ECONOMETRICS PROBLEM SET

Problem #4 – Suppose that you have the data on x and y supplied below and you wish to estimate the following regression model:

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

Using what you have learned, compute the estimates of the regression coefficients, their standard errors and the standard error of the regression. Then add the regression line to the scatterplot below.

obs	x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	x_i^2	$(x_i-\bar{x})^2$	$(x_i-\bar{x})(y_i-\bar{y})$	$\widehat{y}_i$	ε_i	ε_i^2
#1	+1	+3	−1	+2	+1	+1	−2	−1	+4	16
#2	+2	−3	0	−4	4	0	0	+1	−4	16
#3	+3	+3	+1	+2	9	1	+2	+3	0	0
#4	+1	−2	−1	−3	1	1	+3	−1	−1	1
#5	0	−3	−2	−4	0	4	+8	−3	0	0
#6	+5	+8	+3	+7	25	9	+21	+7	+1	1
sum	+12	+6	0	0	40	16	+32			34
mean	+2	+1								

$\widehat{\alpha} = \bar{y} - \widehat{\beta}\bar{x} = 1 - 2 \cdot 2 = -3$

$\widehat{\beta} = \dfrac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\sum (x_i-\bar{x})^2} = \dfrac{32}{16} = 2$

$\widehat{\sigma} = \sqrt{\frac{1}{N}\sum\left(y_i - \widehat{\alpha} - \widehat{\beta}x_i\right)^2} = \sqrt{\frac{1}{6} \cdot 34} = 2{,}38$

$s.e.\ (\widehat{\alpha}) = \widehat{\sigma}\sqrt{\dfrac{\frac{1}{N}\sum x_i^2}{\sum (x_i-\bar{x})^2}} = 2{,}38 \cdot \sqrt{\dfrac{\frac{1}{6} \cdot 40}{16}} = 1{,}54$

$s.e.\ (\widehat{\beta}) = \widehat{\sigma}\sqrt{\dfrac{1}{\sum (x_i-\bar{x})^2}} = 2{,}38 \cdot \sqrt{\dfrac{1}{16}} = 0{,}60$