \text{government saving}$$

- **Now if we define:** $s_P \equiv \dfrac{Y-T-C}{Y}$ $\quad s_G \equiv \dfrac{T-G}{Y}$ **then** $I = (s_P + s_G)\cdot Y$
- **These equations give the false impression that taxation has no effect on national saving because I haven't defined the consumption function yet.**
- **If we assume that:** $C = a + b\cdot(Y - T)$ **where:** $0 \le a$ **and** $0 < b < 1$

  then national saving is an increasing function of Tax Revenues since:

$$s = \frac{Y - T - a - b\cdot(Y-T)}{Y} + \frac{T-G}{Y} \quad \Rightarrow \quad \frac{d\,s}{d\,T} = \frac{b}{Y} > 0$$

Homework #5

1. Suppose that an economy was initially in steady state when part of its capital stock is destroyed by war. Assume that none of its residents are killed by the war. Use the Solow Model without technological progress to answer the following questions.
    a. What is the immediate impact on total output?
    b. What is the immediate impact on output per worker?
    c. Assuming that the country's saving rate remains unchanged, what happens to:
        - output per worker in the postwar economy?
        - investment per worker in the postwar economy?
        - consumption per worker in the postwar economy?

       Illustrate your answers with diagrams
    d. Is the growth rate of output per worker in the postwar economy greater or smaller than it was before the war?

1. Suppose that an economy was initially in steady state when many of its residents are killed by a war. Assume that none of its capital stock is destroyed by the war. Use the Solow Model without technological progress to answer the following questions.
    a. What is the immediate impact on total output?
    b. What is the immediate impact on output per worker?
    c. Assuming that the country's saving rate remains unchanged, what happens to:
        - output per worker in the postwar economy?
        - investment per worker in the postwar economy?
        - consumption per worker in the postwar economy?

       Illustrate your answers with diagrams
    d. Is the growth rate of output per worker in the postwar economy greater or smaller than it was before the war?

Lecture 6

Economic Growth: Human Capital

Eric Doviak

Economic Growth and Economic Fluctuations

Why Study Other Growth Models?

- **Because the Solow Model comes up short.**
- **Recall from Lecture 2 that in the Long-Run:**
  - **firms hire labor up to the point where the wage equals price times marginal product of labor (MPL):** $w = p \cdot MPL$
  - **firms hire capital up to the point where the rental rate on capital equals price times marginal product of capital (MPK):** $r = p \cdot MPK$
- **Next recall from Lecture 4, the production function that incorporates technological progress:**

$$Y = K^{\alpha} \cdot (AL)^{1-\alpha}$$

- **The MPL of this production function is:**

$$MPL = (1-\alpha) \cdot K^{\alpha} \cdot A^{1-\alpha} \cdot L^{-\alpha}$$

- **The MPK of this production function is:**

$$MPK = \alpha \cdot K^{\alpha-1} \cdot (AL)^{1-\alpha}$$

- **Finally, recall from Lecture 3, that if economic profit is zero, then:**

$$p \cdot Y = r \cdot K + w \cdot L$$

Why Study Other Growth Models?

- **Now bring all three of those conditions together:**

$$p \cdot Y = r \cdot K + w \cdot L$$
$$p \cdot Y = p \cdot MPK \cdot K + p \cdot MPL \cdot L$$
$$Y = \alpha \cdot K^{\alpha-1} \cdot (AL)^{1-\alpha} \cdot K + (1-\alpha) \cdot K^{\alpha} \cdot A^{1-\alpha} \cdot L^{-\alpha} \cdot L$$
$$Y = \alpha \cdot Y + (1-\alpha) \cdot Y$$

- **This implies that:**
  - **Capital's share of national income is equal to α and**
  - **Labor's share of national income is equal to $(1-\alpha)$**
- **In practice, we know that capital's share of national income is roughly constant across countries and approximately one-third, i.e.** $\alpha \approx 1/3$
- **Next, consider two countries. According to the Penn World Tables, in 2000, real GDP per worker was $64,437 in the US and $1479 in Nigeria.**
- **Now if the US and Nigeria share the same technology and if differences in capital per worker explain differences in output per worker and if** $\alpha \approx 1/3$, **then capital per worker is 83,077 times higher in the US than it is in Nigeria.**
- **Does that look right to you? … I didn't think so.**

What's wrong with the Solow Model?

- **The Solow Model that we examined in the previous lecture assumes that output is produced using:**
  - **physical capital** (i.e. machinery, buildings, etc.)
  - **human labor**
- In an extension of that model, we also incorporated technological progress
- **If we were to use the Solow Model to examine levels of output across countries, then we would have to assume that there are no difference in human labor across countries** (i.e. that human labor is homogenous)
- **Is that assumption realistic? No.**
- **If you ever work with a poorly educated person, you'll notice that they're much less productive than you are.**
- **When a complication arises in the task that they are performing, they don't know what to do and often make bad decisions.**
- **Models of economic growth that incorporate "Human Capital" attempt to examine differences in education levels.**

What is Human Capital?

- **The Solow Model that we examined in the previous lecture assumes that the Marginal Product of Labor is positive.**
- **But what would be the marginal product of a person without any child-rearing or education at all?** (i.e. someone who was "raised by wolves").
- **It would be virtually zero.**
- **In this sense, all of the returns to human labor must reflect returns to human capital.**

- **If we assume that there is some minimum level of human capital that human beings acquire more or less automatically** (e.g. the ability to walk and talk, etc.)**, then:**
  - **we can interpret this minimum level as the input of "raw labor"**
  - **and separately examine differences in output levels that occur because some societies have higher average levels of human capital than others**

the Mankiw-Romer-Weil Model

- **In 1992, N. Gregory Mankiw, David Romer and David N. Weil published a variant of the Solow Model that better explains cross-country differences in GDP per worker.**
- **In particular, they explored the relationship between cross-country differences in human capital per worker and cross-country differences in output per worker.**
- **Like Solow, they divide output among consumption and investment:**

$$Y = C + I$$

- **but they further divide investment into investment in physical capital and investment in human capital:**

$$I = I_K + I_H$$

- **and they assume that output is produced using physical capital,** K**, human capital,** H**, and effective labor,** AL

$$Y = K^{\alpha} \cdot H^{\beta} \cdot (AL)^{1-\alpha-\beta} \quad \text{where:} \quad \begin{matrix} 0 < \alpha < 1 \quad 0 < \beta < 1 \\ 0 < \alpha + \beta < 1 \end{matrix}$$

Physical and Human Capital

- **The underlying theory behind the Mankiw-Romer-Weil Model:**
  - **countries with higher levels of:**
- **technology,**
- **physical capital per worker and**
- **human capital per worker**
  - **have higher levels of output per worker.**

- **If their theory is correct, then all we have to do to increase output per worker – and lift billions of people out of poverty – is:**
  - **provide them with technological "know-how"**
  - **increase the amount of physical capital they have to work with and**
  - **provide them with more schooling – to increase their levels of human capital**

MRW Steady State

- **In the Mankiw-Romer-Weil Model, the economy must converge to a steady state where:**
  - **physical capital per unit of effective labor is constant over time**
  - **human capital per unit of effective labor is constant over time**

- **For example, imagine a country devastated by war and emigration:**
  - **its physical capital stock was destroyed by bombing campaigns**
  - **its human capital stock was depleted by the emigration of its best and brightest to America**
- **Is the country now doomed to perpetual poverty? No.**

- **If the economy devotes a large share of its** (substantially reduced) **output to investment in new physical and human capital, then:**
  - **over time it will replace its lost physical capital stock**
  - **over time it will replace its lost human capital stock**
  - **over time it will converge to a higher steady state level of output per unit of effective labor**

- **This is an incredibly optimistic model!**
- **All a country needs is high saving rates!**

MRW Steady State

- **Now, imagine a very rich country:**
  - **it has so many factories and machines that its physical capital stock** (per unit of effective labor) **is the highest in the world**
  - **it boasts the best universities in the world and its human capital stock** (per unit of effective labor) **is also the highest in the world**
- **Will this country always be the richest in the world? Not necessarily.**
- **If the residents of this country suddenly become decadent and consume all of their output and stop investing new physical and human capital, then over time:**
  - **its physical capital stock** (per unit of effective labor) **will diminish**
  - **its human capital stock** (per unit of effective labor) **will diminish**
  - **over time it will converge to a lower steady state level of output per unit of effective labor**

"Lazy hands make a man poor, but diligent hands bring wealth."
– Proverbs 10:4

So what do we want?

- **If we want our economy's living standards to be higher, then we want:**
  - **a higher physical capital saving rate,** s_K
  - **a higher human capital saving rate,** s_H
  - **a lower rate of physical and human capital depreciation,** δ
  - **a lower labor force growth rate,** n
  - **a HIGHER rate of technological progress,** g
- All of these should be intuitive, although the last "want" – a higher rate of technological progress – can be confusing.
- After all, doesn't a higher rate of technological progress reduce the steady state level of output per unit of effective labor? Yes, but …
  - A person doesn't consume output per unit of effective labor
  - A person consumes output per worker
- Recall from Lecture 4 that when we incorporate technological progress into the model the steady state growth rate of output per worker is equal to the rate of growth of technological progress.
- **A faster rate of growth of technological progress implies a rapidly rising standard of living for the residents of that economy**

What factors affect a country's level of economic development?

	GDP per cap. at PPP USD	Human Dev. Index*	saving as % of GDP	annual pop. growth rate	net secondary school enrollment	Gender-Empower. Measure**	Imports as % of GDP	Exports as % of GDP	Gini Index***
	2002	2002	1990-2000	1975-2002	1990-91		2002	2002	
Norway	36,600	95.6%	33.8%	0.4%	88%	0.908	27%	41%	25.8%
Australia	28,260	94.6%	22.5%	1.3%	79%	0.806	22%	20%	35.2%
Sweden	26,050	94.6%	24.6%	0.3%	85%	0.854	37%	43%	25.0%
Canada	29,480	94.3%	26.4%	1.1%	89%	0.787	39%	44%	33.1%
Netherlands	29,100	94.2%	27.3%	0.6%	84%	0.817	56%	62%	32.6%
United States	35,750	93.9%	19.8%	1.0%	85%	0.769	14%	10%	40.8%
Japan	26,940	93.8%	33.8%	0.5%	97%	0.531	10%	11%	24.9%
Ireland	36,360	93.6%	30.4%	0.8%	80%	0.710	83%	98%	35.9%
United Kingdom	26,150	93.6%	17.4%	0.2%	81%	0.698	28%	26%	36.0%
Switzerland	30,010	93.6%	30.7%	0.5%	80%	0.771	38%	44%	33.1%
Finland	26,190	93.5%	27.7%	0.4%	93%	0.820	30%	38%	26.9%
Denmark	30,940	93.2%	26.8%	0.2%	87%	0.847	39%	45%	24.7%
New Zealand	21,740	92.6%	22.4%	0.8%	85%	0.772	32%	33%	36.2%
Greece	18,720	90.2%	13.0%	0.7%	83%	0.523	27%	21%	35.4%
South Korea	16,950	88.8%	37.6%	1.1%	86%	0.377	39%	40%	31.6%
Poland	10,560	85.0%	18.0%	0.5%	76%	0.606	31%	28%	31.6%
Hungary	13,400	84.8%	17.5%	–0.2%	75%	0.529	67%	64%	24.4%
Chile	9,820	83.9%	21.4%	1.5%	55%	0.460	32%	36%	57.1%
Costa Rica	8,840	83.4%	14.3%	2.6%	37%	0.664	47%	42%	46.5%
Mexico	8,970	80.2%	17.5%	2.0%	45%	0.563	29%	27%	54.6%
Panama	6,170	79.1%	20.7%	2.1%	50%	0.486	29%	28%	56.4%
Venezuela	5,380	77.8%	20.4%	2.5%	19%	0.444	17%	29%	49.1%
Paraguay	4,610	75.1%	4.2%	2.9%	26%	0.417	43%	31%	56.8%
Bolivia	2,460	68.1%	3.0%	2.2%	29%	0.524	27%	22%	44.7%
Botswana	8,170	58.9%	24.6%	2.8%	29%	0.562	37%	51%	63.0%
Bangladesh	1,700	50.9%	4.5%	2.4%	19%	0.218	19%	14%	31.8%

*The Human Development Index (HDI) is a composite index measuring average achievement in three basic dimensions of human development – a long and healthy life, knowledge and a decent standard of living.

**The Gender Empowerment Measure (GEM) is a composite index measuring gender inequality in three basic dimensions of empowerment – economic participation and decision-making, political participation and decision-making and power over economic resources.

***The Gini index measures inequality over the entire distribution of income or consumption. A value of 0% represents perfect equality, and a value of 100% perfect inequality.

Sources: Human Development Report (2003) and Penn World Table 6.1

What factors affect a country's level of economic development?

correlation coefficients	GDP per cap. at PPP	Human Dev. Index	Saving as % of GDP	annual pop. growth rate	net secondary school enroll.	Gender-Empower Measure	Imports as % of GDP	Exports as % of GDP	Gini Index
	2002	2002	1990-2000	1975-2002	1990-1991		2002	2002	
GDP per cap. at PPP	1.0000								
Human Dev. Index	0.8227	1.0000							
Saving as % of GDP	0.6778	0.6247	1.0000						
annual pop. growth rate	–0.7298	–0.7616	–0.5280	1.0000					
net secondary school enroll.	0.8487	0.8774	0.6824	–0.9011	1.0000				
Gender Empower Measure	0.8057	0.7345	0.4797	–0.5768	0.6580	1.0000			
Imports as % of GDP	0.1513	0.1477	0.1430	–0.1844	0.1082	0.1599	1.0000		
Exports as % of GDP	0.2904	0.2039	0.3832	–0.2348	0.1736	0.2921	0.9312	1.0000	
Gini Index	–0.6134	–0.5406	–0.3995	0.8162	–0.7285	–0.4320	–0.0829	–0.1168	1.0000

- As predicted by the Mankiw-Romer-Weil Model, saving (investment in physical capital) and education (investment in human capital) are positively correlated with a country's level of economic development, as measured both by per capita Gross Domestic Product (GDP) and the Human Development Index (HDI).
- As predicted by the Solow Model and the Mankiw-Romer-Weil Model, higher rates of population growth (which translate to higher rates of labor force growth) are negatively correlated with per capita GDP and HDI.
- The definition of women's empowerment that we used when discussing Homework #6 doesn't match this measure of empowerment. This measure looks at estimates of income based on gender as well as the percentage of female legislators, senior officials, managers, professionals etc. Nonetheless, this measure of women's empowerment is positively correlated with per capita GDP and HDI.
- Notice that women's empowerment is negatively correlated with the population growth rate.
- Larger shares of trade in GDP – as measured by both exports an imports – increases a country's level of economic development.
- Lastly, income inequality – as measured by the Gini Index – is highly correlated with both per capita GDP and HDI. The more unequal is a country's income distribution, the lower is its level of economic development.

Homework #6

1. According to a study released in 1997 by the National Center for Health Statistics, a woman's educational level is the best predictor of how many children she will have. The study found a direct relationship between years of education and birth rates, with the highest birth rates among women with the lowest educational attainment.

Assume that this finding is true for women all over the world and **comment** on why the UN Millennium Project – a body commissioned by the UN Secretary-General to advise on development strategies – recommended that the UN should:

> "Focus on women's and girls' health (including reproductive health) and education outcomes, access to economic and political opportunities, right to control assets, and freedom from violence."

a. In your answer, discuss how empowering women meets two of the conclusions of the Mankiw-Romer-Weil Model about the ways to improve living standards within a country.

b. In your answer, discuss two ways that empowering women can increase the level of human capital within the economy.

2. The 1983 Economic Report of the President contained the following statement: "Devoting a larger share of national output to investment would help restore rapid productivity growth and rising living standards." Do you agree with this claim? Explain.

Lecture 7

Economic Growth: Technological Progress

Eric Doviak

Economic Growth and Economic Fluctuations

Increasing Living Standards

- **If we want our economy's living standards to be higher, then we want:**
  - **a higher physical capital saving rate,** s_K
  - **a higher human capital saving rate,** s_H
  - **a lower rate of physical and human capital depreciation,** δ
  - **a lower labor force growth rate,** n
  - **a HIGHER rate of technological progress,** g

- Recall from Lecture 4's discussion of the Solow Model with technological progress that when the economy converges to steady state, the growth rate of output per worker will be equal to the rate technological progress.

$$\text{in steady state: } \frac{\dot{\tilde{y}}}{\tilde{y}}=0 \quad \text{which implies that: } \frac{\dot{(Y/L)}}{(Y/L)}=g$$

- **So a faster rate of growth of technological progress implies a rapidly rising standard of living for the residents of that economy**

So how can we increase the growth rate of technological progress?

- **The answer to that question is worth an instant Nobel Prize.**
- **Economists have developed other models that attempt to answer that question.** Below is a summary of some of a few theories.

Research and Development Models

- **Some models of R&D predict that the long-run growth rate of output per worker is an increasing function of the growth rate of the labor force**
- **But that's a little odd.**
- **On average, the growth rate of output per worker is <u>not</u> higher in countries with faster population growth.**

- **As a model of worldwide economic growth however, such models are more plausible. If the variable A in our models:**
  - **represents knowledge that can be used anywhere in the world and**
  - **if the growth rate of that knowledge, g, depends on the growth rate of the labor force**
- **then the larger the world population is the more people there are to make discoveries that advance the rate of technological progress.**

So how can we increase the growth rate of technological progress?

"Learning by Doing" in AK Models

- **In AK models, the source of technological progress:**
  - **does <u>not</u> depend on a Research and Development sector, but rather**
  - **depends on how much new knowledge is generated by everyday economic activity**
- **The underlying theory behind these models is that:**
  - **learning occurs as new capital is produced, so**
  - **producing new capital has benefits that are not captured by the conventional return on capital investment, r**
- **Increased capital therefore raises output through:**
  - **its direct contribution to output**
  - **by indirectly contributing to the development of new ideas**
- **There's no steady state in these models. Instead the long-run growth <u>rate</u> of output per worker is proportionate to the saving rate.**
- **The implication of such models is that the government should intervene to subsidize the accumulation of new capital.**

So how can we increase the growth rate of technological progress?

International Trade and Foreign Direct Investment

- **Trade enables a less developed trade partner to learn from the more developed trade partner how to implement the managerial** (and other) **practices best suited to using new technologies.**
- **Also in the absence of trade, domestic producers may seek government protection from competition – licensing requirements, etc.**
- **When a country opens to trade however:**
  - **international competition forces domestic producers to cease their** (socially wasteful) **protective activities and**
  - **allocate resources towards becoming more productive by adopting new technologies.**
- **Recipients of FDI acquire knowledge of foreign managerial practices, which they can compare with their own to find more efficient methods of production**

- **Trade and FDI cannot affect the growth rate of technological progress**
- **It enables less developed countries to import a whole level of technology**

So how can we increase the growth rate of technological progress?

political structure

- **A country's political structure affects the rate at which new technologies are adopted.**

- **If there's a high risk that the government will infringe upon the returns to technology adoption by expropriating industrial capacity, businesses will be less likely to undertake an investment in new technology.**

- **Similarly, if the government:**
  - **redistributes tax revenues to a minority of the elite rather than**
  - **allocating tax revenues to public goods that are necessary for business development** (such as roads, communications, sewage, etc.)
- **then businesses will be less likely to undertake an investment in new technology**

So how can we increase the growth rate of technological progress?

political structure

- **If the adoption of new technology is costly, but use of that new technology greatly reduces the cost of producing a good, then the entry of firms using the new technology will lower the market price.**
- **Producers who continue to use the older, less productive technology may find it more profitable to lobby government to block the use of the new technology rather than adopting it.**
- **Such lobbying benefits the users of the old technology at the expense of the majority of society.**
- **In theory, a democratic government should protect property rights and act in the interest of the majority of the society and not in the interest of an elite minority or a vested interest.**

Eric Doviak
Economic Growth and Economic Fluctuations

Review for the Mid-term Exam

The first exam will cover Lectures 1 through 7.

The best way to study for the exam is to review the Homework sets. Another good way to prepare is to make sure you understand the concepts that we discussed in those lectures, so make sure you understand the terms listed below.

terms you should know

- independent variable
- dependent variable
- ceteris paribus
- absolute advantage
- comparative advantage
- opportunity cost
- relative price
- gains from trade
- production possibilities frontier
- increasing opportunity cost
- factor demand
- factor payments
- wage rate
- rental rate on capital
- marginal product of labor
- marginal product of capital
- perfect competition
- short-run
- long-run
- economic profit
- total revenue
- total cost
- average cost
- marginal cost
- constant returns to scale
- Gross Domestic Product
- national income
- depreciation
- consumption
- investment
- government purchases
- net exports
- economic growth
- technological progress
- steady state
- saving rate
- labor force growth rate
- depreciation rate
- growth rate of technological progress
- Golden Rule
- human capital
- transition dynamics

Lastly, know the relationship between economic growth and:

- Research and Development
- learning by doing
- international trade
- Foreign Direct Investment
- political structure

Lecture 8

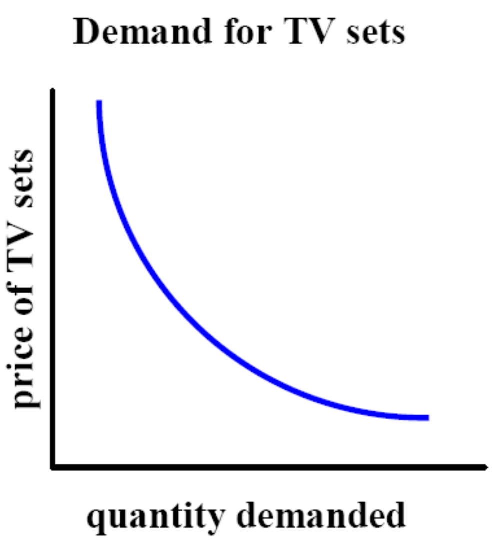
Supply, Demand and Equilibrium

Eric Doviak

Economic Growth and Economic Fluctuations

Demand

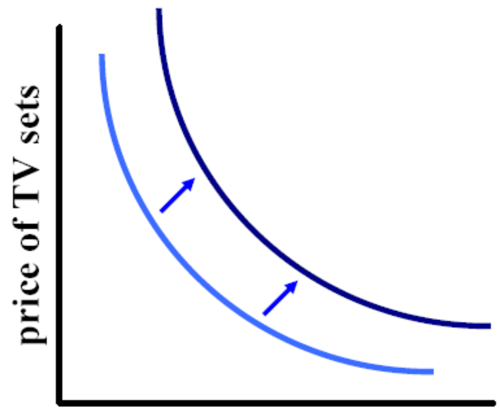
- How many TV sets do you have in your house?
- One in the kitchen, one in the bedroom, one in the living room ....
- Back in the 1950's, most families only had one TV, if they had one at all.
- Q.: Why do families have so many more TV sets today?
- A. # 1: They're cheaper.



If price of TV sets rises, then, ceteris paribus, the quantity of TV sets demanded will fall, and vice versa.

Demand

- **Q.: Why do families have so many more TV sets today?**
- **A. # 2: Families' real incomes are larger.**
- **An increase in income changes relationship between price and quantity demanded.**
- **Demand curve shifts out (up and right).**



When income rises, but the price of TV sets doesn't change (i.e. ceteris paribus)**, there's more demand for TV sets at every price level.**

Movement along Demand Curve vs. Shift of Demand

Movement along:

- **Only if there is change in the good's price (shift of supply curve)**

Shift of demand,

due to changes in:

- ***Income***
- **Accumulated *wealth***
- **Tastes and preferences (ex. fewer smokers)**
- **Prices of other goods**
- **Expectations (of future income, wealth and prices)**

Income – ***a flow*** – **sum of all earnings** (wages, salaries, profits, interest payments, rents, etc.) **in a given period of time**

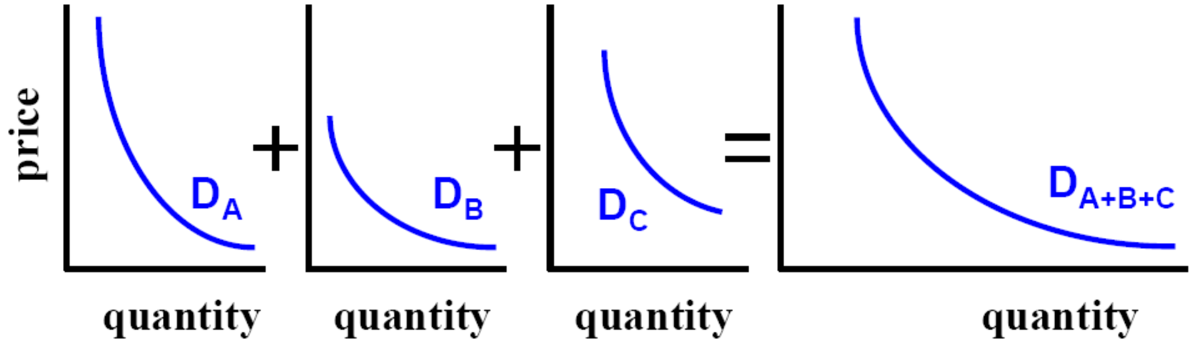
Wealth – ***a stock*** – **total value of what household owns minus what it owes**

Substitutes – **when price of good X rises, demand for good Y rises** (ex. cigs & rolling tobacco)

Complements – **when price of good X rises, demand for good Y falls** (ex. pasta & sauce)

Market Demand

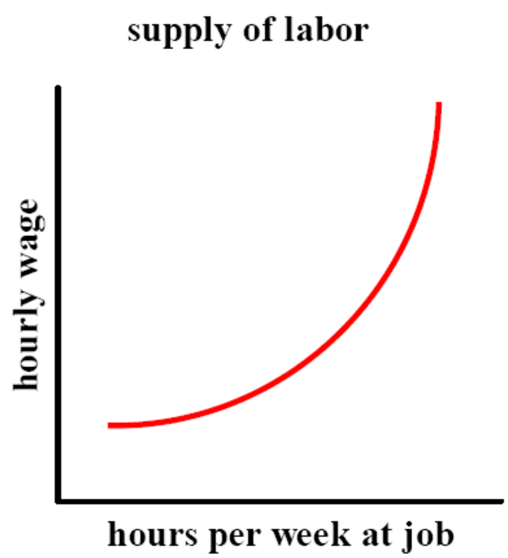
Sum of all individual demand curves (sum of all individual quantities demanded at each price)



price	quantity demanded by: A		B		C	market demand
$4	4	+	0	+	4	= 8
$2	8	+	3	+	9	= 20

Supply

- **If you were offered a job for $3 an hour, how many hours a week would you work?**
- You wouldn't take a job that pays so little.
- **At $50 an hour, how many hours a week?**
- **40, 50, 60 hours?**
- **At $100 an hour, how many hours a week?**
- **40, 50, 60 hours? There's a limit to how much you can work in a week.**
- ***Ceteris paribus*** – ex. if there's very high inflation, increase in wage reflect inflation and therefore will not increase output (hours worked)

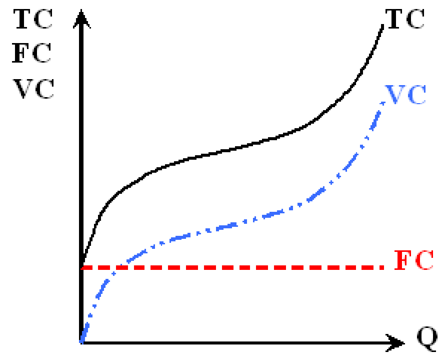


An increase in price – wage is the price of labor – **will *ceteris paribus* increase the quantity of labor supplied and vice versa.**

Firm Supply

Total Cost: TC = FC + VC

- **Fixed costs (FC)** – repayment of loans, lump sum taxes, etc.
- **Variable costs (VC)** – labor, raw materials, electricity, etc.

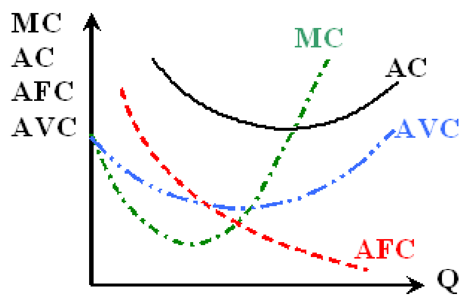


Average Cost: AC = AFC + AVC

- **Average fixed cost (AFC) decreases as output increases**
- **Average variable cost (AVC) increases as output increases** (at least at higher output levels)

Marginal Cost (MC):

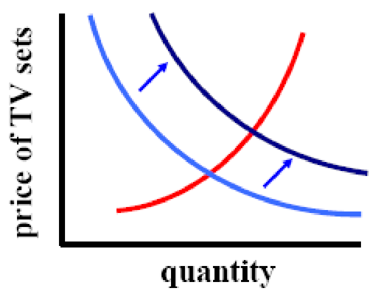
- **Rate of change in total costs from extra unit of output**
- **Is the supply curve when MC > AC**



Movement along Supply Curve vs. Shift of Supply

Movement along:

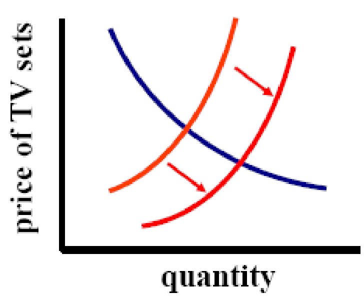
- **Only if there is a change in the good's price** (shift of demand curve)



Shift of supply,

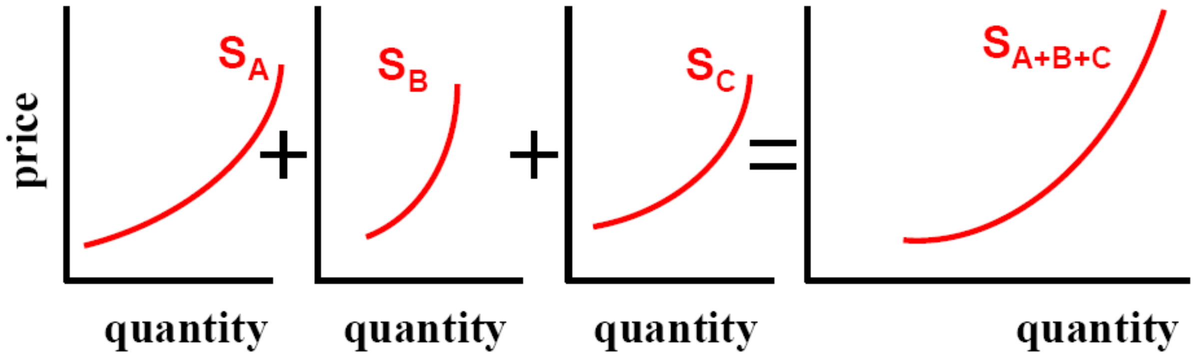
due to changes in:

- **Costs:**
  - wages
  - dividend payments
  - raw materials
- **Technology**
  - more productive machines
  - increased efficiency with which firm uses its inputs into its production



Market Supply

Sum of all individual supply curves (sum of all individual quantities supplied at each price)

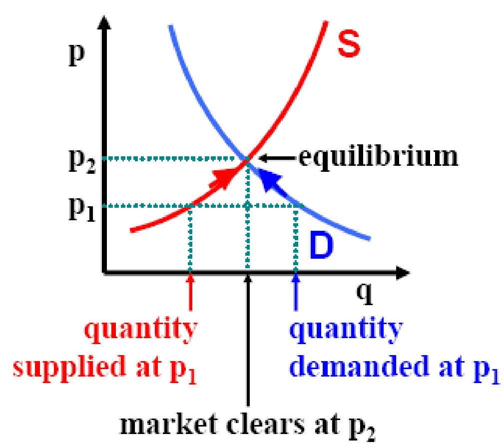


price	quantity supplied by: A		B		C	market supply	
$4	30	+	10	+	25	=	65
$2	10	+	5	+	10	=	25

Market Equilibrium

Excess Demand

- The absolutely must-have Christmas present
- Parents bid up the price of the present, some parents drop out of the market
- Factories increase production and ask higher price
- Shortage eliminated – "price rationing"

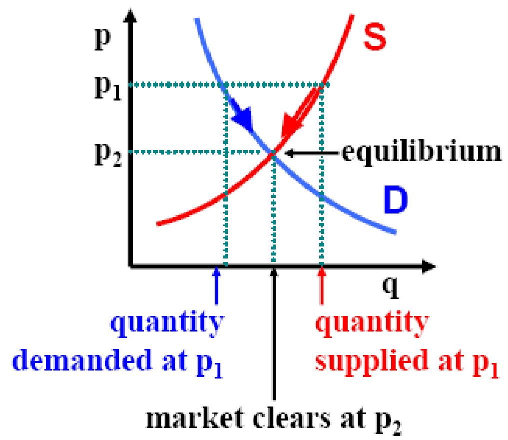


At equilibrium there is no natural tendency for further adjustment.

Market Equilibrium

Excess Supply

- **Car sales at the beginning of a recession**
  - Think back to end-2001
  - Every other ad on TV/radio was a car commercial
- **Buyers know of excess supply, offer lower prices and increase quantity demanded**
- **Automakers decrease production and accept a lower price**

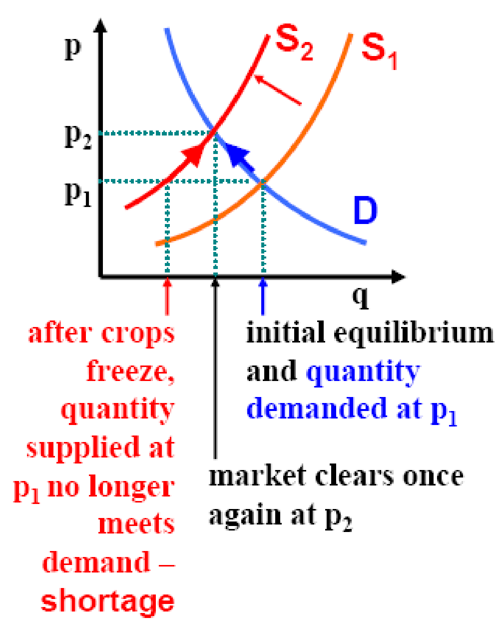


At equilibrium there is no natural tendency for further adjustment.

Changes in Equilibrium

Fall in Supply

- **Some crops freeze**
- **Initially at equilibrium**
- **After freeze, market supply is more limited**
- **Supply curve shifts in**
- **Shortage at initial price**
- **Price bid up, some drop out of market**
- **Other farms harvest more of frozen crop**
- **New equilibrium**



Scalpers

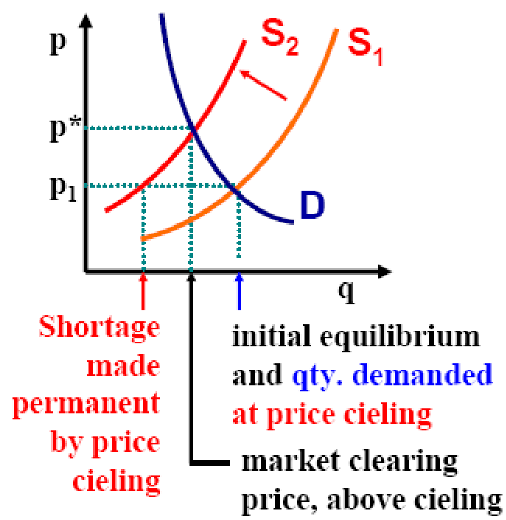
- How much would you pay for the best seats in the house?
- Limited supply
- Tickets priced below market equilibrium
- Excess demand at list price
- **Arbitrage** – whoever obtains tickets at list price, can profit by reselling them

Stupidity

If "pro–" is the opposite of "con–," what's the opposite of "progress?"

Price ceilings

- 1974 – OPEC imposed oil embargo on US
- Supply shock
- Congress imposed a price cieling on the price of gasoline
- Result: long lines and shortage of gasoline



<u>Ways to beat a cieling</u>

1. **Queuing** – be first in line
2. **Favored customers** – bribe the retailer
3. **Ration coupons** – buy coupons from friends

Homework #8

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Lecture 9

Unemployment in the Long Run

Eric Doviak

Economic Growth and Economic Fluctuations

Quick History of Macroeconomics

- **Prior to the Great Depression of the 1930's, the term "economics" referred entirely to what we now call "microeconomics"**
- **Economists were concerned almost exclusively with the decision-making of individuals, households and firms**
- **To the extent that they studied economy-wide problems** (such as the Great Depression)**, they always used micro analysis**
- **For example, they explained the phenomenon of unemployment as an excess supply of labor induced by a <u>real</u> <u>wage</u> that was set above the market-clearing level** (i.e. the prevailing wage was set too high and not justified by the price of output, yet unemployed workers stubbornly refused to accept the lower wage at which employers would hire them)**.**
- **Such models failed to explain the persistently high unemployment that occurred during the Great Depression.**
- **It's not realistic to assume that workers endure unemployment over long periods of time because they're unwilling to accept a lower wage**
- **Economists also argued that high <u>interest</u> <u>rates</u> discouraged firms from investing in new capital during the Great Depression**

The Keynesian Revolution

General Theory of Employment, Interest and Money (Keynes, 1936)

- **John Maynard Keynes was the first economist to analyze the problem of unemployment using a framework other than the price/wage rate framework discussed above.**
- **Keynes argued that prices and wages don't determine the level of employment in the short run**
- **Keynes argued that:**
  - **if wages fell, employers would be able to hire more workers, but**
  - **employed workers would have to consume less and**
  - **the <u>net</u> effect would be a reduction in aggregate demand for the economy's output**
  - **which would worsen the unemployment problem**
- **He was also skeptical that lower interest rates would stimulate investment in times of depression**
- **Instead he argued for demand management:**
  - **in times of recession, the government should increase its purchases and lower taxes to stimulate aggregate demand**
  - **to balance the budget, the government would have to reduce its purchases and raise taxes during economic expansions**

M. Friedman and Monetarism

A Monetary History of the U.S.… (Friedman and Schwartz, 1963)

- **In contrast to Keynes, Milton Friedman argued that money, prices and interest rates can affect aggregate demand in the short run**
- **He stressed that money is a commodity:**
  - **just as you can substitute a commodity** (like orange juice) **for another commodity** (like a box of cookies)
  - **you can substitute money for a wide range of other commodities**
  - Ex.: after being paid, people living in a country with an extremely high inflation rate purchase commodities (US dollars, food … anything!) as soon as possible in order to store the value of the money they received
- **Since money is a commodity, monetary policy** (how much money a central bank injects into the economy) **can affect aggregate demand**
  - **If a central bank suddenly injects more money into the economy and thus increases the inflation rate, a firm that borrows at a fixed nominal interest rate will find it easier to pay back the loan**
  - **thus firms will want to undertake more investment projects**
  - **the resulting increase in investment increases aggregate demand**
- **The central bank must however control the amount of money that it injects into the economy otherwise inflation will get out of control**

a synthesis

- **Although the topics discussed above were once the subject of vigorous debate, most economists now believe that the conflicting points of view can be reconciled with each other**
- **In the long run, any unemployment will be eliminated by falling wages and/or higher aggregate demand**
- **In the short run however, there may be nominal rigidities that keep the wage rate above the market-clearing level**

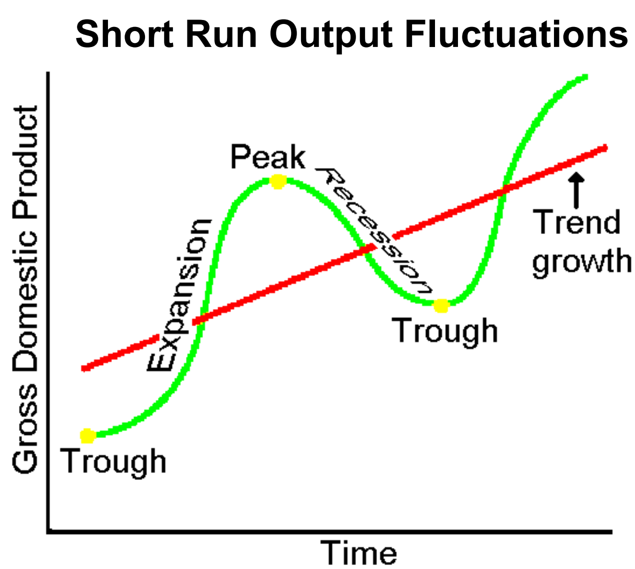
- **Most economists have accepted Keynes' aggregate demand framework**
- **Most believe that in the short run, both fiscal policy and monetary policy can affect the level of output, the price level and employment**

the main concerns of macro

- If you don't understand everything that I wrote in the "Quick History…," **don't worry!** You will by then end of the course.
- The "Quick History…" is just a preview of what we will be discussing in the remaining lectures: output growth, unemployment, money and inflation

output growth

- **output growth is increase in the total qty. of goods and services produced in given period**
- **Short Run output growth is subject to fluctuations – called the business cycle**
    - **Expansions** – aggregate output is increasing
    - **Recessions** – aggregate output is decreasing
- **Long Run output growth**
    - **sustained increase in real output per worker**
    - **shifts out economy's PPF**

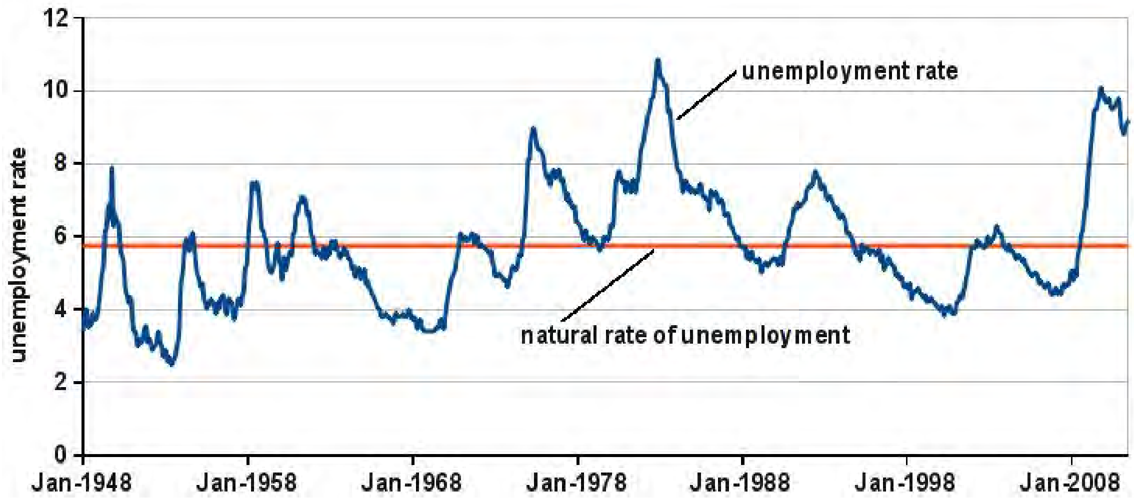


- **We already discussed long run output growth at length and we'll discuss short run output fluctuations later, so this lecture and the next one will focus on unemployment, money and inflation in the long run.**

Unemployment

- **Cyclical unemployment** refers to the year-to-year fluctuations in unemployment around its natural rate.
- **Cyclical unemployment** is a **short run** phenomenon associated with short-term ups and downs of the business cycle

- Every economy normally experiences some amount of unemployment
- The **natural rate of unemployment** is unemployment that does not go away on its own even in the **long run**



How Is Unemployment Measured?

- Based on responses to the Current Population Survey, the Bureau of Labor Statistics (BLS) places each adult into one of three categories:
  - **Employed – if he or she has spent most of the previous week working at a paid job**
  - **Unemployed – if he or she is on temporary layoff, is looking for a job, or is waiting for the start date of a new job**
  - **Not in the labor force – a person who is neither employed nor unemployed, such as a full-time student, homemaker or retiree**

- The **labor force** is the sum of the employed and the unemployed.
- The **unemployment rate** is the percentage of the labor force that is unemployed.
- The **labor-force participation rate** is the percentage of the adult population that is in the labor force.

Labor-Market Experiences by Race and Gender

2010	unemployment rate	labor-force participation rate
Adults (age 20-64)		
Asian female	6.4	71.3
Asian male	6.7	89.3
black female	12.3	76.2
black male	16.3	82.2
white female	7.0	75.5
white male	8.5	90.5
Teenagers (age 16-19)		
Asian female	23.7	22.0
Asian male	25.8	22.1
black female	40.5	25.1
black male	45.4	25.8
white female	20.0	38.0
white male	26.3	37.4

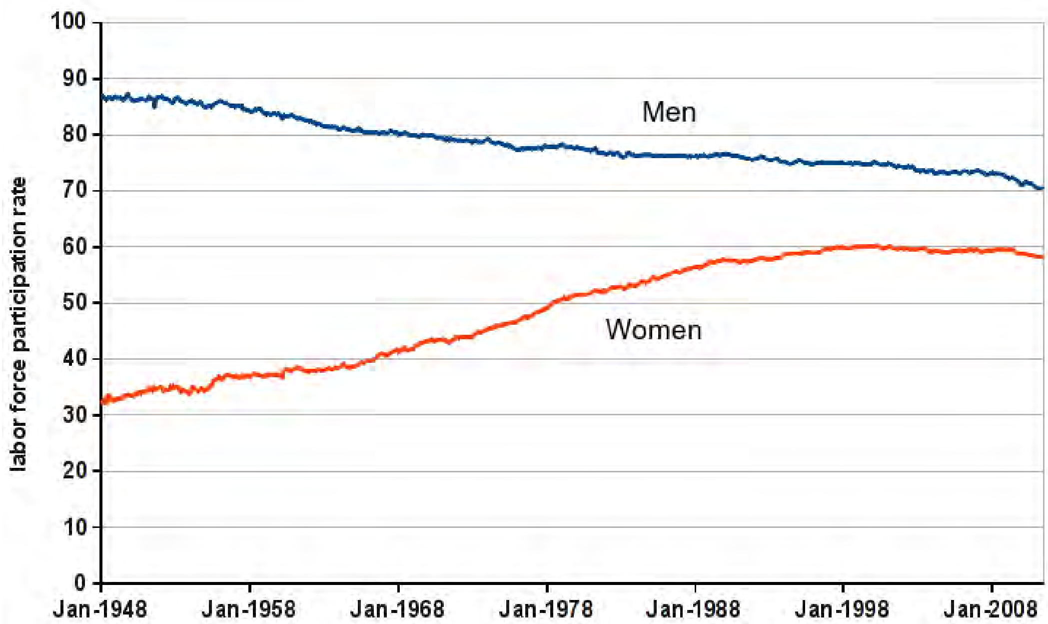
Source: Bureau of Labor Statistics, Current Population Survey

- **The table above implies discrimination, but it does NOT prove that discrimination exists.**
- **A better measure of discrimination examines wage differentials.**

Does Discrimination Exist?

- **The data above does not compare "apples to apples," because the groups may differ in more than just skin color and gender.**
- **To see if discrimination exists we can use data from the Current Population Survey to examine wage differentials while accounting for differences in each survey recipient's:**
  - level of schooling
  - work experience
  - hours worked
  - occupation
  - industry
  - region of the country
  - city size and
  - location (urban, suburban or rural)
- **By one estimate:**
  - **Asian females earn 12.8% less than white males** std. err.: 1.5%
  - **Asian males earn 8.2% less than white males** std. err.: 1.1%
  - **black females earn 8.8% less than white males** std. err.: 0.9%
  - **black males earn 11.1% less than white males** std. err.: 0.7%
  - **white females earn 19.9% less than white males** std. err.: 0.4%
- **The estimates above are comparisons between comparable individuals** (i.e. they take account of differences in the factors listed above)
- **The estimates above indicate that racism and sexism do indeed exist**

Why do Increasingly More Women Work?



- **Do more women work today because they have more employment opportunities?**
- **Do more women work today because it is more expensive to maintain a certain standard of living?**

Does the Unemployment Rate Measure What We Want It To?

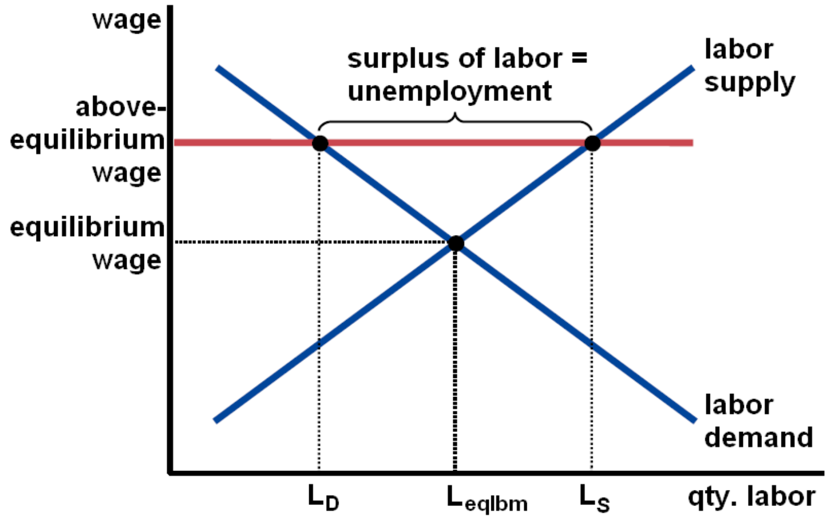
- **It is difficult to distinguish between a person who is unemployed and a person who is not in the labor force.**
- **Discouraged workers**, people who would like to work but have given up looking for jobs after an unsuccessful search**, don't show up in the unemployment statistics.**
- **Other people may claim to be unemployed in order to receive financial assistance, even though they aren't looking for work.**

How Long Are the Unemployed without Work?

- **Most spells of unemployment are short.**
- **Most unemployment observed at any given time is long-term.**
- **Most of the economy's unemployment problem is attributable to relatively few workers who are jobless for long periods of time.**

Why Are There Always Some People Unemployed?

- **In an ideal labor market, wages would adjust to balance the supply and demand for labor, ensuring that all workers would be fully employed.**
- **Structural unemployment is the unemployment that results because the number of jobs available in some labor markets is insufficient to provide a job for everyone who wants one.**
  - **occurs when the qty. of labor supplied exceeds the qty. demanded**
  - **explains longer spells of unemployment**



- **Structural unemployment could be caused by:**
  - **Minimum-wage laws** – create unemployment when the minimum wage is set above the equilibrium level since the labor supplied at that wage rate exceeds the labor demanded at that wage rate
  - **Unions** – By acting as a cartel with ability to strike or otherwise impose high costs on employers, unions usually achieve above-equilibrium wages for their members.
  - **Efficiency wages** – above-equilibrium wages paid by firms in order to increase worker productivity.

Efficiency Wage Theory

- **Firms may pay an** (above-equilibrium) **efficiency wage if the marginal product of workers depends in part on the wages they are paid.**
  - **Turnover Model –** A higher paid worker is less likely to look for another job.
  - **Shirking Model –** if a firm pays a higher wage rate and fires poorly performing workers, then workers will work hard to keep their jobs
  - **Superior Job Applicant Pool –** If a firm cannot discern which of its applicants are the most productive, it may offer a higher wage to encourage the most productive to apply (and leave other firms), so that on average it will recruit a more productive workforce.

Why Are There Always Some People Unemployed?

- **Frictional unemployment – the unemployment that results from the time that it takes to match workers with jobs**
- **During a job search it takes time for qualified individuals to be matched with appropriate jobs given their tastes and skills**

- **Frictional unemployment is different from the other types since it is NOT caused by an above-equilibrium wage rate**
- **It is caused by the time spent searching for the “right” job.**

- **The economy’s composition of labor demand among industries or regions is always changing.**
- **Such sectoral shifts make some frictional unemployment inevitable because it takes time for workers to search for and find jobs in new sectors.**

Homework #9

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Lecture 10

Money and Inflation in the Long Run

Eric Doviak

Economic Growth and Economic Fluctuations

Three market arenas

Goods and Services Market

- **Households, government and the rest of the world demand goods and services**
- **from firms and the rest of the world who supply those goods and services**

Labor market

- **Firms and government demand labor**
- **from households who supply labor**

Money market

- **Households and the rest of the world supply funds**
- **to other households who demand funds to finance various purchases, especially housing** (and other expensive items)
- **to firms who demand funds to finance investment expenditures**
- **to government who demands funds to finance budget deficits**
- **and to the rest of the world**

The Money Market

- **Of the three market arenas described on the previous page, the only one we have not discussed yet is the money market.**

- **Money is the set of assets in an economy that people regularly use to buy goods and services from other people**
- **Money has three functions in the economy:**
    - **Medium of exchange – buyers give money to sellers in exchange for goods and services**
    - **Unit of account – the measure people use to post prices and record debts**
    - **Store of value – people can use money to transfer purchasing power from the present to the future**

- **Liquidity is the ease with which an asset can be converted into the economy's medium of exchange.** For example, a checking account is more liquid than a money market account because:
    - a checking account can be quickly converted into currency, whereas
    - a money market account cannot be converted as easily because there are limits to the number of checks a money market account holder can write in a month

The Federal Reserve System

- **The Federal Reserve (Fed) serves as the nation's central bank.**
    - **It is designed to oversee the banking system.**
    - **It regulates the quantity of money in the economy.**
- **The primary elements in the Federal Reserve System are:**
    1. the Board of Governors
    2. the 12 Regional Federal Reserve Banks
    3. the Federal Open Market Committee

- **Of the seven members of the Fed's Board of Governors, Chairman Ben Bernanke is the most important.**
- **The chairman directs the Fed staff, presides over board meetings and testifies about Fed policy in front of Congressional Committees.**

"Helicopter Ben" Bernanke

The Functions of the Fed

- **The Fed has three primary functions.**
  1. **The Fed regulates banks to ensure they follow federal laws intended to promote safe and sound banking practices**
  2. **The Fed acts as a lender of last resort – makes loans to banks when they find themselves in trouble**
  3. **The Fed controls the money supply – primarily through open-market purchases and sales of U.S. government bonds**
     - To increase the money supply, the Fed buys bonds from the public
     - To decrease the money supply, the Fed sells bonds to the public
- **The Federal Open Market Committee (FOMC) serves as the main policy-making organ of the Federal Reserve System.**
  - The FOMC is made up of the members of the Board of Governors and the presidents of the regional Federal Reserve banks
  - The meetings are chaired by the Chairman of the Board of Governors
- **The FOMC meets about every six weeks to review the economy and determine the course of monetary policy – i.e. they decide how much money to supply to the economy**

How the Fed Controls the Money Supply

- **The Money Supply is equal to the sum of deposits inside banks and the currency in circulation outside of banks**

Measures of Money

- **C** – currency outside the U.S. Treasury, Federal Reserve Banks, and the vaults of depository institutions
- **M1** – the sum of currency, traveler's checks, demand deposits and other checkable deposits
- **M2** – the sum of M1, savings deposits, small-denomination time deposits and balances in retail money market mutual funds.
  - "savings deposits" include money market deposit accounts
  - "small denomination" refers to amounts under $100,000
  - M2 does not include individual retirement account (IRA) and Keogh balances

- **The Federal Reserve controls the money supply by:**
  1. increasing or decreasing the **required reserve ratio**
  2. increasing or decreasing the **interest rate** it pays **on reserves**
  3. increasing or decreasing the **discount rate**
  4. buying or selling government bonds on the **open market**

Bank Balance Sheet

- **Consider a bank's balance sheet.**
- **A bank's assets consist of:**
  - **Loans** – the most important asset of a bank
  - **Reserves** – vault cash and deposits with the Fed
- **A bank's liabilities consist of:**
  - **Deposits** – most important liabilities of a bank
  - **net worth ≡ assets – liabilities**

- "Net worth" is also called "bank capital."
- Bank capital mostly consists of shareholders' equity.
- When a bank suffers a loss on its portfolio of loans, the loss is also subtracted from the bank's capital.
  - The two sides of the balance sheet remain equal.
  - Owners of the bank are punished for the losses. Not depositors.

- The examples below simplify the discussion by setting the bank's net worth to zero. In practice, the bank would be closed if its net worth were zero.

The Required Reserve Ratio

- **The balance sheet of a bank:**
  - the sum of assets must equal
  - the sum of liabilities

Assets		Liabilities	
Reserves	20	100	Deposits
Loans	80	0	Net Worth
Total	100	100	Total

- **If the Fed has set the required reserve ratio at 20 percent and if the bank receives $100 in deposits, then:**
  - it holds $20 in reserve as vault cash or as deposits at the Fed
  - the other $80 can be used to make loans
- **The required reserve ratio determines the amount of money that is created from a deposit**
- **Imagine that there's only one bank in the economy, that bank receives $100 in deposits, that the required reserve ratio is 20 percent and the Fed pays <u>no interest on reserves</u>.**
  - The bank loans out $80, but when those $80 are used to make a purchase, they will be ultimately be deposited with the bank
  - So the bank will hold $16 of that new $80 deposit in reserve (20 percent of $80 is $16) and loan out the other $64
  - Since those $64 will be deposited with the bank, the bank will hold $12.80 of that $64 deposit in reserve and loan out the other $51.20

Creation of Money

Assets		Liabilities	
Reserves	100	500	Deposits
Loans	400	0	Net Worth
Total	500	500	Total

- **The process described above continues until $400 worth of new deposits are created from the initial $100 deposit.**
- The fact that there's more than one bank in the economy need not concern us. The assets and liabilities of our example's one bank represent the sum of the assets and the sum of the liabilities of all of the banks in the economy
- **The key thing to notice is that there is a relationship between:**
  - **the initial deposit ($100)**
  - **the total deposits created from that initial deposit ($500) and**
  - **the required reserve ratio (20 percent)**
- **The $100 initial deposit is 20 percent of the $500 in total deposits created**

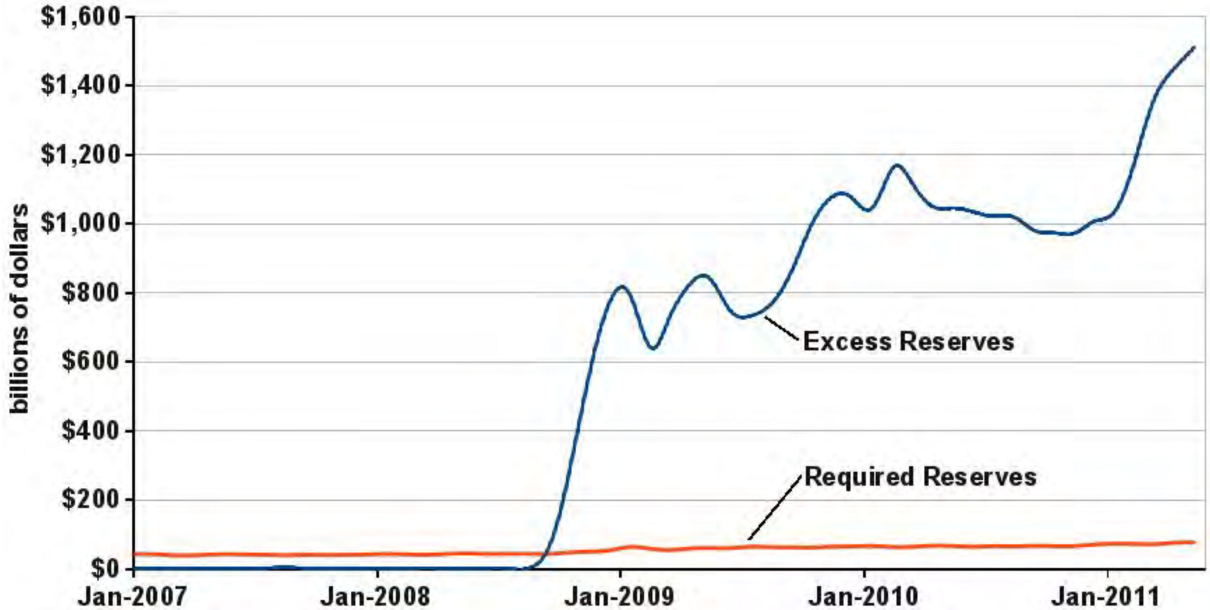
$$\text{money multiplier} = \frac{1}{\text{required reserve ratio}}$$

- **The Money Multiplier is the multiple by which deposits can increase for every dollar increase in reserves.**
- The Money Multiplier in this example is 5

Money Supply and the Required Reserve Ratio

- **What would happen if the Fed reduced the required reserve ratio?**
- **To continue the previous example, imagine that the Fed lowers the required reserve ratio to 10 percent.**
  - The bank now only has to hold in reserve $50 of the $500 worth of deposits and it has $50 in excess reserves.
  - So it loans out those $50 in excess reserves,
  - but those $50 will be ultimately be deposited, so the bank will hold $5 of that new $50 deposit in reserve and loan out the other $45
  - This process will continue until $500 in new deposits are created
- **Recall that the Money Supply is equal to the sum of deposits inside banks and the currency in circulation outside of banks**
- **So by lowering the required reserve ratio, the Fed can increase the money supply**
- **Conversely, if the Fed raises the required reserve ratio, it would decrease the money supply** – banks would cease to make new loans until enough outstanding loans were repaid that the bank has excess reserves to loan out

Required Reserves and Excess Reserves



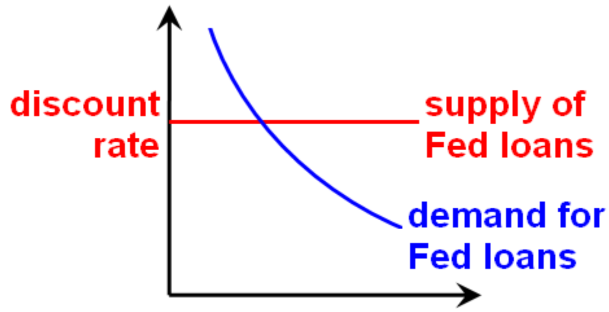
Source: Federal Reserve Statistical Release H.3, Table 1.

Money Supply and the Interest Paid on Reserves

- In response to the recent financial crisis, Congress authorized the Fed to pay interest on reserves. The Fed began paying interest in October 2008.
- **When the Fed pays interest on reserves, a bank with excess reserves can make new loans or leave the excess reserves on deposit at the Fed.**
- If the Fed pays a high interest rate on reserves, banks will hold more excess reserves. If the interest rate is low, banks will make more loans.
- **Note that paying interest on reserves reduces the money multiplier.**
- What's the purpose of paying interest?
    - When the Fed makes a loan to a bank at the discount window (see below), the bank will have excess reserves.
    - If the Fed does not pay interest, the bank will lend out its new excess reserves and push interest rates below the Fed's target level.
    - The Fed could push interest rates back up by selling some of its bonds (see below), but – if many banks borrow from the Fed – then the Fed will run out of bonds to sell before interest rates return to target level.
    - **Paying interest on reserves enables the Fed to lend to many troubled banks without altering the Fed's target interest rate or triggering a round of hyperinflation** (see below)**.**

Money Supply and the Discount Rate

- **the discount rate is the interest rate banks pay when they borrow from the Fed**
- **the money supply increases as banks borrow more from the Fed, since banks use the loans from the Fed to make loans themselves**
- **The discount rate is the cost of borrowing**
  - Banks borrow less when the discount rate is high
  - Banks borrow more when the discount rate is low
  - The discount rate is the price of a loan



- The Fed rarely uses the discount rate to control the money supply. The discount rate cannot be used to control the money supply with great precision, because its effects on banks' demand for reserves are uncertain.
- **Moral Suasion – pressure exerted by the Fed on member banks to discourage them from borrowing heavily from the Fed.**
- After all, the Fed regulates the banking system and no sane bank manager would want Fed regulators examining every detail of the bank's business

Money Supply and Open Market Operations

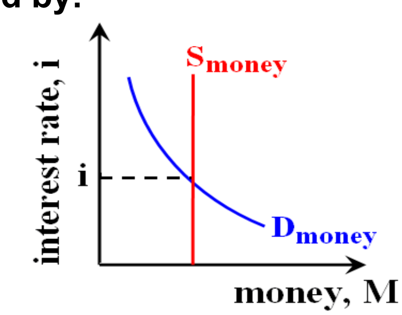
- **The main way the Fed conducts monetary policy is through purchases and sales of government bonds**
- **So let's say the required reserve ratio is 10 percent**
- **and let's say the Fed sells Jane Q. Public a $10 bond and that to pay for the bond, Jane writes a check to the Federal Reserve**
  - **that sale reduces the Fed's assets** (bond holdings) **by $10**
  - **there's no change in Jane's total assets, but her deposits at her bank decline by $10**
  - **her bank passes $10 to the Fed by reducing its reserves at the Fed**
  - **the bank's liabilities fell by $10 and its assets also fall by $10**
- **Notice that the bank now has a deficit of reserves, so it must further reduce its deposits by another $90 by "calling in loans"** (it ceases to issue new loans until over $90 in outstanding loans have been repaid)
- **By selling Jane a $10 bond, the Fed reduces the money supply $100**
  - the Money Supply is equal to the sum of deposits inside banks and the currency in circulation outside of banks
  - total bank deposits have fallen by $100 – through a $10 reduction in Jane's deposits and a $90 reduction in deposits created through loans

Money Supply and Open Market Operations

- **An open market sale of government bonds**
  - results in a decrease in bank reserves and
  - **results in a decrease in the money supply** – equal to the money multiplier times the value of bonds sold (which is equal to the change in bank reserves)
- **An open market purchase of government bonds**
  - results in an increase in bank reserves and
  - **results in an increase in the money supply** – equal to the money multiplier times the value of bonds purchased (which is equal to the change in bank reserves)
- **Open market operations are the Fed's preferred means of controlling the money supply because:**
  - they can be used with some precision
  - are extremely flexible and
  - are fairly predictable

Supply and Demand for Money

- **If the Fed chose a quantity target for the money supply, then a vertical supply curve would represent the quantity of money that the Fed chooses to supply regardless of the resulting interest rate**
- **But this isn't what the Fed does.**
- **When you hear about the Fed in the news you** (usually) **hear that the Fed has raised or lowered the Federal Funds Rate**
  - banks with excess reserves lend to banks in need of reserves
  - the Federal Funds Rate is the interest rate banks are charged when they borrow reserves from other banks
  - the Fed's buys and sells government bonds through its open market operations to keep the Federal Funds Rate at the level it has set
  - **the money supply is implicitly determined by:**
  - **the choice of Federal Funds Rate** (which affects all other interest rates in the economy)
  - **the demand for money**
- To introduce the relationship between money and interest rates, we'll focus on the case where the Fed directly chooses the money supply.



The Demand for Money

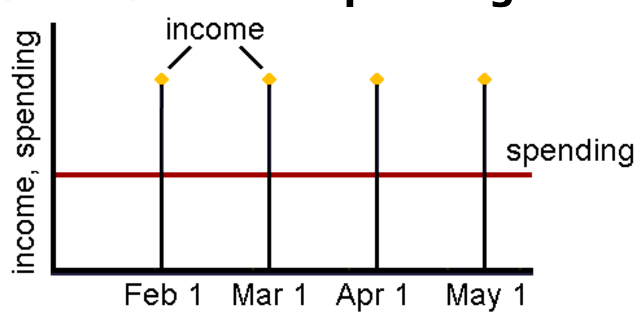
- **people hold money because it facilitates the purchase goods and services, but when you hold money you forgo the interest you could have received by holding interest bearing assets**
- **The interest rate is opportunity cost of holding money, so our main interest in studying the demand for money is:**
  - **How much of your financial assets you want to hold in the form of money,** (which does not earn interest)**,**
  - **versus how much you want to hold in interest-bearing assets?**
- **In this lecture, we'll examine two theories that seek to explain why the demand for money is a decreasing function of the interest rate:**
  - **transactions motive –** this theory focuses on how money facilitates the purchase of goods and services
  - **speculation motive** – this theory focuses on forecasts of future interest rates
- Both theories predict that:
  - people will want to hold less money when the interest rate on interest bearing assets is higher
  - people will want to hold more money when the interest rate on interest bearing assets is lower

The Transactions Motive

- **There's a trade-off between:**
  - **the liquidity of money** and
  - **interest income offered by other assets** – the interest rate is the opportunity cost of holding money
- **For simplicity, we'll assume that there's only one alternative financial asset to money – government bonds – and that there is no inflation.**
- **We'll also assume that:**
  - a typical household's income arrives once a month, but
  - the typical household spends the same amount each day
  - spending over the month is exactly equal to income for the month

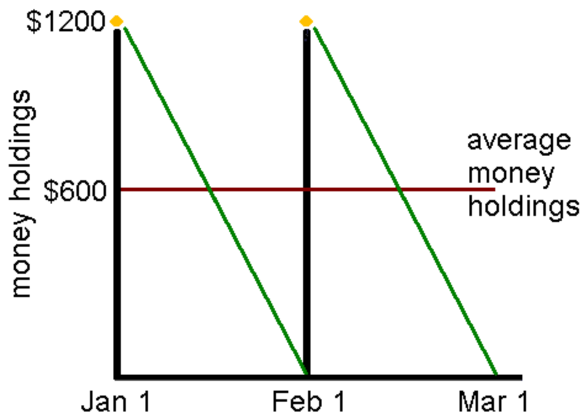
Non-synchronization of Income and Spending

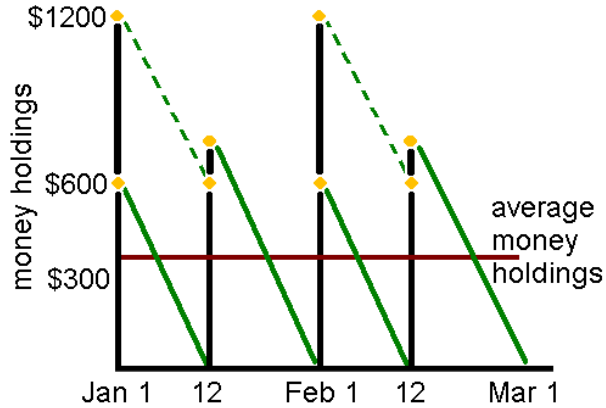
- **The assumptions above imply a mismatch between the timing of money inflow and the timing of money outflow**
- **Income arrives once a month, but spending takes place continuously**



Money Management and the Transactions Motive

- Consider a household that receives a $1200 paycheck at the beginning of the month
- The household could deposit the entire $1200 paycheck into a checking account at the start of the month and run the balance down to zero by the end of the month.
- The household's average monthly money holdings would be $600



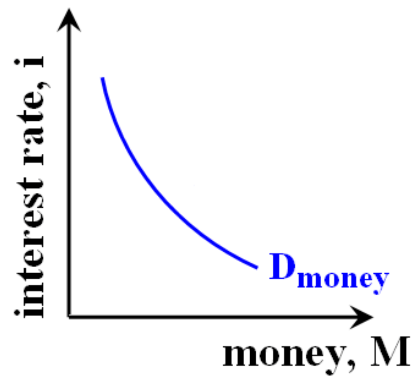


- The household could also deposit half of the paycheck ($600) into a checking account and use the other half to buy $600 in bonds
- Shortly before mid-month, the household could sell the bonds and deposit the $600 and the interest earnings into the checking account
- Average money holdings in this case would be slightly higher than $300

Optimal Balance in the Transactions Motive

- **Why not hold the entire paycheck in bonds** (where it earns interest) **and make transfers from bonds to money every time the household wants to make a purchase?**
- **If there were no cost to transferring the bond into money and if there were no cost involved in making the necessary trips to the bank, then the household would never hold money for more than an instant.**
- **At the very least however, there's a cost in terms of lost time and convenience. These are *figuratively* called shoeleather costs** (since more frequent trips to the bank wear out your shoes more quickly)
- **Switching wealth from bonds to money more often:**
  - **earns the household a higher level of interest income, but**
  - **increases shoeleather costs**
- **The optimal balance is the level of average money holdings that earns the household the most profit after taking into account:**
  - **the interest earned on bonds and**
  - **the cost paid for switching from bonds to money**

Optimal Balance in the Transactions Motive



- **A higher interest rate:**
  - corresponds to a lower optimal money balance
  - people want to take advantage of the high return on bonds, so they choose to hold very little money
- **A lower interest rate:**
  - corresponds to a higher optimal money balance
  - people choose to hold more money because interest rate is too low to make frequent transfers from bonds to money worthwhile

- **The Speculation Motive that we will examine below also explains why the quantity of money that households choose to hold**
  - falls when interest rates rise and
  - rises when interest rates fall
- **In other words, the Speculation Motive also provides an explanation of the downward sloping money demand curve.**

The Speculation Motive

- **Consider a risk-free bond at a price of \$100 that pays a \$5 return.**
- **If the price of the bond remains constant, then the "yield" on the bond will remain at 5 percent.**
- **But what if another risk-free bond is offered at a price of \$100, but this other bond offers a \$10 return?**
  - Since both bonds are risk-free it would be preferable to hold the bond that offers a 10 percent rate of return.
  - So the demand curve for the bond that pays a \$5 return will shift inward and drive the price of the bond that pays a \$5 return to \$50.
  - Why \$50? Because at a price of \$50, the **rate of return** (the interest rate) is equalized across bonds – i.e. \$5 is 10 percent of \$50

$$\text{rate of return} = \frac{\text{return on bond (in dollars)}}{\text{price of bond (in dollars)}}$$

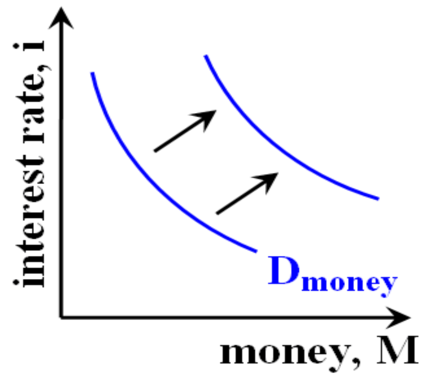
- **If rates of return are high and you expect them to fall, you will buy bonds and hold lower money balances – you are speculating that the price of the bond will rise.**
- **If rates of return are low and you expect them to rise, you will sell bonds and hold higher money balances – you are speculating that the price of the bond will fall.**

The Total Demand for Money

- **The total quantity of money demanded is the sum of the demand for checking account balances and cash by both households and firms**
- **The quantity of money demanded at any moment depends on:**
  - **the interest rate** – a higher interest rate raises the opportunity cost of holding money and thus reduces the quantity of money demanded
  - the **total dollar volume of transactions made** which depends on:
    - **the total number of transactions and**
    - **the average transaction amount**

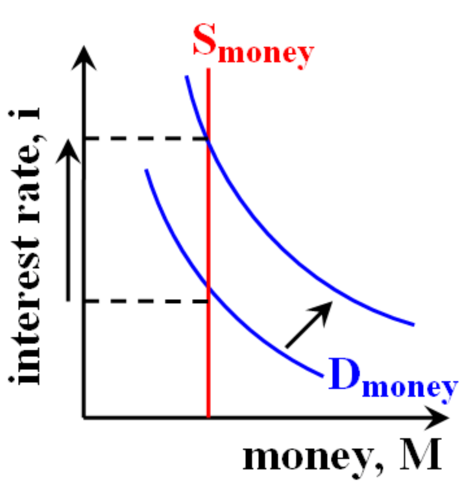
Transactions Volume – Output and Prices

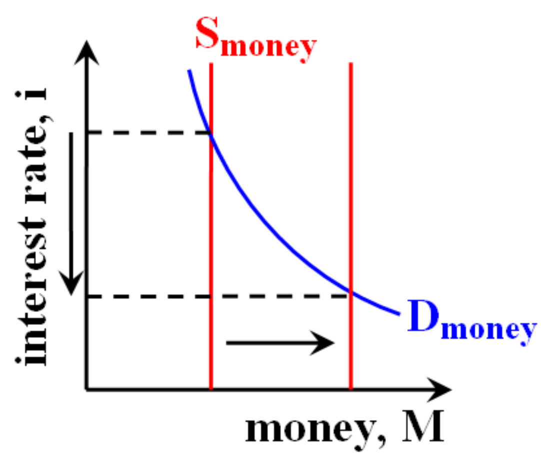
- **the money demand curve shifts out when:**
  - **output (income) rises – the total number of transactions rises** – to purchase the higher amount of goods and services produced when output rises individuals need more to hold more money
  - **the price level rises – the average dollar amount of each transaction rises**, so individuals need more money to make each transaction



Money and Interest Rates

- **If the money demand curve shift out** (due either to higher GDP or a higher price level)**, then:**
  - **the equilibrium interest rate will rise**
  - **ceteris paribus** – so long as the money supply remains constant





- **If the Fed increases the money supply, then:**
  - **the equilibrium interest rate will fall**
  - **ceteris paribus** – so long as the money demand curve remains unchanged

The Quantity Theory of Money

- **The quantity theory of money asserts that:**
  - **the quantity of money available determines the price level**
  - **the growth rate of the quantity of money available determines the inflation rate – the percentage increase in the price level** (in the U.S.. the inflation rate averaged about 2 percent per year during the 1990s)
- **The quantity of money available is closely related to the number of dollars exchanged in transactions.**
- **The quantity equation links Money and Transactions:**

$$\text{money} \times \text{velocity} = \text{price} \times \text{transactions}$$
$$M \times V = P \times T$$

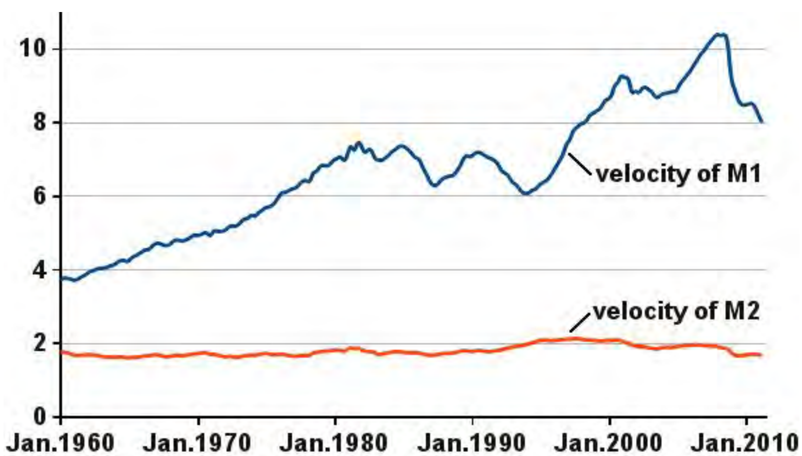
- the version of velocity given here is the transactions velocity of money – the rate at which a dollar bill circulates in the economy
  - **If the economy only produces hamburgers and the economy produces** (and therefore sells) **100 hamburgers per year, then there are 100 transactions in the economy every year.**
  - **If the price of a hamburger is $1 and there are quantity of money available in the economy is $20, then the velocity is 5 per year – each dollar bill changes hands five times per year**

The Quantity Equation

- **In practice, it's difficult to measure the number of transactions conducted in a year.** As we observed above however, the number of transactions increases when output (income) rises, **so the dollar value of transactions is roughly proportionate to the dollar value of output.**
- **with a definitional change we can rewrite the quantity equation as:**

$$\text{money} \times \text{velocity} = \text{price} \times \text{output}$$
$$M \times V = P \times Y$$

- the version of velocity here is the income velocity of money – the number of times a dollar enters a person's income in a year
- **We'll assume that velocity is constant, but this might not be a good assumption**



source: Federal Reserve H.6 and BEA NIPA Table 1.1.5

Money Demand and the Quantity Equation

- **When we introduced the demand for money we examined the demand for nominal money balances** – holdings of dollar bills
- **however, our ultimate goal is to analyze the way in which money affects the economy, so it's convenient to express the quantity of money in terms of the quantity of goods and services money can buy**
  - For example, if you hold a nominal money balance of $50 and the price of a hamburger is $2, then you can buy 25 hamburgers
  - the 25 hamburgers is your real money balance
  - **real money balances** are equal to M/P
- using the assumption that velocity is constant, $\overline{V}$, **we can use the quantity equation to derive a simple demand function for real money balances:**

$$M \cdot \overline{V} = P \cdot Y \;\Rightarrow\; \frac{M}{P} = \frac{Y}{\overline{V}} \;\Rightarrow\; (\mathbf{M/P})_{\mathbf{D}} = \mathbf{kY} \qquad \text{where}: \overline{V} = 1/k$$

- **The money demand function we just derived states that the economy's demand for real money balances is proportionate to GDP** – which is consistent with the demand function described earlier
- The interest rate doesn't appear in this demand function, but don't worry. We'll put it back in later.

Money, Prices and Inflation

- **Nominal variables are variables measured in monetary units**
  - Nominal GDP is the total value of final goods and services produced in a country in a given period – it is measured in terms of the price level that prevailed during that period
- **Real variables are variables measured in physical units**
  - Real GDP figures can be used to measure differences in output levels over time, because the prices used to calculate Real GDP are constant over time

♦ ♦ ♦

- **Recall that the quantity theory of money asserts that the quantity of money available determines the price level**
  - If velocity is constant: $\overline{V}$
  - If the levels of physical capital, human capital, labor and technology determine the level of output, then real GDP, at a given moment in time is constant: $\overline{Y}$
  - Then the quantity equation becomes:

$$M \cdot \overline{V} = P \cdot \overline{Y}$$

  - and the quantity of money available determines the price level and nominal GDP, $P \cdot Y$

Money, Prices and Inflation

- **Recall also that the quantity theory of money asserts that the growth rate of the quantity of money available determines the inflation rate – the percentage increase in the price level**
- **To see this, we'll use our calculus tricks:**

$$M(t)\cdot V(t)=P(t)\cdot Y(t) \quad \Rightarrow \quad \frac{\dot{M}}{M}+\frac{\dot{V}}{V}=\frac{\dot{P}}{P}+\frac{\dot{Y}}{Y}$$

$$\%\Delta \text{ money}+\%\Delta \text{ velocity}=\%\Delta \text{ prices}+\%\Delta \text{ output}$$

  - **The percentage change in the quantity of money available is under the control of the Fed.**
  - **We've assumed that velocity is constant, so the percentage change in velocity is zero.**
  - **The percentage change in prices is the inflation rate.**
  - **The percentage change in output depends on the growth rate of the factors of production and technological progress.**
- **So the inflation rate depends on the rate of growth of output and the growth rate of the money supply.**
- **The quantity theory of money therefore implies that the Fed has the ultimate control over the inflation rate.**

The Fisher Equation

- **If you saved \$100 by purchasing a one-year government bond that provides a \$10 return** (i.e. a 10 percent rate of return)**, then will you be \$10 richer at the end of the year?**
- **Not necessarily.**
  - **If the inflation rate was 15 percent, then your end-of-year \$110 would not be able to buy as much as the beginning-of-year \$100**
  - **In fact, your purchasing power would have fallen by 5 percent**
  - **Only if the inflation rate was <u>less</u> than 10 percent, would your end-of-year \$110 be worth more than the beginning-of-year \$100**

- **nominal interest rate – rate of return that you receive on a government bond or other financial asset**
- **real interest rate – the percentage change in your purchasing power that you receive after accounting for inflation**

$$\text{real interest rate} \quad = \quad \text{nominal interest rate} \quad - \quad \text{inflation rate}$$
$$r=i-\pi$$

- **the real interest rate is the difference between the nominal interest rate and the rate of inflation**

The Real Interest Rate

- **the real interest rate, r, is NOT the rental rate on capital, BUT the two concepts are related to each other.**
    - the real interest rate is expressed as a percentage
    - the rental rate on capital is expressed in dollars (per unit of capital)
- **Lecture 3 mentions a normal rate of return on capital, which equals** (or exceeds) **the real interest rate on risk-free government bonds**

$$\text{normal rate of return on capital} = \frac{\text{rental rate on capital (in dollars)}}{\text{purchase price of capital (in dollars)}}$$

- **Since firms hire capital until the rental rate on capital equals the price of output times the marginal product of capital, i.e. $p \cdot MPK$,**
    - an increase in the price of output will not affect the normal rate of return on capital
    - so long as the purchase price of capital increases at the same rate as the price of output
- **the real interest rate therefore corresponds to the normal rate of return on capital and should be fairly constant**
- **in the long run, the real interest rate should only change if the marginal product of capital changes**

The Real Interest Rate

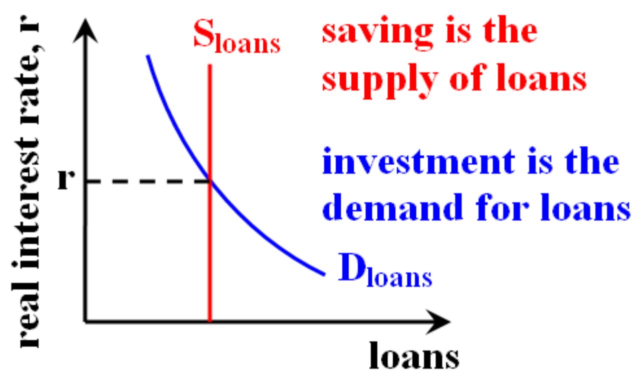
- **So what would cause the marginal product of capital to change?**
- **As we saw in the lectures on economic growth the marginal product of capital shifts outward when:**
    - the stock of human capital increases
    - the labor force increases
    - the level of technology increases
- **We also saw that** – for given levels of the stock of human capital, the labor force and the level of technology – **the marginal product of capital is a decreasing function of the stock of physical capital**
    - an underdeveloped economy – i.e. an economy with less capital per unit of effective labor – will have a higher marginal product of capital and a higher real interest rate than
    - a more developed economy – i.e. an economy with more capital per unit of effective labor – since the more developed economy will have a lower marginal product of capital

♦ ♦ ♦

- **Since the real interest rate reflects $p \cdot MPK$, the real interest rate determines how much capital firms wish to hire:**
    - Firms invest less in new capital when the real interest rate is higher
    - Firms invest more in new capital when the real interest rate is lower

The Real Interest Rate

- **Since investment in new capital is a decreasing function of the real interest rate, we can draw a downward sloping investment demand schedule**
- **In the lectures on economic growth, I asserted that the saving rate is exogenous and does not depend on the real interest rate**
  - if the saving rate is exogenous, then the supply of loanable funds needed to finance new investment would be represented by the vertical line in the graph shown below
  - if the saving rate depends on the real interest rate, then the supply of loanable funds would have the usual upward slope
- **In a closed economy, the equilibrium interest rate adjusts to ensure that saving equals investment.**
- I've asserted that firms use loans to finance capital investment, but similar supply and demand schedules describe other sources of finance.

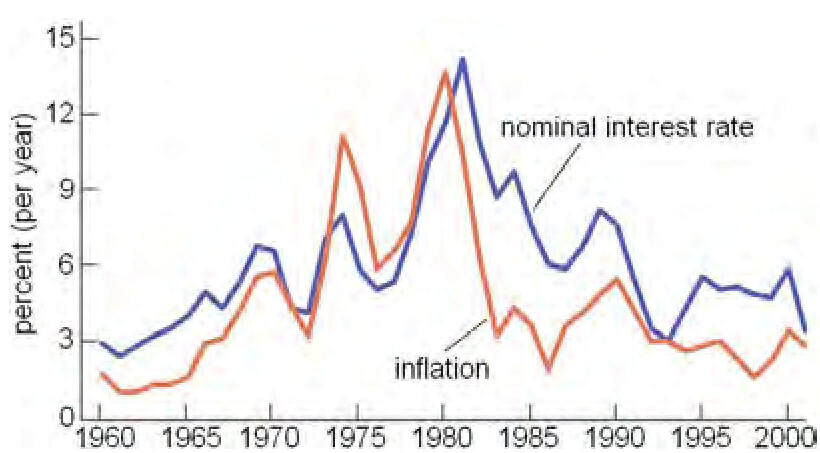


The Fisher Effect

- **Rewriting the Fisher Equation:**

$$i = r + \pi$$

  shows that the nominal interest rate can change for two reasons:
  - because the real interest rate changes or
  - because the inflation rate changes
- **since the real interest rate is fairly constant, there should be a one-to-one adjustment of the nominal interest rate to the inflation rate**



- **the figure above illustrates the Fisher Effect – a one percentage point increase** (decrease) **in the inflation rate <u>should</u> increase** (decrease) **the nominal interest rate by one percentage point**

The Fisher Effect

- **So why isn't there a perfect correlation between the nominal interest rate and the inflation rate?**
  - one explanation is a shift in the marginal product of capital
  - a better explanation involves expectations

Two Real Interest Rates: Ex-Ante and Ex-Post

- **when a borrower and a lender agree on a nominal interest rate, they do not know what the inflation rate will be over the term of the loan**
- **so they try to predict what the inflation rate will be based on:**
  - past inflation rates and
  - the statements and behavior of the Fed
- **the ex-ante real interest rate is the nominal interest rate minus the expected inflation rate:** $i - \pi^e$, where π^e is the expected inflation rate
- **the ex-post real interest rate is the nominal interest rate minus the actual inflation rate:** $i - \pi$, where π is the actual inflation rate
- **Since the nominal interest rate can only adjust to expected inflation, the Fisher Effect is more precisely written as:**

$$i = r + \pi^e$$

The Demand for Money

- **When we discussed the Transactions Motive and the Speculation Motive, we saw that the demand for money is a decreasing function of the nominal interest rate – the opportunity cost of holding money**
- **When we discussed the Quantity Theory of Money we derived a demand for money that depends on income**
- **So, in general, the demand for real money balances is a function of the nominal interest rate and income:**

$$(M/P)_D = L(i, Y)$$

where L is used to denote the demand for the liquid asset – money

- **Equating the supply of real money balances to the demand for real money balances and using the Fisher Equation, we can write:**

$$M/P = L(r + \pi^e, Y)$$

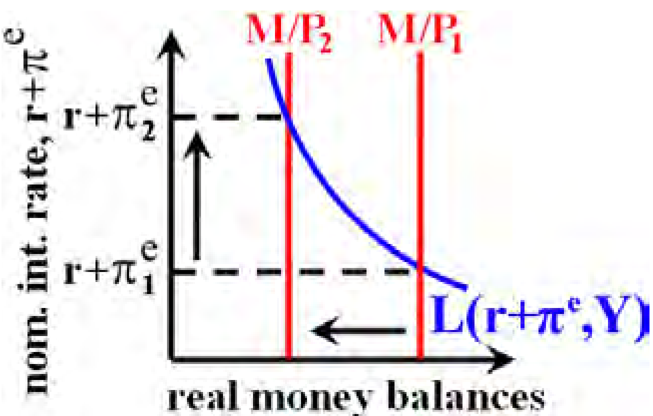
- **The equation above tells us that when the nominal interest rate and output are held constant, the quantity of money available will determine the price level – as the Quantity Theory of Money asserts**
- **However, this is not the end of the story …**

The Demand for Money

- **The nominal interest rate is not constant over time. It depends on expected inflation which depends on:**
  - the rate of money growth
  - past inflation rates and
  - the behavior and statements of the Fed
- **So the price level depends not just on today's money supply, it also depends on the money supply that is expected to prevail in the future**
- **For example, if the Fed were to announce that it would increase the money supply *next year*, then:**
  - people would expect higher inflation
  - nominal interest rates would rise *today* and
  - people would *immediately* reduce their holdings of real money balances
- **Today however, the Fed still has not increased the money supply, so the only way the level of real money balances can fall is if the price level rises**
- **The expectation of future inflation causes inflation – in the form of an immediate increase in the price level.**

The Demand for Money

- **Below is a graphical illustration of how the expectation of inflation causes inflation** (discussed in the previous slide)**:**



- **Remember that we're discussing money and inflation in the long run.**
- **In the long run, the real interest rate is fairly constant, but inflationary expectations can change.**
- In Lecture 12 (when we discuss the money market in the short run), we'll assume that – in the short run – inflationary expectations are constant and allow the real interest rate to change.

The Costs of Inflation

- **shoeleather costs – the resources wasted when inflation encourages people to reduce their money holdings** (and make more frequent shifts from interest-bearing assets, like bonds and savings accounts, into money)
- **menu costs – the costs of adjusting prices**
  - During periods of high inflation, it is necessary to update price lists and other posted prices more frequently
  - This is a resource-consuming process that takes resources away from other productive activities
- **relative price variability** –
  - if a firm issues a new catalog every January and the annual inflation rate is 12 percent, then the real price of the firm's products relative to other products will be 12 percent lower at the end of the year
  - Since inflation distorts relative prices, consumer decisions are distorted and markets are less able to allocate resources to their best use
- **tax distortions – inflation exaggerates the size of capital gains**
  - If you were to buy a stock for $100 today and sell it for $112 a year from now, you would have to pay a tax on your $12 capital gain
  - but if the inflation rate over the year were 12 percent, then your capital gain merely reflects inflation. Nonetheless, you get taxed anyway.

The Costs of Inflation

- **confusion and inconvenience** –
  - inflation erodes the real value of the unit of account because dollars at different times to have different real values
  - inflation therefore makes it more difficult to compare real revenues, costs and profits over time
- **arbitrary redistribution of wealth** –
  - When inflation is unexpected, ex-post real interest rates are lower, thus transferring wealth from lenders and to borrowers
  - Such transfers redistribute wealth in a way that has nothing to do with either need or merit

So why would a central bank ever allow its country to endure hyperinflation?

- **One reason why countries may experience hyperinflation – a rate of inflation that exceeds 50 percent per month – is because the government receives revenue from printing money**
- **Another reason is because disinflation – the process of reducing inflation – can cause high unemployment in the <u>short</u>-<u>run</u>** (we'll discuss this more in Lecture 14)

Monetary Neutrality

- **On the last page, I wrote that disinflation can cause unemployment in the short-run. Can it cause unemployment in the long run? NO.**
- **In the long run, the level of unemployment depends on the natural rate of unemployment., which is caused by:**
  - frictional unemployment and
  - structural unemployment

- **In this lecture, we also saw that the quantity of money available determines the price level and Nominal GDP**
- **Can the quantity of money available or the rate of growth of the money supply ever affect Real GDP? Yes, but only in the short run.**
- **In the long run, physical capital, human capital, labor and technology determine Real GDP – the level of output**

- **monetary neutrality – In the long run, the money supply does not affect real economic variables – such as output and the level of employment**

The Classical Dichotomy

- **According to the classical dichotomy:**
  - different forces influence real and nominal variables
  - changes in the money supply affect nominal variables but not real variables

- **Most economists believe that the classical dichotomy holds in the long run, but not in the short run**
- **As we'll see in Lectures 12, 13 and 14, the economy is subject to fluctuations in the short-run which can be affected by** (or caused by) **changes in the money supply**

Homework #10

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Do this too!

Suppose that the demand for real money balances is given by:

$$\frac{M}{P} = 100 - 100 \cdot \left(r + \pi^e\right) + Y \quad \text{where:} \begin{cases} M = \text{quantity of money available} \\ P = \text{price level} \qquad \pi^e = \text{expected inflation rate} \\ r = \text{real interest rate} \quad Y = \text{income} \end{cases}$$

a. Assume that $Y = 50$. Graph the demand for real money balances by placing real money balances on the horizontal axis and placing the nominal interest on the vertical axis.

b. Suppose that $M = 12{,}000$ and $P = 100$. On the same graph, draw the supply of real money balances. What is the equilibrium nominal interest rate? If $\pi^e = 0$, what is the real interest rate?

For the remainder of the problem, assume that this value of the real interest rate corresponds to the normal rate of return on capital.

c. Now suppose that income increases to $Y = 75$, while the supply of real money balances remains unchanged. What is the new equilibrium nominal interest rate?

d. Now suppose that the Fed is unhappy with the increase in the nominal interest rate that it has just observed. If it increases the money supply to $M = 15{,}000$ and if the price level remains unchanged, what will the new nominal interest rate be?

Now suppose that people believe that the increase in the money supply will cause prices to rise. Specifically, the money supply has increased 25 percent, so people believe the price level will increase 25 percent (i.e. $\pi^e = 0.25$).

e. Using the value of the real interest rate that you found in part **a.** of this problem, calculate the value to which the nominal interest rate will converge.

f. If the money supply and income remain unchanged, then how will the money market converge to this value of the nominal interest rate? Do you see any interesting relationships among the values you calculated?

Lecture 11

Economic Fluctuations: the Goods Market

Eric Doviak

Economic Growth and Economic Fluctuations

Short Run versus Long Run

- **In the previous lectures, we studied output, employment, money, inflation and interest rates in the long run. In studying these concerns, we followed the Classical Dichotomy in assuming that:**
  - different forces influence real and nominal variables
  - money affects nominal variables, not real variables
- **In the long run, the level of output is determined by:**
  - the stock of physical capital
  - the size of the labor force
  - the stock of human capital,
  - and technology
- **In the long run, unemployment is:**
  - frictional – because it takes time to match workers to jobs
  - and/or structural – because the wage rate is set at a level above the one that clears the labor market
- **In the long run, money the quantity of money available determines the price level, while the rate of money growth determines the inflation rate**
- **We'll now turn our attention to the short run, where:**
  - output does not entirely depend on the available factors of production
  - there's a trade-off between unemployment and inflation

Aggregate Output and Income

- **Aggregate output – is the total quantity of goods and services produced by an economy in given period.**
- **Aggregate output refers to <u>real</u> output, NOT <u>nominal</u> output:**
  - This lecture assumes that **prices are "sticky"** in the short run.
  - Unless the inflation rate is extremely high, firms do not immediately adjust their prices in response to changes in the money supply.
- **Aggregate income – is the total income received by all factors of production in a given period**
- **We'll use Y to denote both aggregate output and aggregate income to remind us of the exact equality between the two.**
- **To begin this lecture, we'll examine an economy in which:**
  - **there is no international trade** (so net exports equal zero) **and**
  - **government purchases equal zero**
- **So – when the goods market is in equilibrium – output is divided among consumption and investment:**

$$Y = C + I$$

- **Later on, we'll add taxation and government purchases to the model**

The Allocation of Aggregate Income

- **In the absence of government, the income of the households in the economy, Y, is divided among:**
  - **consumption** of goods and services, denoted: C
  - **saving,** denoted: S
- **Saving is the income that not consumed in a given period, i.e.:** $S \equiv Y - C$
- In reality, aggregate consumption is determined by households' income, households' wealth, interest rates and expectations about the future
- **For simplicity however, we'll only focus on income in this lecture**

The Keynesian Consumption Function

- **For simplicity, we'll assume that aggregate consumption is a linear function of income:**

$$C = a + b \cdot Y \qquad \text{where: } 0 \leq a \quad \text{and} \quad 0 < b < 1$$

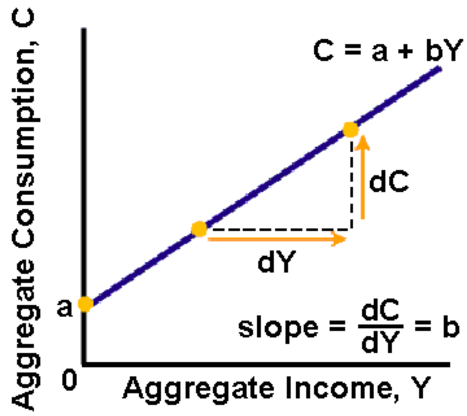
- The microeconomic foundations of such a consumption function are weak at best. However, it does capture two important elements of reality:
  - households only save after surpassing a certain level of income, a
  - when households' income increases (ex. after receiving a tax cut) some of that additional income will be saved and some of it will be consumed

The Keynesian Consumption Function

- The slope of the consumption function, b, is the **marginal propensity to consume** (MPC) – the fraction of an additional increment in income that is consumed.
- For example, if the consumption function were given by:

$$C = 100 + 0.75 \cdot Y$$

then for every \$100 increase in income, consumption rises \$75 and saving rises \$25



- Since income can only go to either consumption or saving, the fraction of an additional increment in income that is not consumed is the fraction saved – which we'll call the **marginal propensity to save** (MPS).

$$MPC + MPS = 1 \quad \rightarrow \quad MPS = 1 - MPC \quad \rightarrow \quad MPS = 1 - b$$

- Once we know how much consumption will result from a given level of income, we also know how much will be saved since: $S \equiv Y - C$

Investment

- Recall from Lecture 4 that investment consists of:
  - goods that firms and households purchase for future use
    - new plants and equipment
    - new housing
  - and **inventory investment** – investment to meet future demand
    - this is positive when firms add to their inventories
    - this is negative when firms run down their inventories
- inventory investment is partly determined by how much households decide to buy, which is not under the complete control of firms

$$\text{change in inventory} = \text{production} - \text{sales}$$

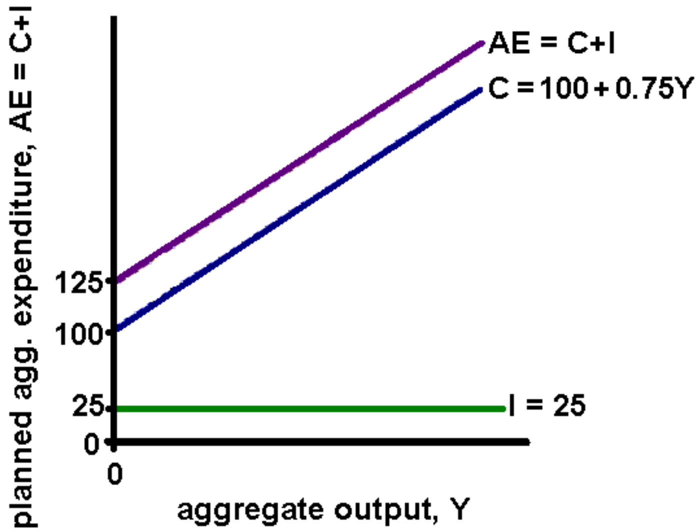
- **planned investment** – is the planned component of additions to the capital stock and inventory
- **actual investment** – includes unplanned changes in inventories
  - sometimes demand falls short of the amount that firms predicted. In such a case, firms add more to their inventories than they had planned
  - other times demand exceeds the amount that firms predicted. In such a case, firms run down their inventories more than they had planned

Planned Aggregate Expenditure

- **For now, assume that planned investment is fixed. It does not respond to changes in income, interest rates, etc.** In the example which follows, we'll assume that planned investment is given by: I = 25
- **Also, continue to assume that consumption is given by:** C = 100 + 0.75·Y
- **Planned aggregate expenditure, AE, is the sum of consumption and planned investment** (at each level of aggregate income)**:**

  AE = C + I

  AE = 125 + 0.75·Y



Equilibrium Aggregate Output

- **Equilibrium in the goods market occurs when planned aggregate expenditure equals aggregate output, i.e.:**

  Y = AE

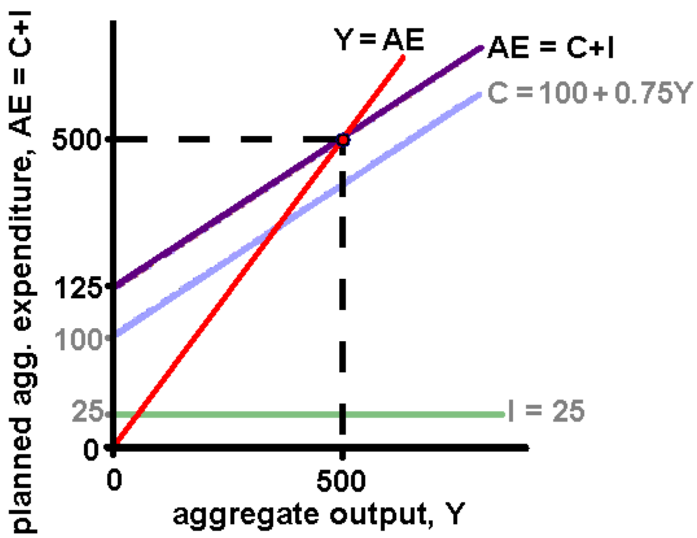
- **In our example, this occurs when** Y = AE = 500 **because:**

  Y = 125 + 0.75·Y

  (1 – 0.75)·Y = 125

  0.25·Y = 125

  $Y = \frac{125}{0.25} = 500$



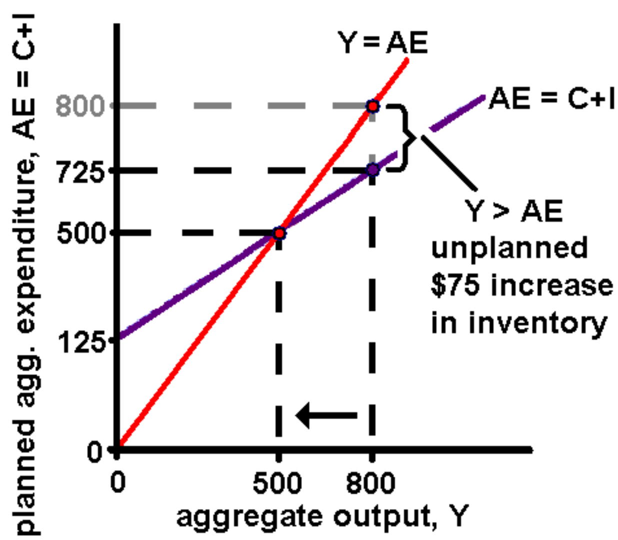
- **The red line in the graph above represents equilibrium in the goods market.**
- **At each point along the red line planned aggregate expenditure equals aggregate output.**

unplanned inventory investment

- **Now remember that there is no unplanned consumption**
- **And remember that firms plan to invest a fixed amount in a given year**
- **What happens if aggregate output exceeds planned aggregate expenditure? That is: What happens if firms produce more than consumers plan to purchase – given their** (aggregate) **income?**

$$Y > AE \quad \rightarrow \quad Y > C + I$$

- **In such a case, firms' inventory would increase by more than they had planned, i.e.** they face unplanned inventory investment
- **To restore equilibrium, firms need to reduce their production level – reducing real GDP – from \$800 to \$500**
- **When firms reduce their production level, they lay off workers.**

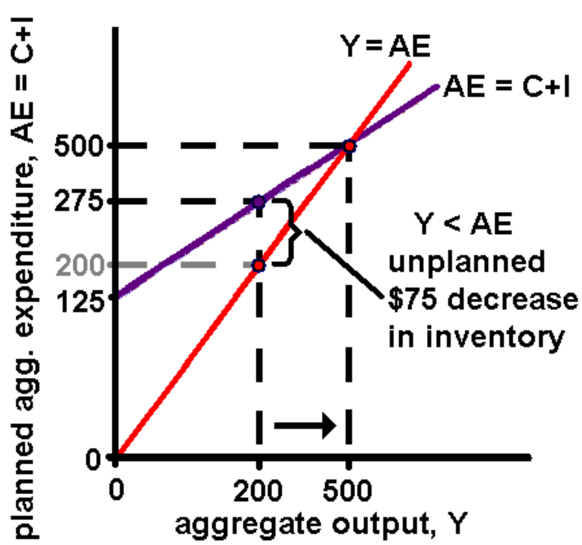


unplanned inventory dis-investment

- **What happens if aggregate output falls short of planned aggregate expenditure? That is: What happens if firms produce less than consumers plan to purchase – given their** (aggregate) **income?**

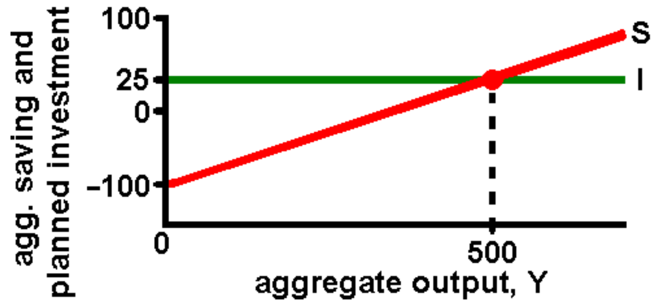
$$Y < AE \quad \rightarrow \quad Y < C + I$$

- **In such a case, firms' inventory would decrease by more than they had planned, i.e.** they face unplanned inventory dis-investment
- **To restore equilibrium, firms need to raise their production level – raising real GDP – from \$200 to \$500**
- **When firms raise their production level, they hire more workers.**



Saving and Investment

- **Aggregate output will only be equal to planned aggregate expenditure when saving equals planned investment, i.e.:** $S = I$



- **To see this recall that:**

  $S \equiv Y - C$
  $S = Y - 100 - 0.75 \cdot Y$
  $S = 0.25 \cdot Y - 100$

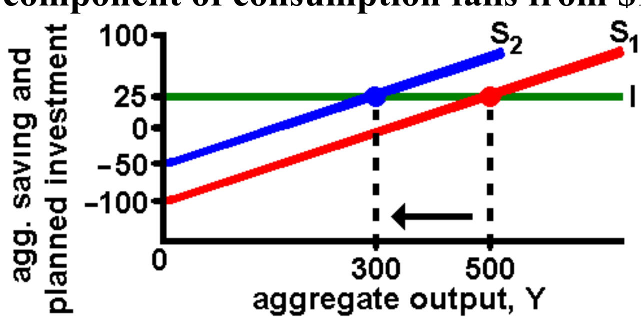
- **then equate saving to planned investment to find that:**

$$\begin{aligned} I &= S \\ 25 &= 0.25 \cdot Y - 100 \\ 125 &= 0.25 \cdot Y \\ Y &= 500 \end{aligned}$$

- **which is the same equilibrium level of aggregate output as before.**

The Paradox of Thrift

- **When households are concerned about the future, they save more.**
- **As we saw in the lectures on economic growth, increased saving leads to a higher level of output per worker in the long run.**
- **But what happens in the short run? Would equilibrium output rise?**
- **To examine this question, suppose the consumption function changes from** $C = 100 + 0.75 \cdot Y$ **to** $C = 50 + 0.75 \cdot Y$**. Would saving rise because the autonomous component of consumption falls from \$100 to \$50? No.**



- **If planned investment remains unchanged, then equilibrium output would fall and** – in their attempt to save for future hard times – **households have inadvertently created the hard times they feared.**
- **They end up consuming less, but they have not saved any more.**

How Realistic is the Paradox of Thrift?

- **At first glance, the Paradox of Thrift looks very unrealistic.**
  - Could the fear of future hard times really cause a major economic recession?
  - If saving is good for long-run economic growth, then how could it damage the economy in the short run?
- **In reality, it's not that the fear of hard times that makes people save more. It's the fact that they have no choice.**
- **Remember: Repaying a loan is a form of saving.** In other words, a borrower "dis-saved" when he/she took out the loan, so he/she must now save by repaying the loan.
- If a homeowner is struggling to repay his/her home mortgage, then he/she will reduce his/her consumption of "luxury" items in order to save and repay the mortgage.
- So the homeowner cuts back on present consumption to pay for the home that he/she purchased in the past.
- **It's the weight of past debt that causes the recession today!**

Adding a Government Sector to the Model

- **If recessions are caused by an increase in saving, then there is a very simple way to alleviate the short-run problems caused by thrift: tell people to start consuming more.**
- **But is such a solution realistic?**
- **Maybe not, but we haven't introduced a government sector yet.**
- **So now let's divide** (equilibrium) **output among consumption, investment and government purchases:**

$$Y = C + I + G$$

- **Of course, government purchases,** G**, must be financed by tax revenues,** T**, so we now have to allow consumption to be a function of disposable income – income net of taxes:** $Y_d \equiv Y - T$

$$C = a + b \cdot (Y - T) \quad \text{where: } 0 \le a \text{ and } 0 < b < 1$$

- For simplicity, we'll assume that tax revenues and government purchases are **autonomous variables** (i.e. they do not depend on the state of the economy), so that changes in taxes or spending result from deliberate changes in government policy.

Adding a Government Sector to the Model

- **If the government's budget is balanced, then:** $G = T$
- **With the exception of four fleeting years** (1998, 1999, 2000 and 2001)**, the US government has consistently run a budget deficit,** $G > T$**, since 1970.**
- **When the government runs a budget deficit:**
  - it finances the deficit by selling Treasury bills, notes and bonds
  - and some saving goes to the government sector

♦♦♦

- **As before, disposable income can be either saved or spent:**

$$Y - T \equiv C + S \quad \textbf{which implies that:} \quad Y \equiv C + S + T$$

- **Similarly, planned aggregate expenditure can either be consumed by households, invested or consumed by the government:**

$$AE \equiv C + I + G$$

- **Equilibrium in the goods market still occurs when aggregate output equals planned aggregate expenditure,** $Y = AE$
- **but now aggregate output will only be equal to planned aggregate expenditure when:** $S + T = I + G$
  - Equilibrium does not require the government's budget to be balanced
  - But if $G > T$, then $S > I$, so that some saving goes to the government

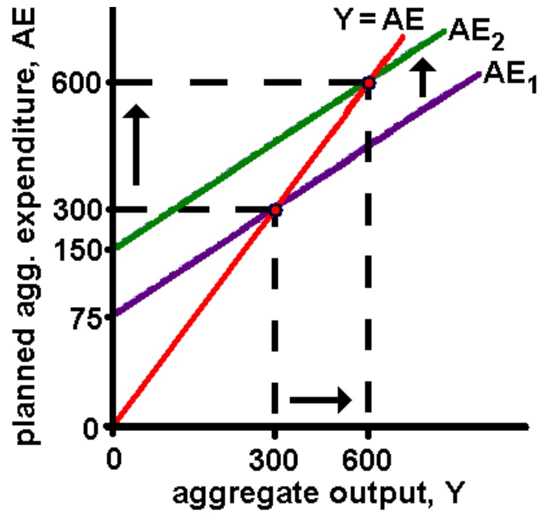
A Solution to the Short-Run Problem of Thrift

- **Now let's say you're the President of the United States.**
  - During the first three years of your administration, the government has not made any purchases, nor has it levied any taxes, i.e. $G = 0$ and $T = 0$
  - Investment has been humming along as planned at $I = 25$ each year.
- **This year, Americans decide that they have borrowed too much and they've decided to increase their saving, by altering their** (aggregate) **consumption function from** $C = 100 + 0.75 \cdot Y$ **to** $C = 50 + 0.75 \cdot Y$**.**
- **As we saw earlier, this causes equilibrium output to fall from \$500 to \$300 as firms reduce the amount they produce by firing workers.**

But you face a re-election campaign this year!

- **The American people will vote you out unless you do something fast!**
- **Fortunately, you read Prof. Doviak's** ***Lecture Notes*** **and you quickly realize that you can increase aggregate output in the short run by increasing government purchases.**
- **Now you don't want to raise taxes in an election year, so you leave** $T = 0$ **and you sell bonds to fund government purchases at a level of** $G = 75$

Why You Win the Election

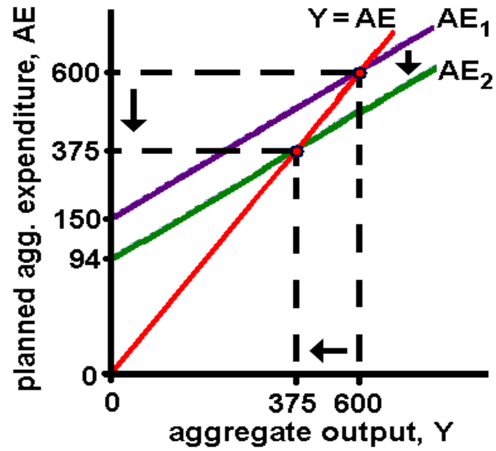


- **Had you left government purchases at G = 0, equilibrium output would have been $300**
- **but because you increased government purchases to G = 75, planned aggregate expenditure rises and equilibrium output rises to $600**
- **to produce that higher level of output, firms have to raise employment**
- **since employment increases, workers think you are a genius and re-elect you in a landslide**
- **Notice that output rose $300 while government purchases only rose $75**
  - output rose four times more than the increase in government purchases
  - the **government spending multiplier** – the ratio of the change in equilibrium output to the change in government purchases – **is four**

After the Election

- **Now notice that because there was a budget deficit of $75, those $75 become part of the government debt.**
- **Unless you balance the budget in the next year, the budget deficit will again be $75 and the government debt will rise to $150** (for simplicity, we'll assume that interest rates are zero)
- **After the election, members of Congress start to worry about the debt and they beg you to balance the budget.**
- **Since you no longer have to face another election, you agree.**
- **There are two ways to balance the budget:**
  - You can cut government purchases back down to zero OR
  - You can raise taxes by $75
- **Because you read Prof. Doviak's *Lecture Notes*, you know that:**
  - the **tax multiplier** – the ratio of the change in equilibrium output to the change in taxes– **is smaller than the government spending multiplier**
  - so a tax increase will have a smaller effect on aggregate output than reducing government spending by the same amount
- **So you decide to raise taxes by $75**

The Effects of the Tax Increase



- **since you leave government purchases unchanged at** G = 75 **and**
- **increase taxes from** T = 0 **to** T = 75
- **equilibrium output now falls from $600 to $375**
- **because the increase in taxes reduces consumption and therefore lowers planned aggregate expenditure**
- **since firms now produce less output, firms have to reduce employment**
- **Notice that output fell $225 while taxes only rose $75**
  - output fell three times more than the increase in taxes
  - the **tax multiplier** – the ratio of the change in equilibrium output to the change in taxes – **is negative three**
  - the tax multiplier is smaller than the government spending multiplier
- **Notice also that the tax increase balances the budget, but it does not eliminate the government debt. To eliminate the debt, another painful tax increase and/or government spending decrease must be passed.**

The Multipliers

- **In the pages above, I told you what the multipliers are, but I did not explain how I obtained them. In equilibrium:**

$$Y = C + I + G$$

- **Plugging the consumption function,** $C = a + b\cdot(Y-T)$, **into the equation for equilibrium output, we have:**

$$Y = a + b\cdot(Y-T) + I + G$$

- **Rearranging terms slightly:**

$$Y\cdot(1-b) = a + b\cdot T + I + G$$

- **we can solve for the equilibrium level of output:**

$$Y = \frac{a - b\cdot T + I + G}{1 - b}$$

- **Notice that the terms on the right-hand side of this last equation are all autonomous variables.**
- **Now that we have a obtained an equation which defines the equilibrium level of output entirely in terms of autonomous variables, we can use our calculus tricks to find the multipliers.**

The Multipliers

- The **government spending multiplier** is the derivative of the last equation with respect to government spending:

$$\frac{dY}{dG} = \frac{1}{1-b}$$

- The **tax multiplier** is the derivative of the last equation with respect to taxes:

$$\frac{dY}{dT} = \frac{-b}{1-b}$$

- **Now recall that** b **is the marginal propensity to consume and** $1 - b$ **is the marginal propensity to save. Therefore:**

$$\frac{dY}{dG} = \frac{1}{MPS} \quad \textbf{and} \quad \frac{dY}{dT} = \frac{-MPC}{MPS}$$

- **We assumed that the consumption function is given by** $C = 50 + 0.75 \cdot Y$, **so we've assumed that** $b = 0.75$. **Therefore:**

$$\frac{dY}{dG} = \frac{1}{1-0.75} = 4 \quad \textbf{and} \quad \frac{dY}{dT} = \frac{-0.75}{1-0.75} = -3$$

Automatic Stabilizers

- **As you should have observed in the discussion of your attempts to ensure victory in the "presidential election," fiscal policy can be used to soften recessions.**
- **In reality, Congress and the President and state lawmakers don't need to suddenly change the course of fiscal policy when they perceive an imminent recession.**
- **Federal and state budgets contain automatic stabilizers – revenue and expenditure items in a budget that automatically adjust to the state of the economy in such a way as to stabilize GDP.**
  - For example, personal and corporate income tax revenues fall when economy goes into recession
  - Governments may also spend more on job placement assistance during recessions
  - Unemployment benefits are **NOT** an automatic stabilizer however, since they merely transfer income from one group of people to another
- **Similarly, when the economy is experiencing a vigorous expansion, there is fiscal drag – average tax rates increase because some taxpayers move into higher income brackets during the expansion**

Homework #11

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Homework #11
(continued)

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Lecture 12

Economic Fluctuations: the Goods and Money Markets

Eric Doviak

Economic Growth and Economic Fluctuations

the Goods and Money Markets

- **In Lecture 10, when we studied the money market in the long run, we saw that the demand for investment is a decreasing function of the real interest rate**
- **In Lecture 11, when we studied the goods market in the short run, we assumed that planned investment is fixed**
- **But it is NOT reasonable to assume that planned investment is fixed in the short run, so we'll relax that assumption in this lecture**

♦ ♦ ♦

- **In Lecture 10, we saw that increases in output increase the demand for real money balances, which increases the nominal interest rate**
- **In Lecture 11, we saw that expansionary fiscal policy** – increases in government spending and decreases in taxes – **increases aggregate output**
- **But in Lecture 11, we did not mention how expansionary fiscal policy would affect interest rates**

♦ ♦ ♦

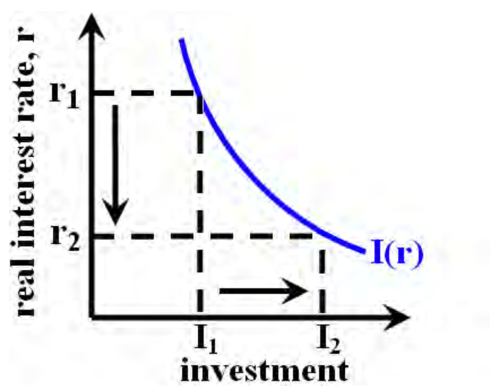
- **The goods market and the money market do not operate independently, so we need to examine the links between the two**

Sticky Prices

- **This lecture assumes that prices are "sticky" in the short run.**
  - Unless the inflation rate is extremely high, firms do not immediately adjust their prices in response to changes in the money supply.
  - Moreover, it may take time for households and firms to change their expectations of future inflation rates in response to changes in the rate of money growth.
- So **expansionary monetary policy** – increases in the money supply, which lower the nominal interest rate – **also lowers the real interest rate in the short run if inflationary expectations do not change quickly**
- **Both the goods and money markets are simultaneously in equilibrium:**
  - at a certain level of aggregate output (income), Y
  - at and a certain level of the interest rate, r
- **As we'll see in this lecture, the requirement that both the goods and money markets be in equilibrium implies that expansionary monetary policy can increase aggregate output in the short run**
- **In this lecture, we'll also see that expansionary fiscal policy** – increases in government spending or a reduction in taxes – **increase aggregate output and increase the real interest rate in the short run**

Real Interest Rate and Investment

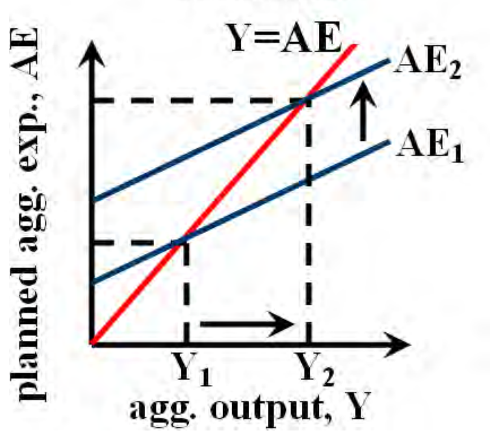
- **Recall from Lecture 10 that planned investment in new capital is a decreasing function of the real interest rate**
- **So if the real interest rate were to fall planned investment in new capital would increase**



- **Next recall from Lecture 11 that planned aggregate expenditure depends in part on planned investment:**

$$AE = C + I + G$$

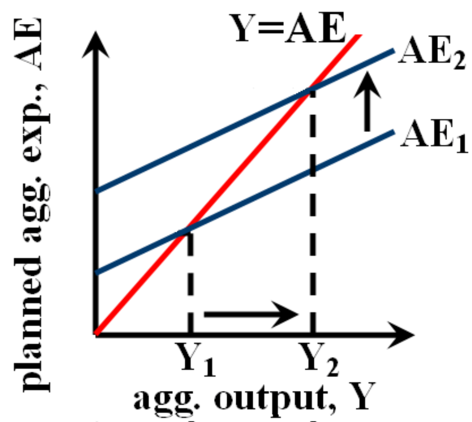
- **So if the real interest rate were to fall, the increase in planned investment would increase equilibrium output**

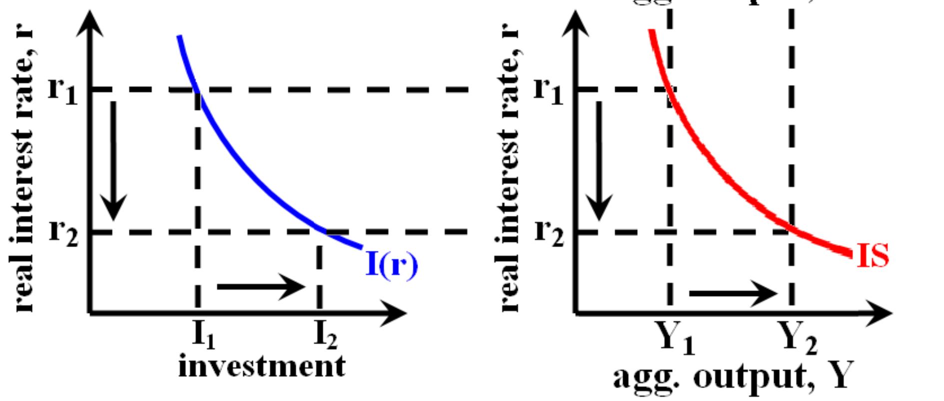


the IS curve

1. A decrease in the interest rate
2. raises planned investment which
3. increases planned aggregate expenditure
4. and raises equilibrium aggregate output
5. The IS curve summarizes these changes

The IS curve shows the combinations of aggregate output and real interest rate for which the goods market is in equilibrium.





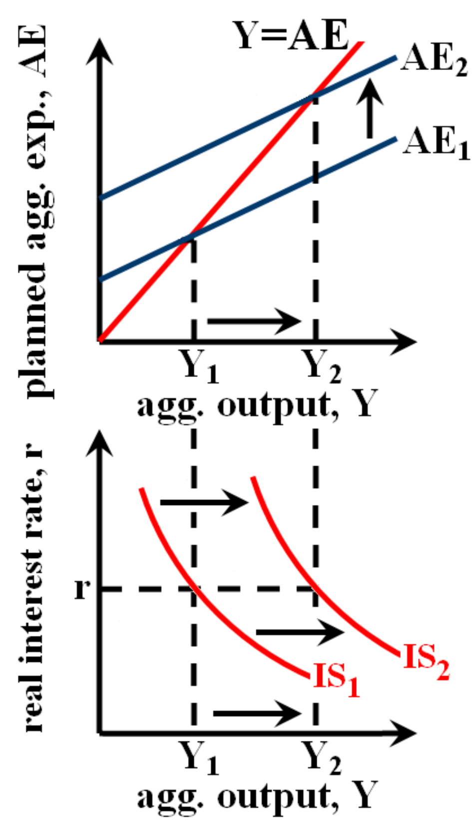
Shifts of the IS curve

- **The IS curve is drawn for a given fiscal policy, i.e. for a given level of government spending and taxation**
- **If the government were to pursue an expansionary fiscal policy –** i.e. increase government spending or decreases taxes
  - **planned aggregate expenditure would increase**

$$
\begin{aligned}
AE &= C + I + G \\
&= a + b\cdot(Y - T) + I + G
\end{aligned}
$$

  - **raising equilibrium output**
  - **and shifting the IS curve outward**

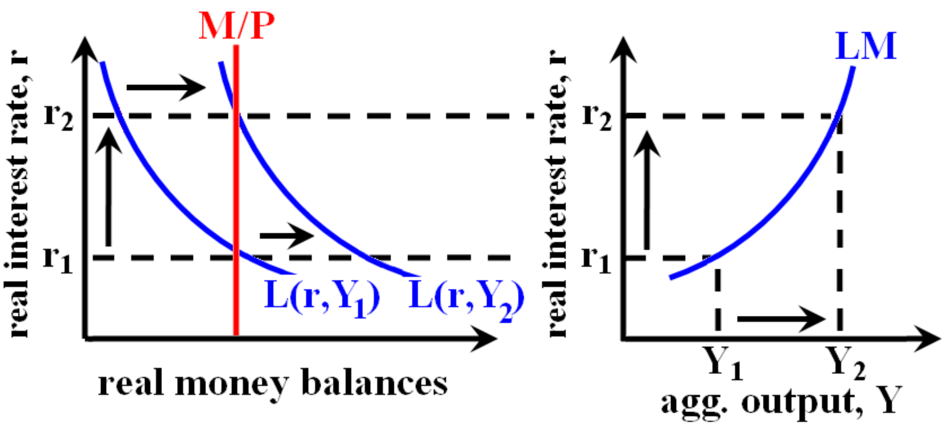
- Similarly, a contractionary fiscal policy – i.e. a decrease in government spending or an increase in taxes – would shift the IS curve inward



Income and Money Demand

- **Aggregate output (income), Y, is determined in the goods market, but in the short run, aggregate output can affect the money market**
- **As we saw in Lecture 10, an increase in aggregate output (income) shifts out the money demand curve, which raises the nominal interest rate**
- **If prices are sticky in the short run, a higher nominal interest rate leads to a higher real interest rate in the short run**

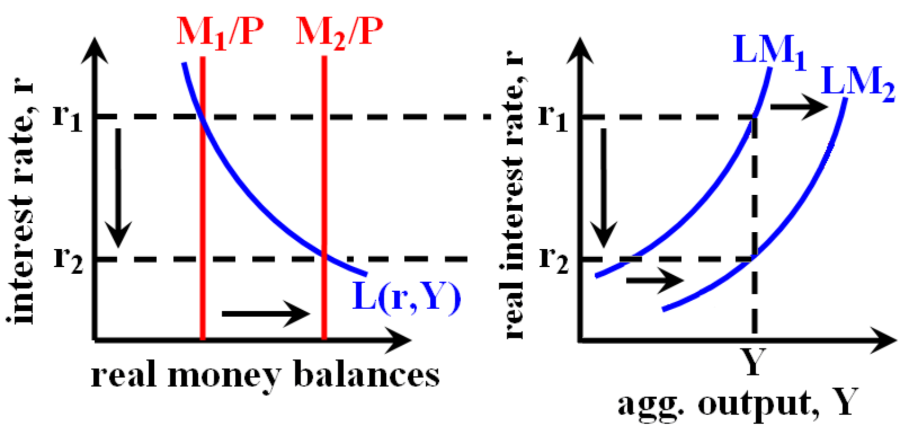
1. **An increase in income raises the demand for real money balances**
2. **increasing the short-run real interest rate**
3. **The LM curve summarizes these changes**



The LM curve shows the combinations of aggregate output and real interest rate for which the money market is in equilibrium.

Shifts of the LM curve

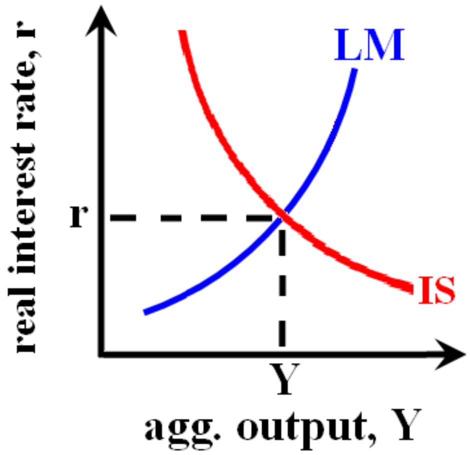
- **The LM curve is drawn for a given supply of real money balances, i.e. the ratio of available money to the price level**
- **If the Fed were to pursue an expansionary monetary policy** – i.e. increase the money supply and lower the nominal interest rate
    - **the real interest rate would fall in the short run**
    - since prices are sticky in the short run
    - **and the LM curve would shift outward**



- Similarly, a contractionary monetary policy – i.e. an increase in the money supply, which lowers the real interest rate – would shift the LM curve inward

The IS-LM Diagram

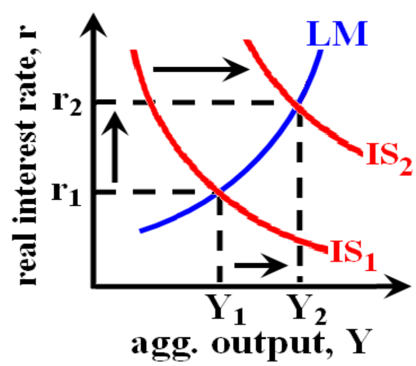
- **The point at which the IS and LM curves intersect corresponds to the point at which the goods market and the money market are both simultaneously in equilibrium.**
  - Each point on the IS curve represents equilibrium in the goods market.
  - Each point on the LM curve represents equilibrium in the money market.

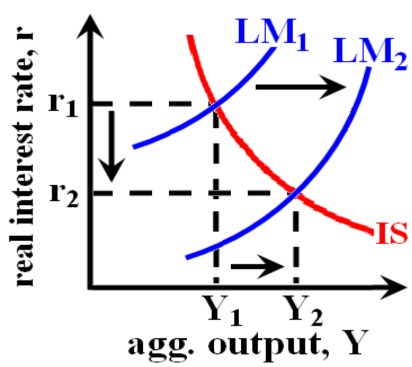


Policy Analysis using IS-LM

expansionary fiscal policy

- **increases aggregate output,** Y
- **increases the real interest rate,** r
- **the higher real interest rate decreases the effectiveness of fiscal stimulus because the higher real interest rate "crowds out" planned investment**



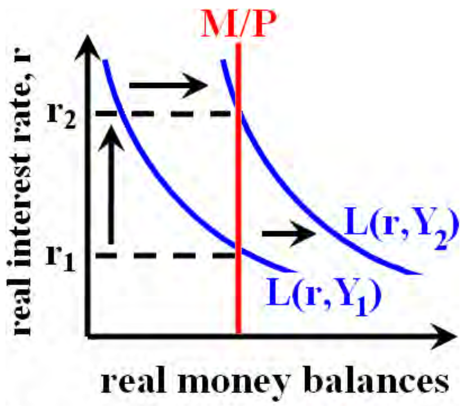


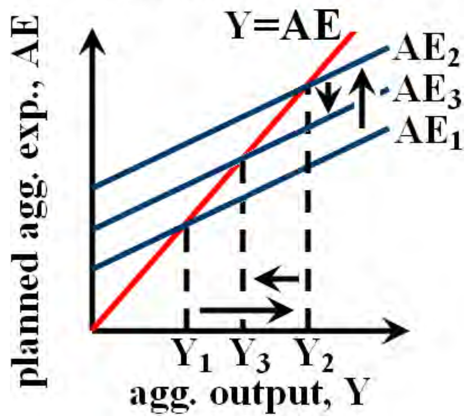
expansionary monetary policy

- **increases aggregate output,** Y
- **decreases the real interest rate,** r
- **the increase in aggregate output "crowds the interest rate" and reduces the ability of the Fed to lower the real interest rate in the short run**

The Crowding-Out Effect

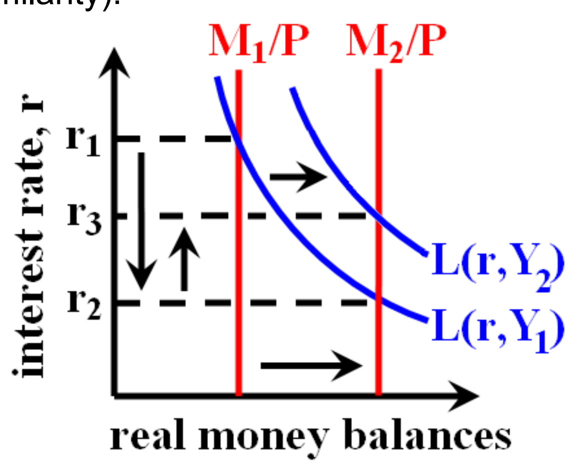
- **The crowding-out effect – refers to the tendency for expansionary fiscal policy to reduce private investment spending**
- **Expansionary fiscal policy increases aggregate output**
- **When aggregate output increases:**
  - the demand for real money balances shifts outward,
  - causing the real interest rate to rise in the short run
- **Because the real interest rate rises, planned investment decreases**
- The higher real interest rate decreases the effectiveness of fiscal stimulus





"Crowding the Interest Rate"

- **The money market also experiences an effect similar to the crowding-out effect discussed above.** (In reality, it's not called "crowding the interest rate." I just made up the term to convey the similarity).
- **An increase in the money supply:**
  - decreases the short run real interest rate
  - and increases investment and aggregate output.
- **However, the higher level of aggregate output increases the demand for real money balances.**
- **The higher demand for real money balances prevents the short run real interest rate from falling as far as it would in the absence of an increase in aggregate output.**



- the increase in aggregate output reduces the ability of the Fed to lower the real interest rate in the short run

What should you do?

- **Now, let's say you've just been elected President of the United States**
- **you've inherited a massive government debt**
- **some of your advisors fear that if you do not eliminate the budget deficit** (and try to finance continued budget deficits by issuing more bonds)**:**
    - **the bond markets will respond by selling off old bonds en masse**
    - **causing interest rates to shoot sky high**
    - **and causing planned investment to collapse**
    - **which, in turn, will provoke a recession**
- **these advisors urge you to focus on long run economic growth**
- **they remind you that cutting government spending and raising taxes will increase government saving** (as discussed in Lecture 7) **and increase the steady state level of output per worker**

♦ ♦ ♦

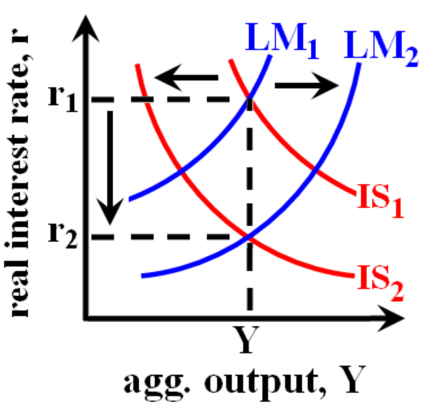
- **other advisors fear the political consequences of the immediate recession provoked by the contractionary fiscal policy**
- **they argue that if you balance the budget, you will lose political support and you will be unable to implement other policies designed to increase the steady state level of output per worker**

Policy Coordination

- **Conflicted, you turn to Fed Chairman Ben Bernanke for help and say:**

  "Ben, I want to balance the budget, but politically, I won't be able to keep the budget balanced for long if the country endures a harsh recession. Could you help me out?"
- **Luckily, Bernanke is sympathetic to your plight. He says to you:**

  "I'm always happy to help Prof. Doviak's students. So let's do this. You balance the budget and I'll increase the money supply. The contractionary effects of your fiscal policy will be offset by the expansionary effects of my monetary policy and the country will not have to endure a nasty recession."



- **Your contractionary fiscal policy shifts the IS curve inward.**
- **The Fed's expansionary monetary policy shifts the LM curve outward.**
- **The short-run real interest rate falls, stimulating planned investment**
- **and equilibrium aggregate output remains unchanged in the short run**

Homework #12

Using IS-LM diagrams, graph the course of fiscal and monetary policy during the following events:

the recession of 1974-75

- The Tax Reduction Act of 1975 provided a tax rebate of $8 billion that was paid out in the second quarter of 1975. The tax rebate and other reductions increased consumer spending, which contributed to the economic recovery that began soon after the new laws went into effect.
- The economic expansion caused demand for money to rise, but the Fed was simultaneously expanding the money supply, so nominal interest rates did not change very much.

♦ ♦ ♦

the recession of 1980-82

- At the urging of Pres. Reagan, Congress passed a huge tax cut in the summer of 1981. The tax cut led to an increase in consumer spending which helped to lift the economy out of recession.
- As the economy recovered in late 1982, demand for money increased. Nonetheless, interest rates actually fell because the Fed (which had been slowing the growth of the money supply since 1979) suddenly began to expand the money supply sharply in the spring of 1982.

♦ ♦ ♦

the recession of 1990-91

- The U.S. economy entered a mild recession in mid-1990. Real GDP began to rise in the second quarter of 1991, but because unemployment remained high, Pres. Bush wanted a tax cut to stimulate the economy. Congress and the Fed were concerned about the effect that a tax cut would have on the already large budget deficit however, so no fiscal stimulus was passed.
- The Fed did reduce interest rates, but the monetary stimulus was small and did not reduce unemployment enough to help Pres. Bush win re-election.

♦ ♦ ♦

Pres. Clinton’s first budget, 1993-94

- In the summer of 1993, Congress passed Pres. Clinton’s first budget which increased taxes and reduced government spending.
- Simultaneously, the Fed pursued an expansionary monetary policy which drove interest rates down to 30-year lows.
- In late 1994, the economy’s period of slow growth had ended and a robust expansion began.

Lecture 13

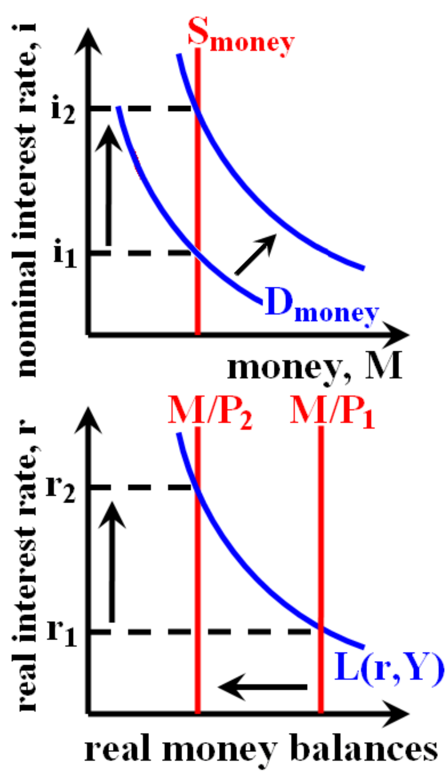
Output and Inflation in the Short Run

Eric Doviak

Economic Growth and Economic Fluctuations

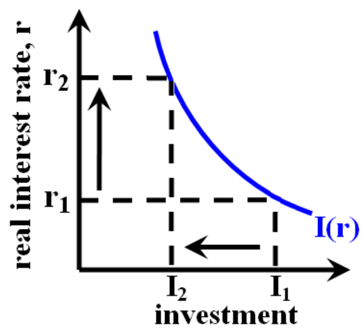
Deriving Aggregate Demand

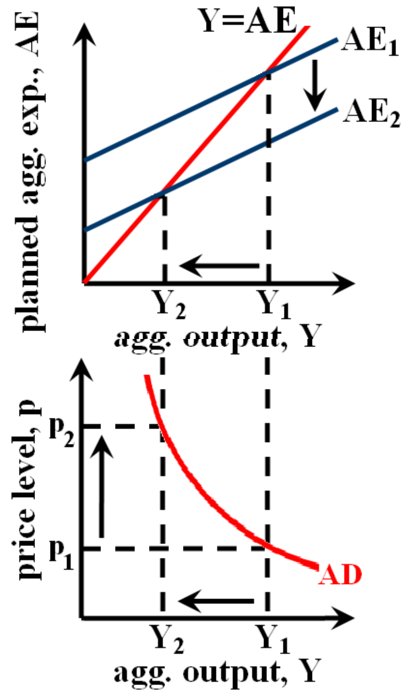
- **Consider an unanticipated increase in the price level,** p
- **As we saw in Lecture 10, when the price level rises, the demand curve for money shifts outward:**
    - **raising the nominal interest rate,** i
    - **and raising the short-run real interest rate,** r (since we assumed that it takes time for expectations of future inflation to change)
- The increase in the short-run real interest rate can also be seen through an analysis of supply and demand for real money balances.
- **For a given money supply,** M**, an increase in the price level,** p
    - **reduces** M/P**, the supply of real money balances and**
    - **raises the short-run real interest rate**



Deriving Aggregate Demand

- **Since an unanticipated increase in the price level raises the short-run real interest rate,** r, **planned investment spending,** I, **falls, which:**
    - **reduces planned aggregate expenditure**
    - **and lowers equilibrium aggregate output.**

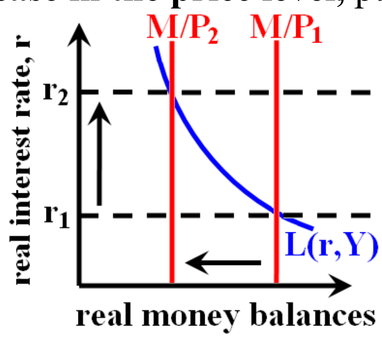


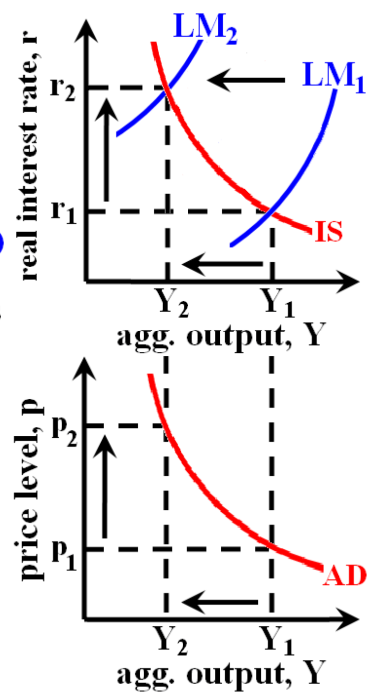


- **The Aggregate Demand curve summarizes these changes. It shows an inverse relationship between the price level,** p, **and aggregate output,** Y.
- It's drawn for a given money supply, M, and for given levels of government spending, G, and taxes T.

Deriving Aggregate Demand

- The same sequence can be depicted using the IS-LM diagram.
- **When the price level rises, the LM curve shifts in because – for a given money supply,** M **– an increase in the price level,** p:
    - **reduces** M/P, **the supply of real money balances and**
    - **raises the short-run real interest rate**

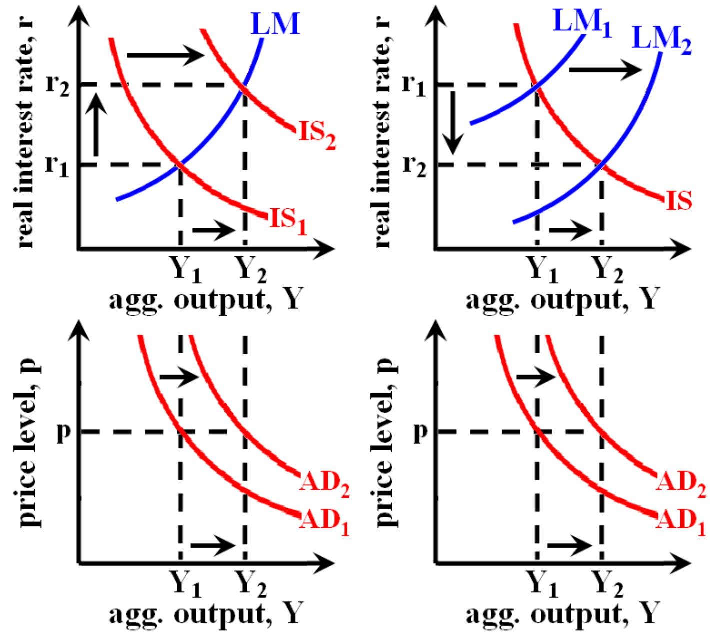




- **Equilibrium aggregate output falls because the higher short-run real interest rate reduces planned investment.**
- **Each point along the aggregate demand curve corresponds to a price level,** p, **and a level of aggregate output,** Y, **for which both the goods market and the money market are in equilibrium.**

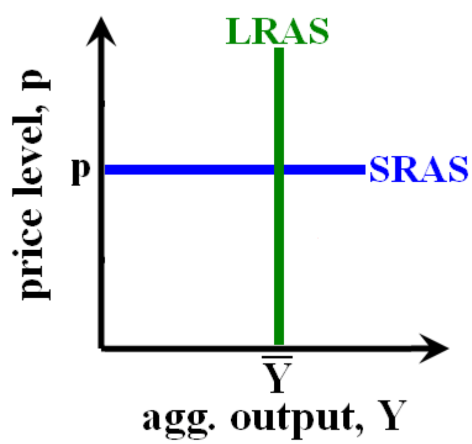
Shifts of Aggregate Demand

- **Expansionary fiscal policy and expansionary monetary policy both shift the aggregate demand curve outward.**
- **Expansionary fiscal policy – an increase in government purchases or a decrease in taxes raise equilibrium aggregate output at each price level.**
- **Expansionary monetary policy – for a given price level, an increase in the supply of money increases the supply of real money balances and raises equilibrium aggregate output.**



Aggregate Supply

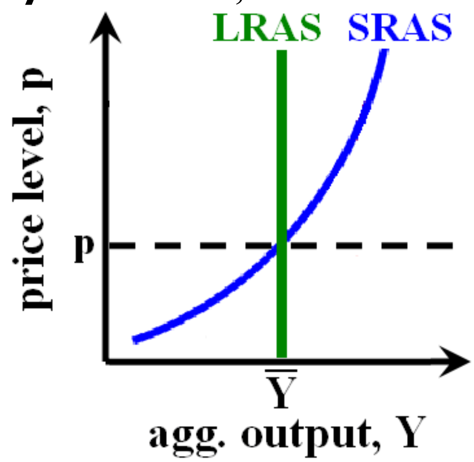
- **When we discussed the goods and money markets in the long run**
    - we assumed that the available stocks of physical and human capital, the size of the labor force and the level of technology determine the level of aggregate output
    - we assumed that the quantity of money available determines the price level, but does not affect the level of output, i.e. money is neutral
    - such assumptions are represented by the vertical long-run aggregate supply curve (LRAS)



- **So far, our discussion of the goods market in the short run has assumed that prices are sticky.**
    - in the extreme case that we have been discussing, firms do not adjust their prices at all in response to fluctuations in aggregate output.
    - such an assumption is represented by a horizontal short-run aggregate supply curve (SRAS)

Aggregate Supply

- **The assumption of extremely sticky prices** (i.e. the assumption that we have been making so far) **may hold in the very short run,**
  - **but it is highly unlikely that all prices would be extremely sticky for a period of time and then suddenly adjust all at once**
  - **it's more likely that some prices change faster than others, so that**
  - **in the short run firms respond to an increase in the price level by supplying more output**
  - **implying an upward-sloping short run aggregate supply curve**

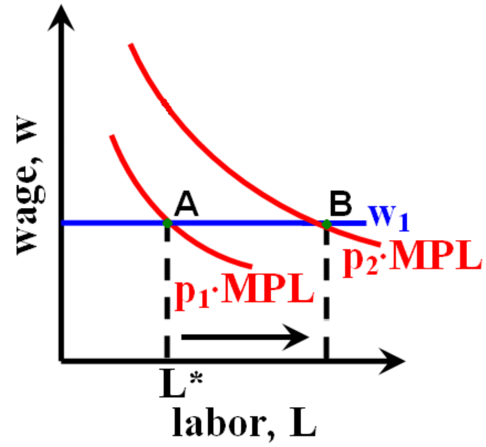


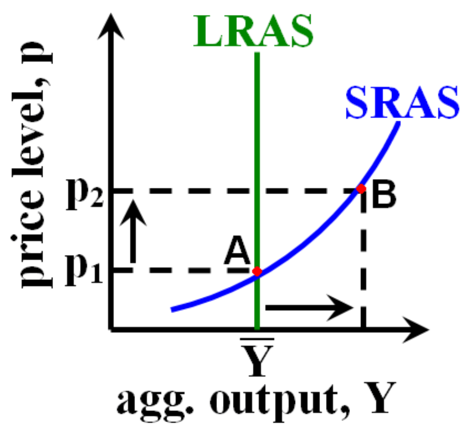
- **In fact, we can draw a short-run aggregate supply curve that traces out the output decisions of all the markets and firms in the economy that correspond to each price level.**
- Many explanations have been offered to explain why the short run aggregate supply curve may slope upward, but – for simplicity – we'll focus on one.

The Sticky Wage Model

- In many industries, particularly those that are unionized, nominal wages are set by long-term contracts, so wages cannot adjust quickly in response to changes in economic conditions.
- **Consider an economy which is producing at the natural rate of output** – i.e. the level of output predicted by the economy's stocks of physical and human capital, the size of the labor force and the level of technology – **this level of output is denoted:** $\bar{Y}$ **. At the natural rate:**
  - **the economy employs** L^* **units of labor**
  - **if units of labor are measured in hours worked, then workers are paid wage rate** w_1 **for each hour that they work**
- **as discussed in Lecture 3, firms hire labor until the wage rate is equal to price times the marginal product of labor, i.e.** $w_1 = p_1 \cdot MPL$
- **When the price level increases:**
  - **the demand for labor shifts outward**
  - **and firms are willing to hire more units of labor at each wage level**
- **So if the price level increases, but the wage rate remains constant, then firms will hire more labor.**
- **Since more labor is employed, more output will be produced.**

The Sticky Wage Model

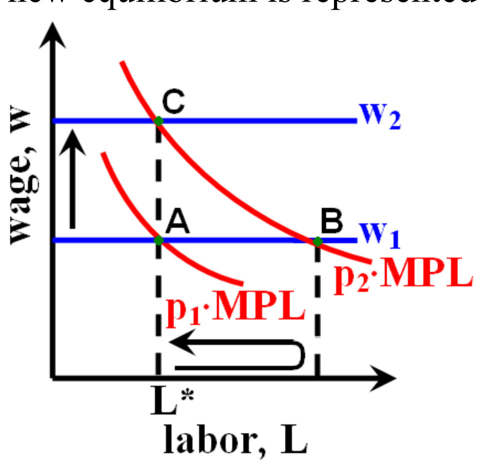


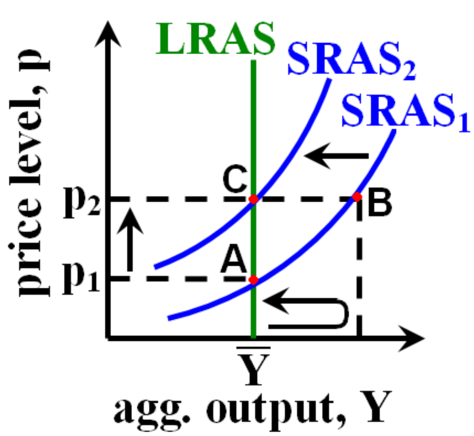


- **At wage rate** w_1**, firms hire labor up to the point where** $w_1 = p_1 \cdot \text{MPL}$
  - **where** p_1 **is the initial price level**
  - **at the initial price level,** L^* **units of labor are employed**
- This initial equilibrium is represented by point A in the figures above
- **If the price level were to unexpectedly rise from** p_1 **to** p_2 **and the wage rate remained at** w_1**, then firms would increase employment to a level above** L^* **and produce output at a level above** $\bar{Y}$
- This new equilibrium is represented by point B in the figures above

The Sticky Wage Model

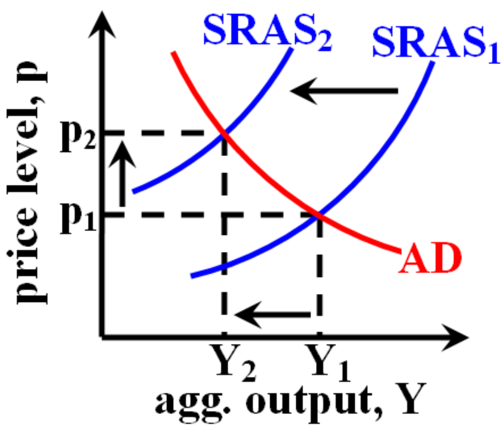
- **With the passage of time, workers would realize that the higher price level has diminished their purchasing power, so they would insist upon a higher nominal wage.**
- **If the nominal wage increased from** w_1 **to** w_2 (and if $w_2 = p_2 \cdot \text{MPL}$),
  - firms would cut the amount of labor that they hire back to L^* units
  - the economy would return to production at the natural rate of $\bar{Y}$
- This new equilibrium is represented by point C in the figures below.





Cost-Push Inflation

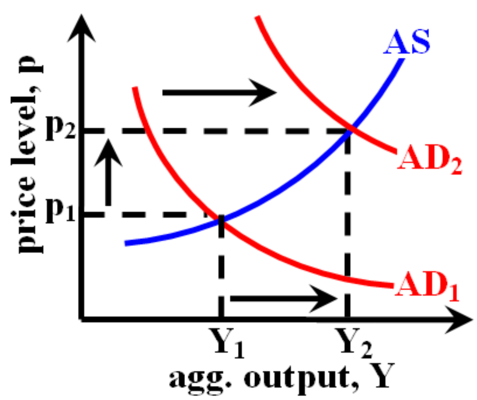
- Notice that the return to the natural rate of output, $\bar{Y}$, at price level p_2 is represented by an inward shift of the short-run aggregate supply curve in the figures above.
- Notice also that the increase in the wage rate causes the inward shift of the aggregate supply curve.
- In fact, whenever **factor input prices** rise, the short run aggregate supply curve shifts inward.
- For example, when OPEC imposed an oil embargo on the U.S. in 1974:
  - **the price of oil jumped 68 percent** (this is called a "cost shock")
  - **the short run aggregate supply curve shifted inward and**
  - **the US experienced "stagflation"** – that is: **"stagnation"** (recession) and **"inflation"** (higher prices)

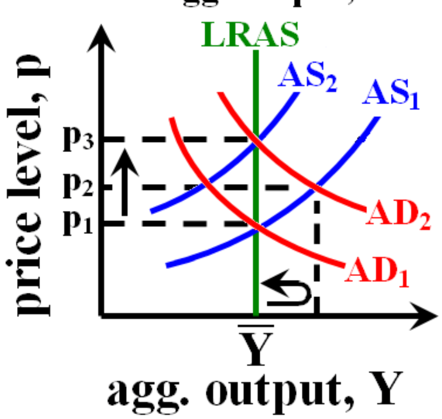


- Conversely, the short run aggregate supply curve shifts outward when factor input prices fall.

Demand-Pull Inflation

- Expansionary fiscal and monetary policy can also cause the price level to increase in the short run through outward shifts of the aggregate demand curve.
- This is an example of demand-pull inflation.
- However, aggregate supply does not depend on the price level in the long run.
- If expansionary policy causes output to exceed its natural rate:
  - **factor input prices will rise**
  - **causing the short run aggregate supply curve to shift inward** (as depicted in the graph on the bottom right)

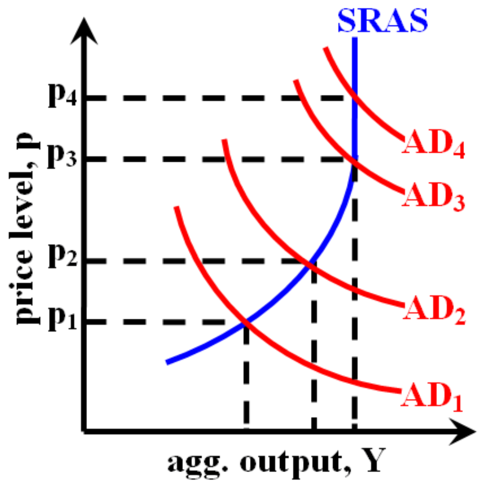




Note: It's also possible that output could return to its natural rate through an inward shift of the aggregate demand curve, which brings the price level back down to its initial level. This rarely occurs however, since the Fed allows the money supply to grow over time.

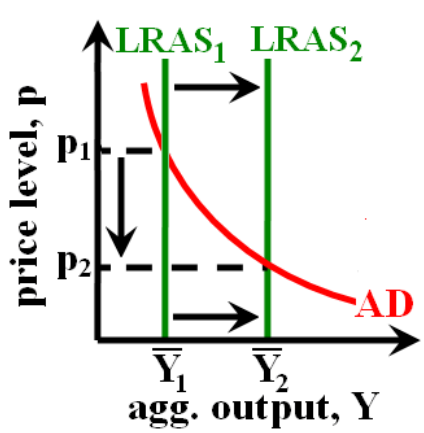
Inflation in the Long Run

- **In the long run, inflation is essentially a monetary phenomenon.**
- **Although expansionary fiscal policy causes some increase in the price level, sustained increases in the price level can only occur when an expanded money supply accommodates the expansionary fiscal policy.**
- **Hyperinflation –** defined as a rate of inflation that exceeds 50 percent per month **– is an extreme example that illustrates the point.**
- Hyperinflation occurs when the government resorts to printing money to finance its spending.
- Hyperinflation ends when the government cuts its spending or raises revenue through traditional taxes.
- As illustrated in the graph at right, continuously expanding the money supply eventually ceases to affect aggregate output. It only increases the price level.



Output Growth and Inflation

- **As we saw in the lectures on long run economic growth, the available stocks of physical and human capital, the size of the labor force and the level of technology determine the level of aggregate output.**
- **This level of output is represented by the vertical long run aggregate supply curve.**
- **If output is growing in the long run, then the long run aggregate supply curve moves rightward over time.**
- **Long run output growth thus decreases the price level** (holding monetary and fiscal variables constant)**.**



Homework #13

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Do this too! Describe the effects of the following events on the price level and on equilibrium GDP in the ***long run*****,** assuming that input prices fully adjust to output prices after some lag. Use aggregate demand curves and aggregate supply curves to illustrate your answers.

a. An increase in the money supply lifts aggregate output above the long-run aggregate supply level.

b. Initially, aggregate output is above the long-run aggregate supply level, but then the government reduces its spending and the Fed contracts the money supply.

c. Initially, the economy is producing at the long-run aggregate supply level, but then a war in the Middle East temporarily increases oil prices. The Fed accommodates the cost-push inflation by increasing the money supply.

Lecture 14

Short-Run Tradeoff between Inflation and Unemployment

Eric Doviak

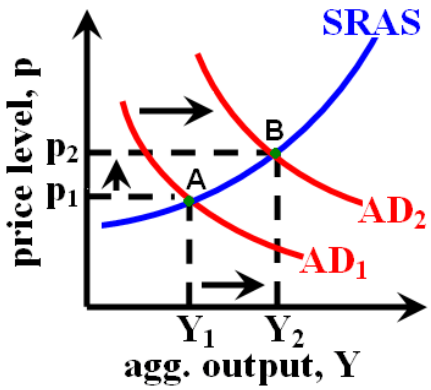
Economic Growth and Economic Fluctuations

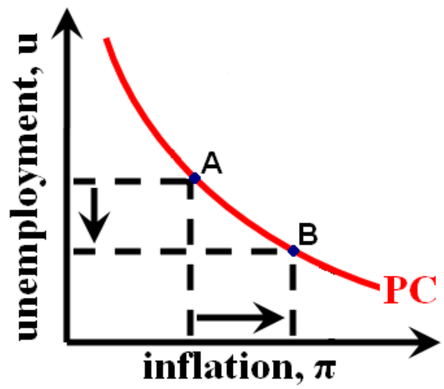
Inflation and Unemployment

- **When we first looked at inflation in Lecture 10, we saw that the inflation rate depends primarily on growth of the money supply, which is controlled by the Fed.**
- **When we first looked at unemployment in Lecture 9, our discussion focused on the natural rate of unemployment – the unemployment that does not go away on its own even in the long run**
- **And we saw that there is always a certain number of people who are unemployed due to:**
  - **frictional unemployment** – the unemployment that results from the time that it takes to match workers with jobs
  - **structural unemployment** – the unemployment that results from a wage rate that is set above the market-clearing level, because of minimum-wage laws, unions and efficiency wages
- **We also saw that there is cyclical unemployment – the year-to-year fluctuations in unemployment around its natural rate that are associated with short-term ups and downs of the business cycle**
- **This lecture focuses on the short-run tradeoff between cyclical unemployment and inflation.**

the Tradeoff

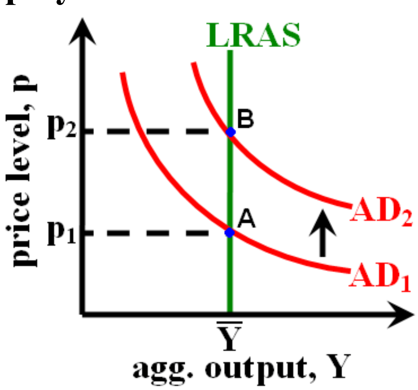
- **Assuming that the short-run aggregate supply curve is upward sloping, expansionary fiscal and monetary policy:**
  - **shifts the aggregate demand curve outward**
  - **which raises the inflation rate** (after all, the inflation rate is just the percentage change in the price level)
  - and **reduces the unemployment rate,** since firms need to employ more labor to produce the higher level of output.
- Contractionary fiscal and monetary policy have the opposite effects.
- **Society therefore faces a tradeoff between unemployment and inflation in the short run. This tradeoff is illustrated by the Phillips Curve.**

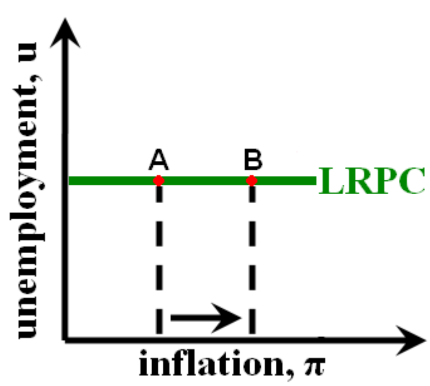




the Long-Run Phillips Curve

- **The Phillips Curve shows the short-run combinations of unemployment and inflation that arise as shifts in the aggregate demand curve move the economy along the short-run aggregate supply curve.**
- **But in the long run, the classical dichotomy holds:**
  - different forces influence real and nominal variables
  - changes in the money supply only affect the price level
  - changes in the money supply do <u>not</u> affect real variables, such as the amount of output produced and the amount of labor hired
- **Therefore, the long-run Phillips Curve is horizontal at the natural rate of unemployment.**





the Phillips Curve

$$u = u^n + \beta \cdot (\pi^e - \pi) + \varepsilon \qquad \text{where:} \begin{cases} u = \text{observed unemployment rate} \\ u^n = \text{natural rate of unemployment} \\ \beta \text{ is a parameter, } \beta > 0 \\ \pi^e = \text{expected inflation rate} \\ \pi = \text{observed inflation rate} \\ \varepsilon = \text{supply shock} \end{cases}$$

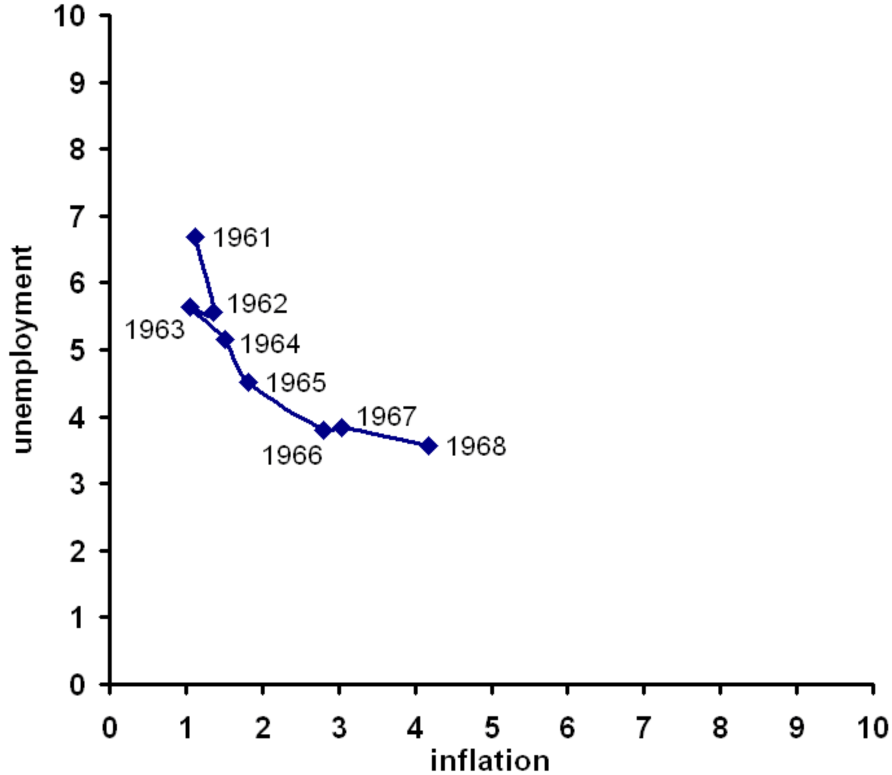
- **The linear Phillips relationship given in the equation above tells us that the observed unemployment rate is a decreasing function of the observed inflation rate.** (Nothing surprising here. This is precisely what is shown in the graphs above).
- **The equation above also tells us that the observed unemployment rate fluctuates around the natural rate of unemployment. This is called the natural-rate hypothesis.**
- **The other two terms in the equation shift the Phillips Curve.**
  - **If people expect that the inflation rate will be higher than it was in the past, then the Phillips Curve will shift outward.**
  - **A supply shock** (such the OPEC oil embargo discussed in Lecture 13) **will also shift the Phillips Curve outward.**

the Phillips Curve

- **The expected inflation rate, π^e, is the inflation rate that people think will prevail. After all, we won't know the rate at which prices are increasing this year until the measurements are released next year.**
- **In the long run, the expected inflation rate adjusts to the inflation rate that we actually observe** (i.e. $\pi = \pi^e$). **For example:**
  - if the Fed says that it will restrict growth of the money supply to ensure that the inflation rate is only 2 percent, then people will expect an inflation rate of 2 percent
  - but if the Fed has a habit of promising 2 percent inflation, but actual inflation is always 4 percent, people will catch on and expect 4 percent
- **So in the absence of supply shocks** (i.e. $\varepsilon = 0$), **when $\pi = \pi^e$, the observed unemployment rate will equal the natural rate of unemployment. This is why the Long-Run Phillips Curve is horizontal at the natural rate.**
- **Now let's say the Fed has consistently promised 2 percent inflation and has consistently delivered 2 percent inflation. This year however:**
  - **the Fed allows the inflation rate to rise to 4 percent, causing**
  - **the unemployment rate to be lower than the natural rate** (so long as there are no supply shocks this year)

So how well does the Phillips Curve work?

The rates of inflation and unemployment in the 1960s traced out a Phillips Curve fairly well.

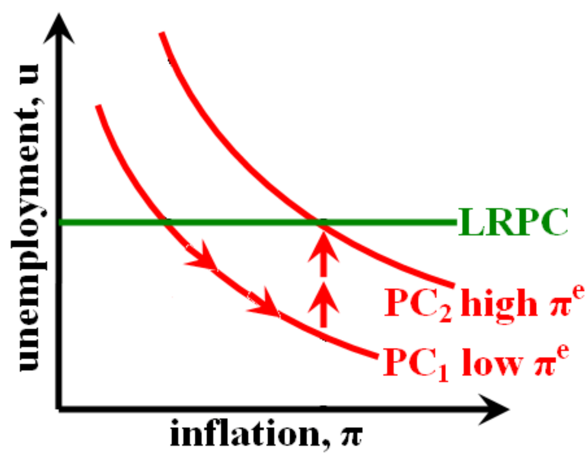


One Shift of the Phillips Curve

- Notice that the inflation rate increased each year from 1963 to 1968.
- As the inflation rate increased, people began to expect higher inflation.
- Now look again at the equation describing the Phillips Curve:

$$u = u^n + \beta \cdot (\pi^e - \pi) + \varepsilon$$

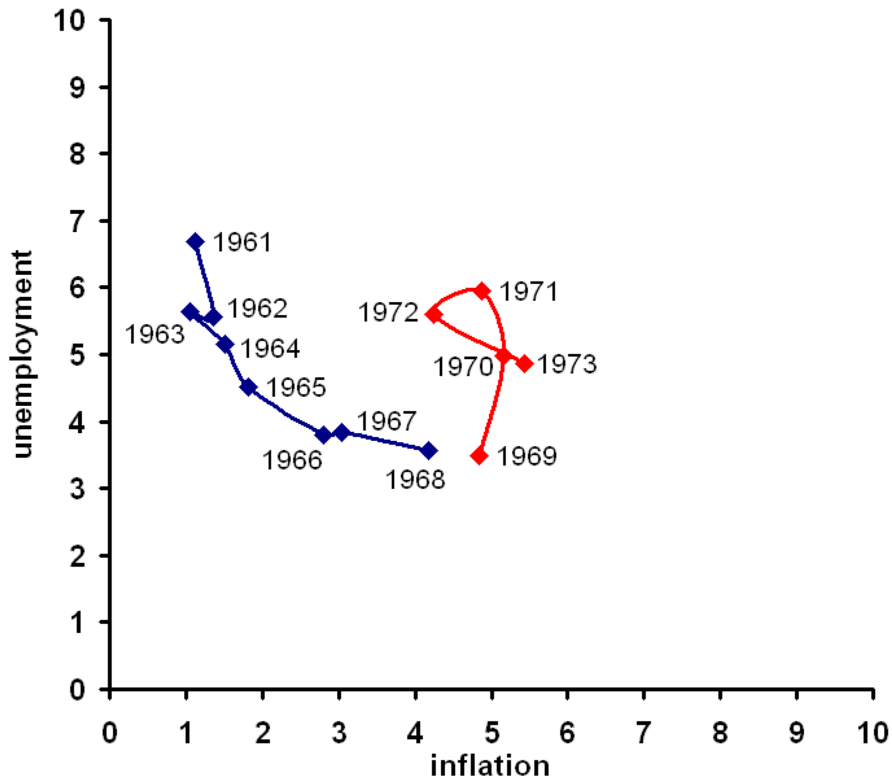
- As mentioned previously, when people expect higher inflation, the Phillips Curve shifts outward.
- People's expectations of inflation didn't change as the inflation rate rose from 2.8 percent in 1966 to 4.2 percent in 1968.
- This caused the unemployment rate to fall temporarily.



- Around 1969 or 1970, people's expectations of inflation began to change.
- As their expectations changed, the Phillips Curve shifted outward and the unemployment rate rose.

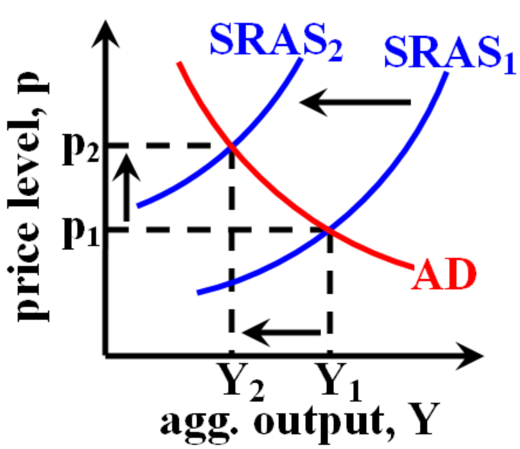
A Shift of the Phillips Curve

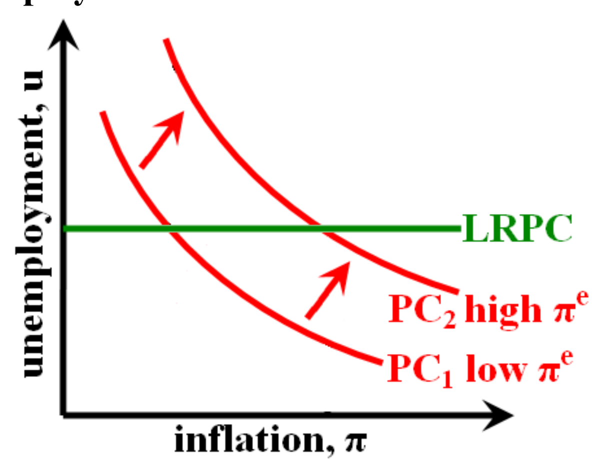
When people's expectations of inflation began to change the Phillips Curve shifted outward and the unemployment rate rose.



Another Shift of the Phillips Curve

- In the 1970s, the price of oil increased tremendously in response to the 1974 OPEC oil embargo, the 1979 Iranian Revolution and the outbreak of war between Iran and Iraq.
- These **supply shocks** shifted the short-run aggregate supply curve inward, which drove up the price level and induced recessions.
- As a result, people began to expect much higher inflation – which shifts the Phillips Curve outward and gives policymakers a less favorable tradeoff between inflation and unemployment.



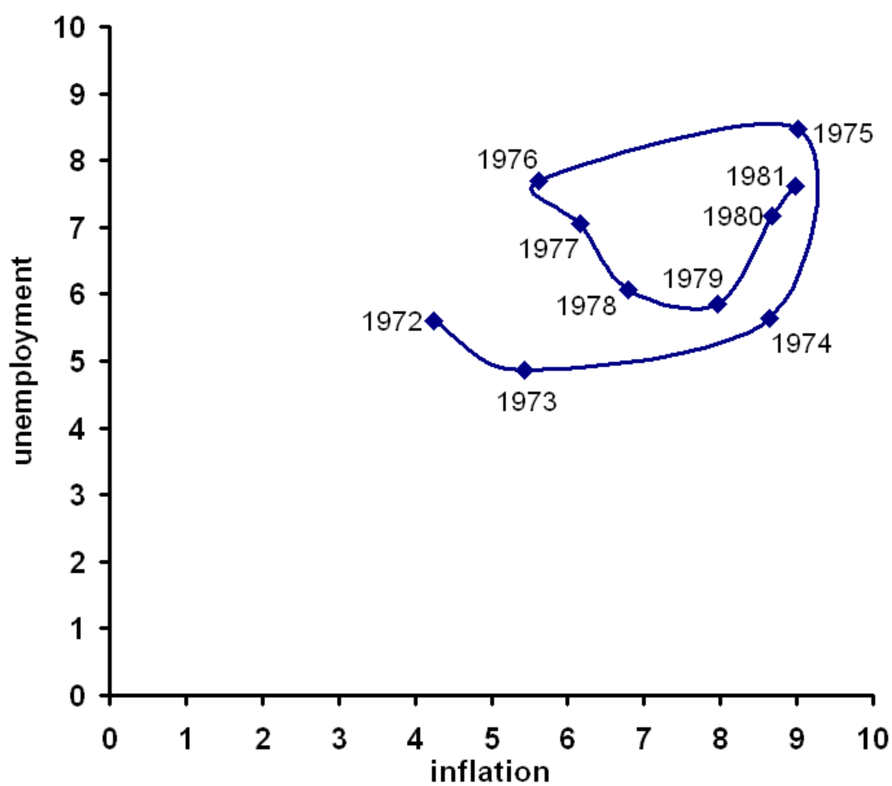


Oil Prices, Inflation and Unemployment

year	percentage change in oil prices	inflation rate (GDP deflator)	unemployment rate
1973	11.0	5.4	4.9
1974	68.0	8.6	5.6
1975	16.0	9.0	8.5
1976	3.3	5.6	7.7
1977	8.1	6.2	7.1
1978	9.4	6.8	6.1
1979	25.4	8.0	5.9
1980	47.8	8.7	7.2
1981	44.4	9.0	7.6
1982	– 8.7	5.9	9.7
1983	– 7.1	3.9	9.6
1984	– 1.7	3.7	7.5
1985	– 7.5	3.0	7.2
1986	– 44.5	2.2	7.0
1987	18.3	5.4	4.9

Another Shift of the Phillips Curve

The 1974 OPEC oil embargo, the 1979 Iranian Revolution and the outbreak of war between Iran and Iraq caused supply shocks which increased the inflation rate and the unemployment rate.



What would you do?

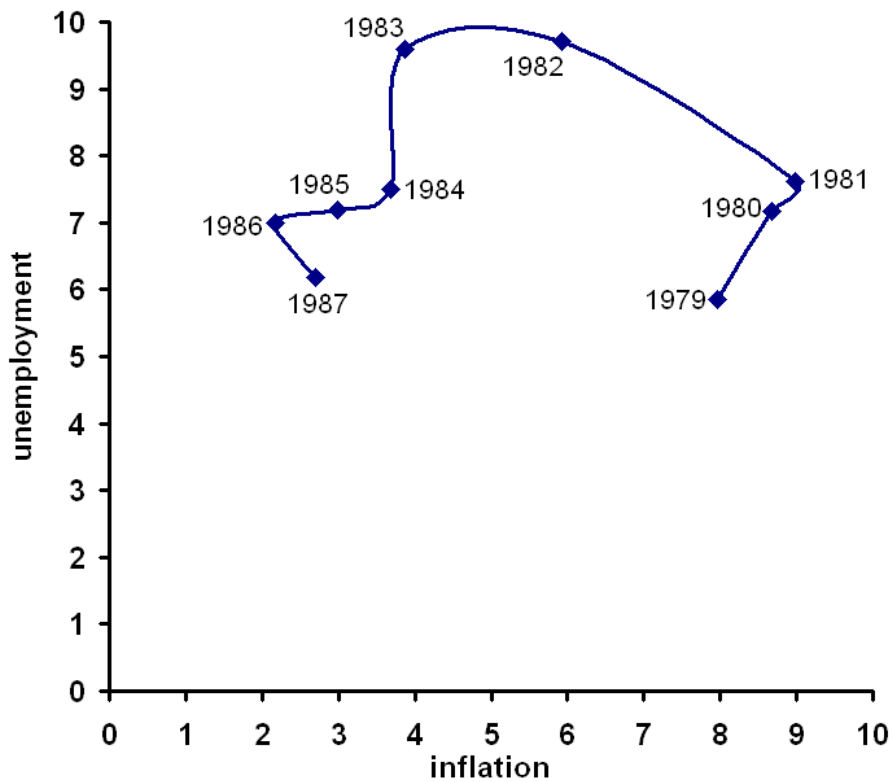
- **Now imagine that you are the chairperson of the Fed in the early 1980s. You face two unappealing choices:**
  - You can reduce unemployment by expanding aggregate demand through increases in the money supply. The trouble with this course of action is that it will accelerate inflation. OR
  - You can fight inflation by contracting aggregate demand through reductions in the money supply. The trouble with this course of action is that the American people will have to endure even higher unemployment.

the sacrifice ratio

- **The sacrifice ratio is the number of percentage points of annual output** (i.e. a year's worth of real GDP) **that is lost in the process of reducing the inflation rate by one percentage point.**
- **In the late 1970s, a typical estimate of the sacrifice ratio was five.**
- **In other words, it was estimated that reducing the inflation rate from 9 percent to 3 percent** (a 6 percentage point reduction) **would require a sacrifice of 30 percent of annual output.**
- **Is that what really happened? No.**

The Volcker Disinflation

In the early 1980s, Fed Chairman Paul Volcker succeeded in reducing inflation – from 9 percent in 1981 to 3 percent in 1985 – **but at the cost of high employment** – about 10 percent in 1982 and 1983.



Expectations

- **Up until this point, we have been assuming that it takes time for households and firms to change their expectations of future inflation rates. This is called the assumption of adaptive expectations.**

- **But what if people used all the information available to them – including information about the likely future course of fiscal and monetary policy – when setting their expectation of the inflation rate?**
- **In such a case, we would say that people have rational expectations.**

- **Remember that the tradeoff between inflation and unemployment in the short run depends on how quickly expectations adjust.**
- **The theory of rational expectations suggests that the sacrifice-ratio could be much smaller than estimated – "disinflation can be painless."**
- **After all, if policymakers are credibly committed to lowering the inflation rate, then rational people will understand their commitment and quickly revise their expectations of inflation downward.**

Sacrifice Ratio during the Volcker Disinflation

- **We mentioned previously that one estimate of the sacrifice ratio is five –** five percent of one year's real GDP must be sacrificed to reduce the inflation rate by one percentage point.
- **But if people have rational expectations, then the sacrifice ratio should be zero.**

♦ ♦ ♦

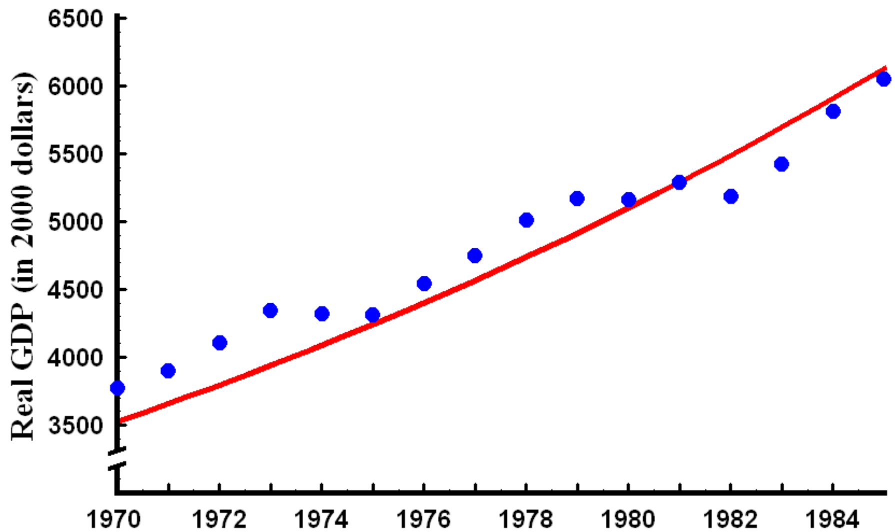
- **Between 1947 and 1980, real GDP grew at about 3.7 percent per year.**
- **If we assume that real GDP would have continued to grow at that rate from 1981 until 1985 in the absence of the disinflation, then we can estimate the loss of real GDP attributable to the Volcker Disinflation.**

year	observed real GDP	predicted real GDP (at 3.7% annual growth)	lost real GDP	inflation rate (GDP deflator)	change in inflation rate
1981	5292			9.0	
1982	5189	5491	– 301	5.9	– 3.1
1983	5424	5697	– 273	3.9	– 2.0
1984	5814	5911	– 98	3.7	– 0.2
1985	6054	6133	– 80	3.0	– 0.7

Observed vs. Predicted values of Real GDP

The blue dots in the graph at right represent the observed values of real GDP between 1970 and 1985.

The red line represents the values real GDP would have taken if it had grown smoothly at 3.7 percent per year.



The predicted value of real GDP is constrained to equal the observed value of real GDP in 1981.

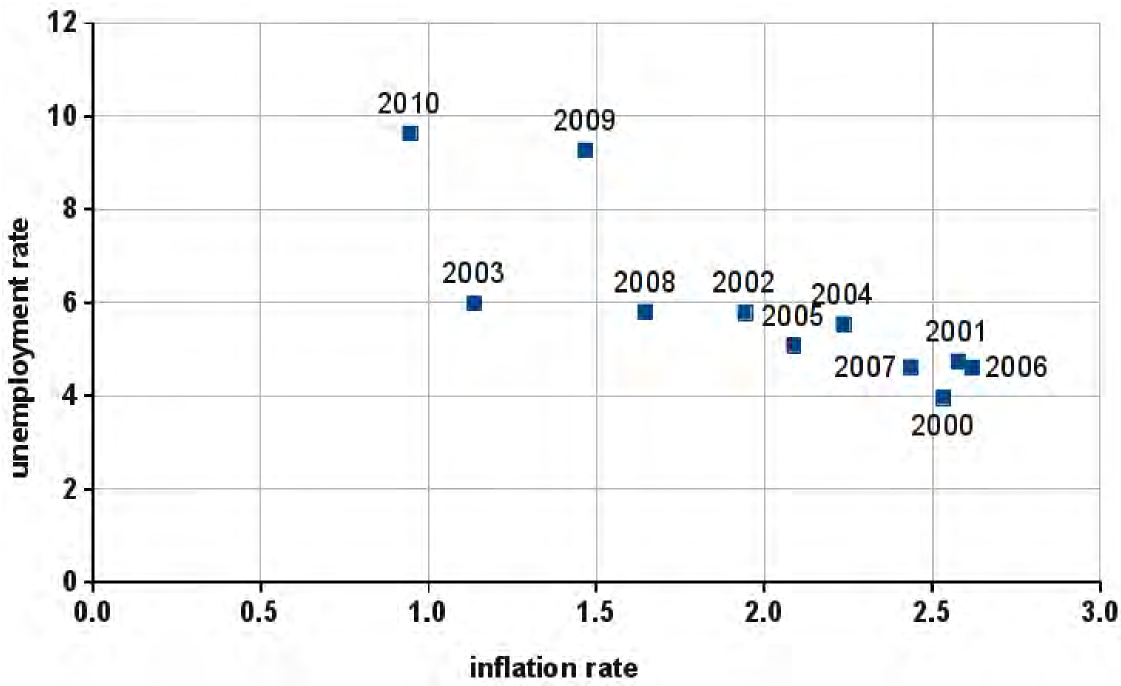
Sacrifice Ratio during the Volcker Disinflation

year	lost real GDP as a percentage of predicted real GDP	change in inflation rate	sacrifice ratio	unemployment rate
1982	– 5.5	– 3.1	1.8	9.7
1983	– 4.8	– 2.0	2.4	9.6
1984	– 1.7	– 0.2	8.3	7.5
1985	– 1.3	– 0.7	1.9	7.2
sum	**– 13.2**	**– 6.0**	**2.2**	–

- **The sacrifice ratio during the Volcker Disinflation was much lower than previously predicted at the time.** (In the 1970s, the estimated value of the sacrifice ratio was five).
- **Did Volcker get lucky? Maybe, but it's more likely that his credible promise to reduce inflation influenced people's expectations of inflation.**
- **Because people's expectations were rational, the loss of real GDP and the unemployment rate turned out to be much lower than predicted.**
- **In fact, the most rapid disinflations tend to have the smallest sacrifice ratios. In contrast to the prediction of the Phillips Curve with adaptive expectations, quick disinflations are less painful than gradual ones.**

What would you do?

- **Now imagine that you are the chairperson of the Fed in 2011.**
- **Would you focus on fighting inflation or unemployment?**



Homework #14

I am rewriting these homework problems. Sorry for the inconvenience. Please check back soon.

Eric Doviak
Economic Growth and
Economic Fluctuations

Review for the Final Exam

The final exam will cover Lectures 8 through 14.

The best way to study for the exam is to review the Homework sets. Another good way to prepare is to make sure you understand the concepts that we discussed in those lectures, so make sure you understand the terms listed below.

Terms to know

- business cycle
- expansion
- recession
- cyclical unemployment
- natural rate of unemployment
- discouraged workers
- efficiency wages
- structural unemployment
- frictional unemployment
- demand for liquidity
- money supply
- required reserve ratio
- discount rate
- open market operations
- transactions motive
- speculation motive
- nominal interest rate
- real interest rate
- observed inflation rate
- expected inflation rate
- quantity theory of money
- real money balances
- Fisher Equation
- monetary neutrality
- classical dichotomy
- aggregate output
- aggregate income
- saving
- investment
- consumption
- disposable income
- taxation
- government purchases
- marginal propensity to consume
- marginal propensity to save
- inventory investment
- planned investment
- actual investment
- planned aggregate expenditure
- disposable income
- government spending multiplier
- tax multiplier
- automatic stabilizers
- expansionary fiscal policy
- contractionary fiscal policy
- expansionary monetary policy
- contractionary monetary policy
- crowding out effect
- "crowding the interest rate"
- aggregate demand
- aggregate supply
- cost-push inflation
- demand-pull inflation
- Phillips Curve
- Long-Run Phillips Curve
- natural rate hypothesis
- supply shocks
- sacrifice ratio
- adaptive expectations
- rational expectations
- disinflation