GRADUATE MICRO
BARUCH COLLEGE

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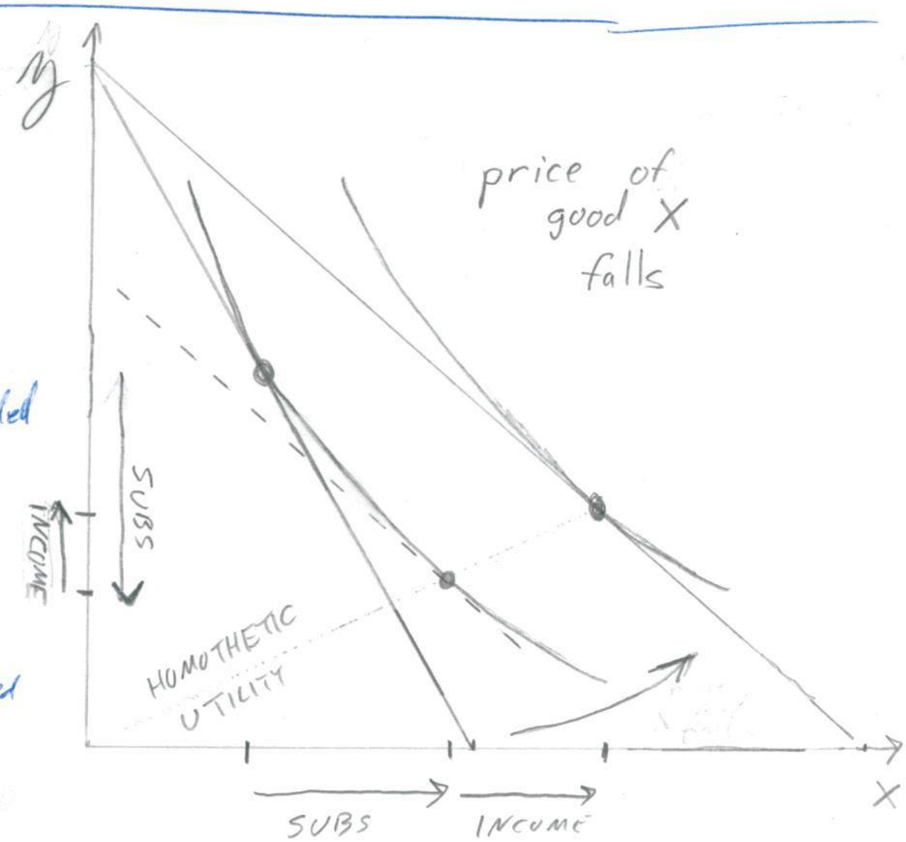
NOTES on INCOME and SUBSTITUTION EFFECTS and the OWN-PRICE and CROSS-PRICE ELASTICITY and the INCOME ELASTICITY and more!! :)

budget constraint

$$M = p \cdot X + q \cdot y$$

income = price of X · X + price of Y · Y



when p falls, qty of X demanded

- rises by SUBS effect
- rises by INCOME effect

when p falls, qty of Y demanded

- falls by SUBS effect
- rises by INCOME effect

OWN PRICE ELASTICITY of DEMAND

$$\varepsilon_{Xp} = \frac{\Delta X_{Demanded}}{\Delta p} \cdot \frac{p}{X_{Demanded}}$$

$$= k_X(\sigma_{XX} - \eta_X)$$

SUBS INCOME

CROSS PRICE ELASTICITY of DEMAND

$$\varepsilon_{Yp} = \frac{\Delta Y_{Demanded}}{\Delta p} \cdot \frac{p}{Y_{Demanded}}$$

$$= k_X(\sigma_{Xy} - \eta_y)$$

SUBS INCOME

$$k_X \equiv \frac{pX}{M}$$ ← share of income spent on X

$$\eta_X \equiv \frac{\Delta X}{\Delta M} \cdot \frac{M}{X}$$

$$k_y \equiv \frac{qy}{M}$$

$$\eta_Y \equiv \frac{\Delta Y}{\Delta M} \cdot \frac{M}{Y}$$

$$\sigma_{XY} \equiv \frac{-\Delta(X/Y)}{\Delta(p/q)} \cdot \frac{(p/q)}{X/Y}$$

$$\sigma_{XX} = \frac{-k_Y}{k_X} \sigma_{Xy}$$

(trust me!)

CROSS PRICE ELASTICITY of DEMAND

$$\varepsilon_{YP} = k_X (\sigma_{XY} - \eta_Y)$$

If the quantity of Y demanded increases in response to an increase in P, then Y is a gross substitute for X

$$\sigma_{XY} > \eta_Y \implies \varepsilon_{YP} > 0$$

SUBS EFFECT > INCOME EFFECT

Y is gross substitute for X

$\not\Leftrightarrow$

Alternatively, if the quantity of Y demanded decreases in response to an increase in Q, then Y is a gross complement to X

$$\sigma_{XY} < \eta_Y \implies \varepsilon_{YP} < 0$$

SUBS EFFECT < INCOME EFFECT

Y is gross complement to X

In the cross-price elasticity, we compared the elasticity of substitution σ_{xy} to the income elasticity. η_x

The own-price elasticity contains a similar comparison, but here we balance the substitution. When moving along an indifference curve decreased consumption of Y corresponds to increased consumption of X.

$$k_x \sigma_{xx} = -k_y \sigma_{xy} \quad \text{(trust me!)}$$

this substitution effect is inside the own-price elasticity

$$\varepsilon_{xp} = k_x(\underbrace{\sigma_{xx}}_{\text{SUBS}} - \underbrace{\eta_x}_{\text{INCOME}})$$

The trade-off between X and Y is measured by σ_{xy} which is defined as positive.

Therefore σ_{xx} is negative (Strictly speaking σ_{xx} is negative because utility maximization requires it to be negative, but that proof requires far more math than what you need to read).

When there are more than two goods

$$M = px + qy + rz$$ budget constraint

and

$$k_x \sigma_{xx} = -1 * (k_y \sigma_{xy} + k_z \sigma_{xz})$$

(+) (−) (−) (+)

Negative OWN SUBSTITUTION (i.e. "out of X")

Positive CROSS SUBSTITUTION (i.e. "into Y and Z")

So when moving along an indifference ~~curve~~ surface decreased consumption of X corresponds to increased consumption of Y or Z or both

When there are more than two goods ~~σ_{xy}~~ σ_{xy} could be negative, so long as σ_{xz} is positive AND $(k_y \sigma_{xy} + k_z \sigma_{xz})$ is positive

σ_{xy} would be negative if X + Y were highly complementary and Z were highly substitutable for the X and Y combination(s).

OWN PRICE ELASTICITY of DEMAND

$$\mathcal{E}_{xp} = k_x(\sigma_{xx} - \eta_x)$$

Because σ_{xx} is negative, $\mathcal{E}_{xp}$ will also be negative if η_x is positive

$$\eta_x = \frac{\Delta X_{Demanded}}{\Delta M} \cdot \frac{M}{X_{Demanded}}$$

INCOME ELASTICITY

A good is a normal good when $\eta_x > 0$
The quantity demanded rises as income rises.

The demand curve of a normal good slopes downward because a higher price reduces quantity demanded through both the income and the substitution effects. Why? Remember that $\mathcal{E}_{xp} = \frac{\Delta x}{\Delta p} \cdot \frac{p}{x}$

That slope, $\frac{\Delta x}{\Delta p}$, is the slope of the demand curve, so if $\mathcal{E}_{xp} < 0$ then $\frac{\Delta x}{\Delta p} < 0$ and the demand curve slopes downward

CASE of a NORMAL GOOD $\eta_x > 0$

$$\mathcal{E}_{xp} = k_x(\sigma_{xx} - \eta_x)$$

(−) (+) (−) (+)

Suppose now that good X is an inferior good, i.e. $\eta_x < 0$

Now quantity demanded falls as income rises, so a reduction in price reduces demand for X via the income effect but increases demand for X via the substitution effect.

$$\sigma_{xx} < \eta_x < 0 \implies \varepsilon_{xp} < 0$$

SUBSTITUTION EFFECT is larger than INCOME EFFECT

Demand curve still slopes Downward

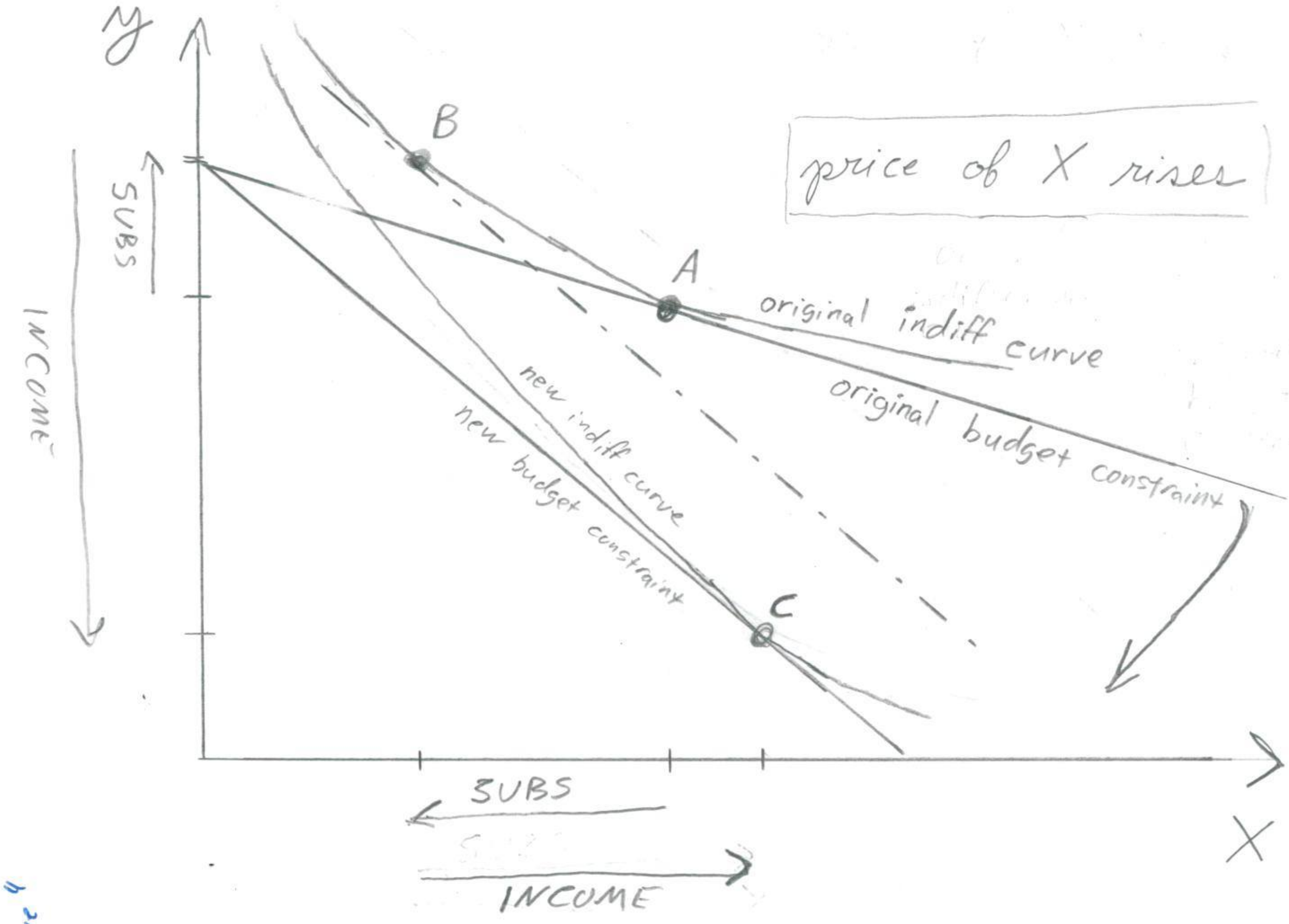
In cases where there are few substitutes for X (ie. σ_{xx} close to zero) and good X is a highly inferior good (ie. $\eta_x < 0$ and far from zero), then:

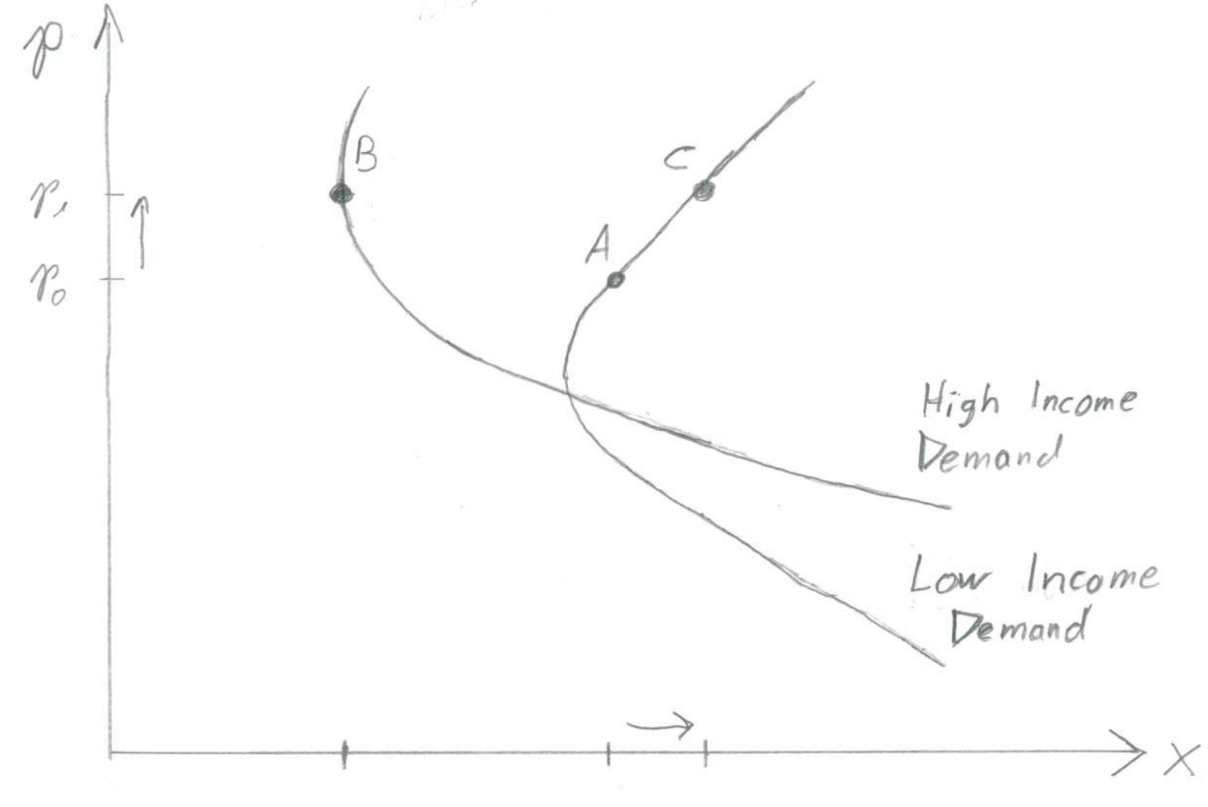
$$\eta_x < \sigma_{xx} < 0 \implies \varepsilon_{xp} > 0$$

INCOME EFFECT is larger than SUBS EFFECT

Demand curve slopes UPWARD

CASE WHERE $\eta_x < \sigma_{xx} < 0$





Note that the "high income demand curve" is MORE ELASTIC than the "low incomed demand curve"

The Single Mother's Decision

→ A single mother divides her total time T between labor L and her child C.

→ The value of her labor (i.e. her income) is $w \cdot L$, the wage rate times the number of hours that she works

→ Her income is used to purchase consumption good X. There are no college savings plans in this model, so $w \cdot L = p \cdot X$

TIME CONSTRAINT $\quad T = L + C$

$$wT = wL + wC$$

"VALUE of TIME CONSTRAINT" $\quad w \cdot T = p \cdot X + wC$

value of her time = value of X consumed + value of her time with child

→ Rearrange terms to write the budget constraint equation:

$$X = \frac{w}{p} \cdot T - \frac{w}{p} \cdot C$$

X: on the vertical axis; $\frac{w}{p} \cdot T$: intercept; $\frac{w}{p}$: slope; C: on the horizontal axis

→ When the single mother **maximizes** her **utility** subject to the "value of time constraint"

Marginal Rate of Substitution (slope of indifference curve)

$$\frac{MU_{child}}{MU_X} = \frac{w}{p}$$

"relative wage" (i.e. the wage rate relative to the price level)

→ Note that the relative wage $\frac{w}{p}$ tells us how much she can purchase per hour worked (i.e. the opportunity cost of hour with child)

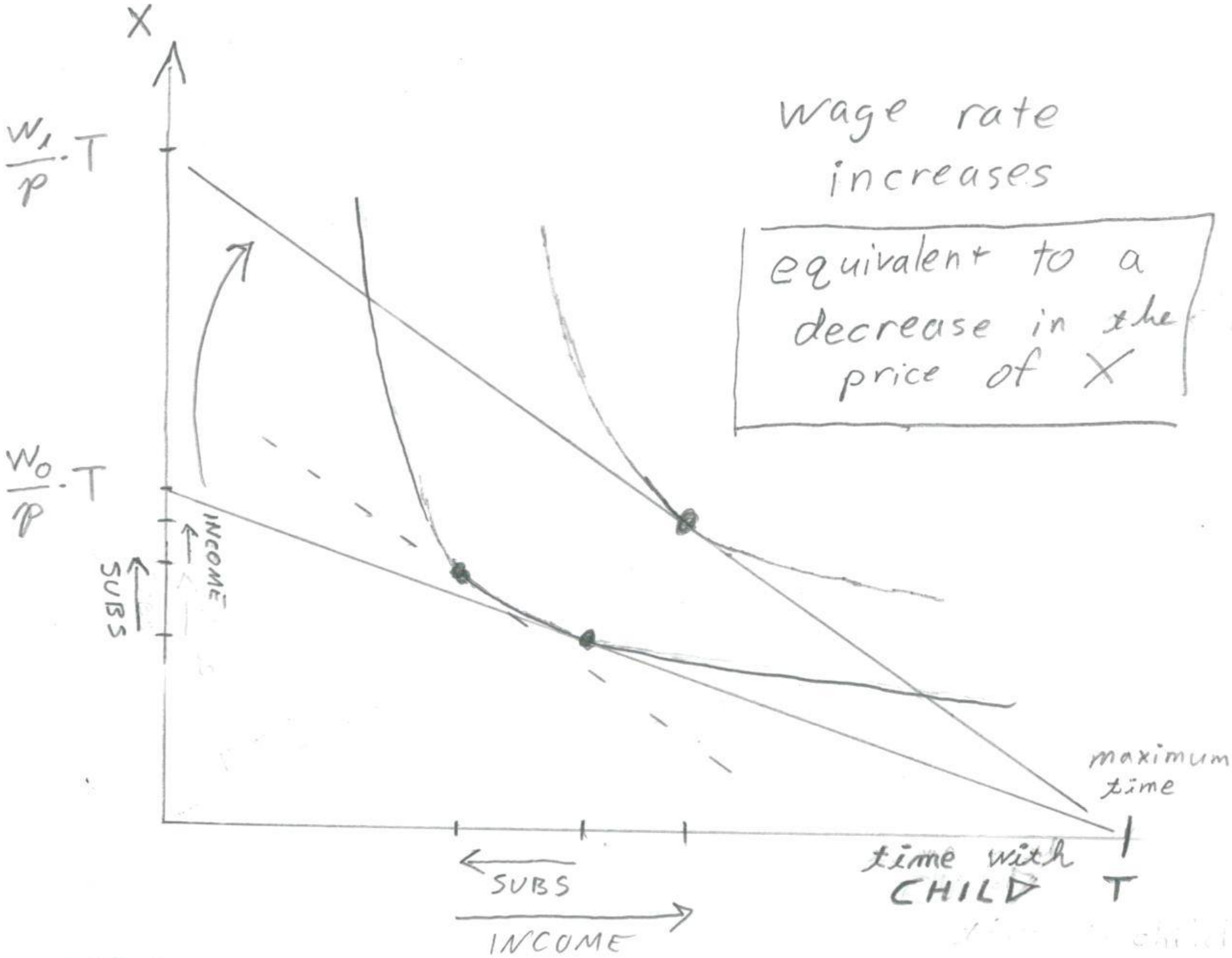
→ What happens when wage rate rises?

- **SUBS Effect** — time with child becomes relatively more expensive so she **substitutes out of child** and **into consumption of X**

- **INCOME Effect** — her purchasing power has increased, so she spends **more time** with **child** and **consumes more X**

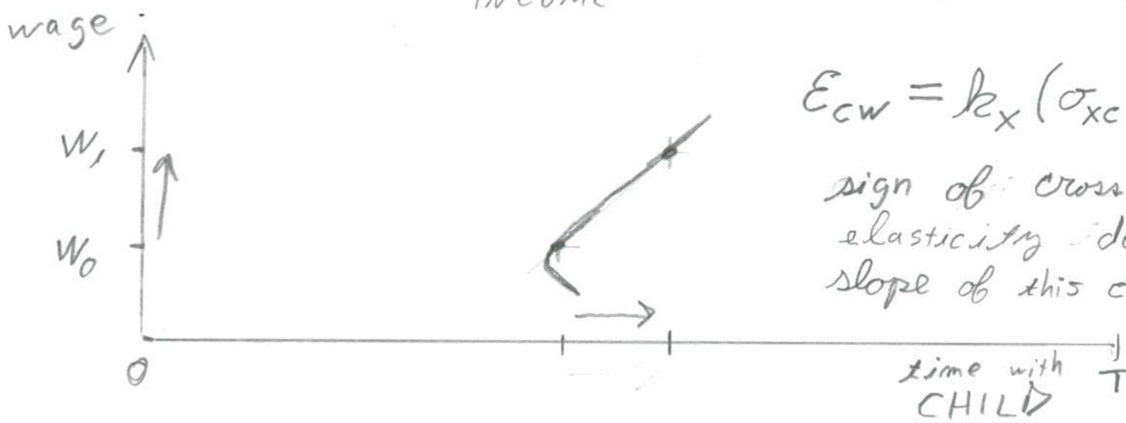
The Single Mother's Decision

budget constraint $X = \frac{w_1}{p} \cdot T - \frac{w_1}{p} \cdot C$



wage rate increases

equivalent to a decrease in the price of X



$$\varepsilon_{cw} = k_x(\sigma_{xc} - \eta_c)$$

sign of cross price elasticity determines slope of this curve

BACKWARDS LABOR SUPPLY CURVE