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Principles of Microeconomics

Notes on Isoquants, Isocosts and the Memo on Land Value Taxation

In Lecture 6, I used isoquants and isocosts to analyze profit-maximization. This is not strictly correct. The point of tangency between an isoquant and an isocost line illustrates the point where cost is minimized for a given level of output. It is not necessarily the point where a firm maximizes its profit.

Obviously, a firm cannot be maximizing its profit unless it's minimizing cost, but the reverse is not necessarily true. The difference occurs because:

- **cost minimization** – occurs when the firm minimizes the cost of producing a given level of output
- **profit maximization** – occurs when the firm minimizes the cost of producing the level of output which maximizes its profit.

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asset returns – the case of perfect substitutes

Imagine that you have the opportunity to buy shares of two equally risky stocks.

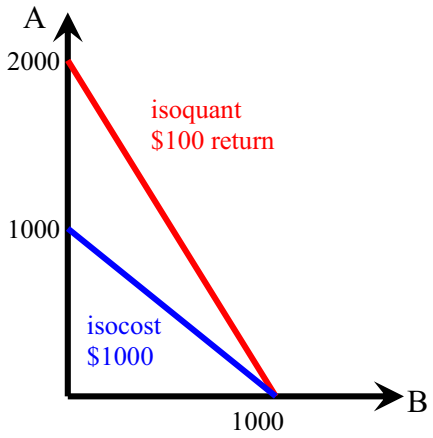
- the price of stock A is $1
- the price of stock B is $1
- the return on stock A is 5%
- the return on stock B is 10%

Isocost – if you have **$1000 to invest**, how much of each stock can you buy?

- If you don't buy any shares of stock B, then you can buy 1000 shares of stock A.
- If you don't buy any shares of stock A, then you can buy 1000 shares of stock B.

Isoquant – how much of each stock would you have to buy to get a **return of $100**?

- If you invest entirely in stock A, then 2000 shares of stock A would give you a $100 return.
- If you invest entirely in stock B, then 1000 shares of stock B would give you a $100 return.

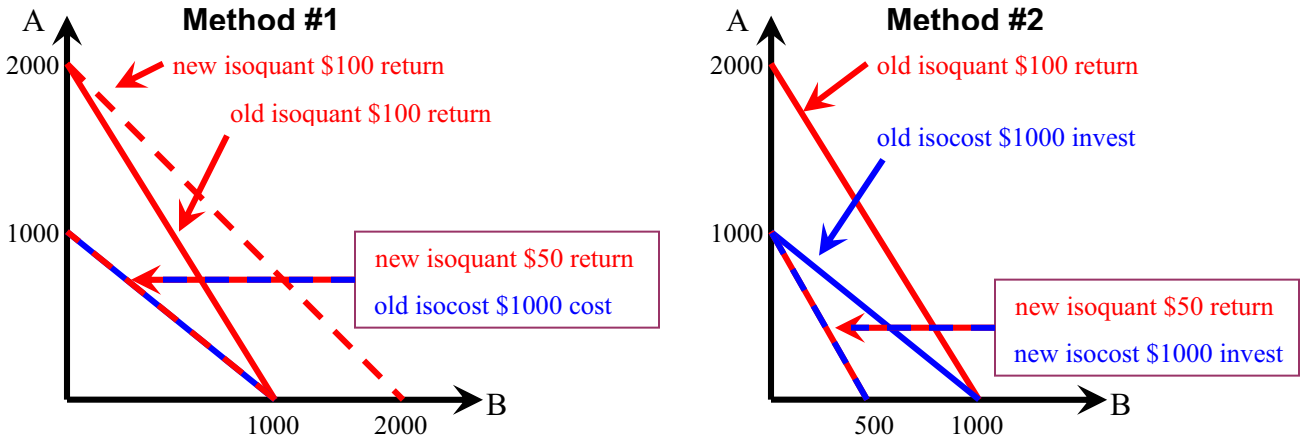


Note that the isoquant is a straight line. Isoquants are always straight lines when the two inputs (in this case: stocks) are perfect substitutes. The same is true of a consumer's indifference curves.

Now let's examine the case where the government imposes a 50% tax on the return to stock B. There are two ways of looking at the change:

- **Method #1** – The return to stock B (net of taxes) has fallen to 5%, so the slope of each isoquant changes. Given the same isocost line, you end up on a lower isoquant since it's no longer possible to earn a $100 return. Instead you can only earn a $50 return.

- **Method #2** – It is now more costly to hold stock B, so you can think of the tax as increasing the price of stock B from \$1 to \$2, while leaving the return unchanged[*]. The slope of each isocost line changes. Once again, you end up on a lower isoquant, since you can no longer earn a \$100 return. You can only earn a \$50 return.



Even though method #1 more accurately describes the case of taxing asset returns, I'll use method #2 in this analysis because my goal is to eventually describe the taxation of land and capital.

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asset returns – the case of perfect complements

Once again, imagine that the price of stock A and the price of stock B are both \$1 and that you have \$1000 to invest, so that the initial isocost line is drawn from 1000 shares of stock A on the vertical axis to 1000 shares of stock B on the horizontal axis.

Now assume that you have to buy stocks A and B in equal quantities – stocks A and B are perfect complements. This is not as absurd as it may seem. For example, if you're buying a house, you have to buy the plot of land it sits on too.

In this case, the isoquant becomes L-shaped.

- You could buy one house and two plots of land, but your return (in terms of the rent you can charge) will be no greater than if you had only bought one house and one plot of land. After all, who's going to pay to live on a vacant plot of land?
- Similarly, if you bought one plot of land and two houses, but the second house had no land to sit on (imagine that the second house was just an unassembled pile of bricks and mortar), then your return (in terms of the rent you can charge) will be no greater than if you had only bought one house and one plot of land.

Since you have to buy the shares in equal quantities:

- you would buy 500 shares of stock A and 500 shares of stock B
- your 5% return on stock A would give you \$25 and
- your 10% return on stock B would give you \$50.

So your initial isoquant is drawn for a \$75 return.

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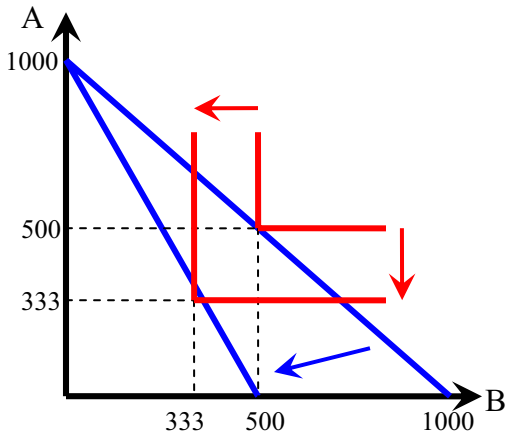
[*] The effective price is equal to $\frac{p_{share}}{1-\tau}$, where: p_{share} is the market price per share and τ is the tax rate on returns to that stock.

If the government imposes a 50% tax on the return on the return to stock B – via method #2: the effective price of stock B rises from $1 to $2 (effectively, a $1 tax per share) – then your isocost line would rotate inward in a clockwise direction.



Since you have to purchase stocks A and B in equal quantities:

- you would now buy 333 shares of stock A and 333 shares of stock B
- your 5% return on stock A would give you $16.67 and
- your 10% return on stock B would give you $33.33.

So your new isoquant is drawn for a $50 return.

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the substitution effect – the case of perfect complements

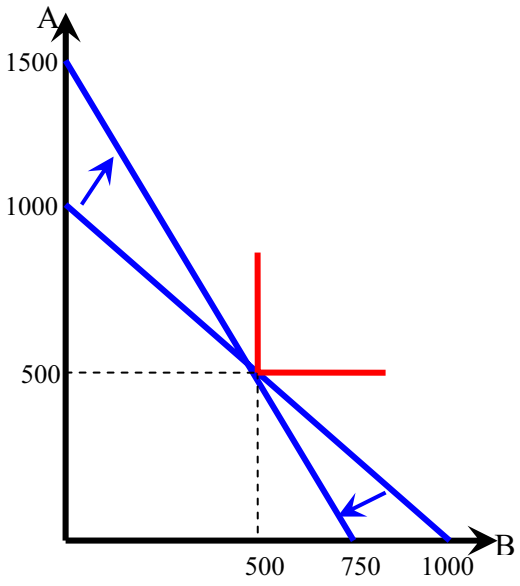
In the case of an individual's consumption of two goods, the pure substitution effect contains a change in relative price, but compensates the consumer for the relative price change by enabling him to consume the same initial bundle of goods. That is: there is a relative price change, but the consumer's initial real income (purchasing power) is left unchanged.

To draw the pure substitution effect for the case of stocks A and B, we'll rotate the isocost line through the initial portfolio – so that the initial allocation of stocks is still possible.

So once again, imagine that the initial price of stock A and the initial price of stock B are both $1 and that you have $1000 to invest. The initial isocost line is drawn from 1000 shares of stock A to 1000 shares of stock B.

Once again imagine that you have to buy stocks A and B in equal quantities, so you initially buy 500 shares of A and 500 shares of B. Your initial return is still $75.

Now imagine that the government imposes 25% tax on the returns to stock B – raising the effective price to $1.33 tax per share of stock B – and gives a 50% subsidy on the returns to stock A – lowering the effective price to $0.67 per share of stock A.



Your new isocost line runs from 1500 shares of stock A to 750 shares of stock B. Notice that you can still purchase 500 shares of stock A and 500 shares of stock B (500*$1.33 + 500*$0.67 = $1000).

Will you change the allocation of stocks in your portfolio? No, you won't. You cannot earn a higher return by changing your allocation because the stocks are perfect complements. Any reallocation would leave you on a lower isoquant – your returns would fall.

But what if the stocks were not perfect complements?

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the substitution effect – the "normal" case

Rarely are two goods or two stocks are perfect complements or perfect substitutes. Normally, they're somewhere in between. For review:

- **perfect substitutes** – indifference curves/isoquants are straight lines
- **perfect complements** – indifference curves/isoquants are L-shaped
- **somewhere in between** – indifference curves/isoquants are curved (convex to the origin).

For example, two stocks are not perfect substitutes when one is more risky than the other. Land and capital are not perfect complements because on a given plot of land you can build a single-family home or a 10-story apartment complex.

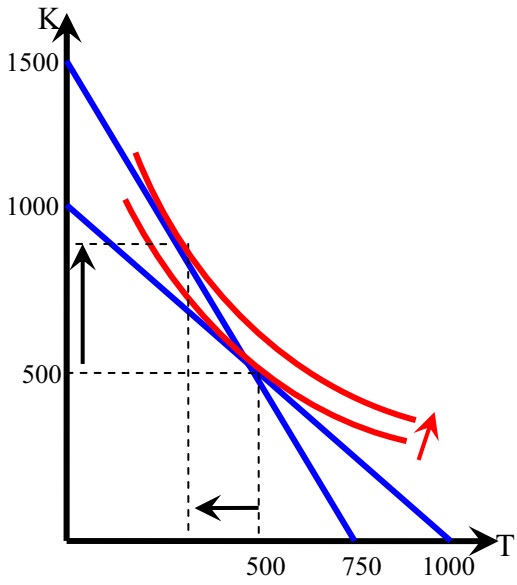
When zoning is present however, land and capital are "near perfect complements," but not "perfect complements." After all, a rental property can be in good condition or poor condition.

If you read my memo on land value taxation, you'll notice that the councilman who advocated land value taxation ignored the issue of zoning, so he thought that you could substitute land for capital more easily. That is: he thought that the isoquant was curved (convex to the origin).

Let's explore his idea in more detail. He proposed that the city should raise the tax on land and lower the tax on capital (buildings) in such a way that the overall tax burden would be left unchanged. That is he proposed a "pure substitution effect."

If not for zoning, his idea would have encouraged owners of property to reduce their holdings of land and increase their holdings of capital – via renovation or other improvements – in order to earn a higher return on their portfolio of land and capital assets.

Their higher return is illustrated by their ability to reach a higher isoquant when they sell off some of their stock of land (abbreviated with a T) and purchase capital (abbreviated with a K).



In the presence of zoning however, the near perfect complementarity between land and capital would have given owners little incentive to substitute land for capital (as depicted in the previous section), since the isoquants would have been nearly L-shaped.