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Econometrics
short explanation of probability models

for mathematical convenience, we'll focus on logit models

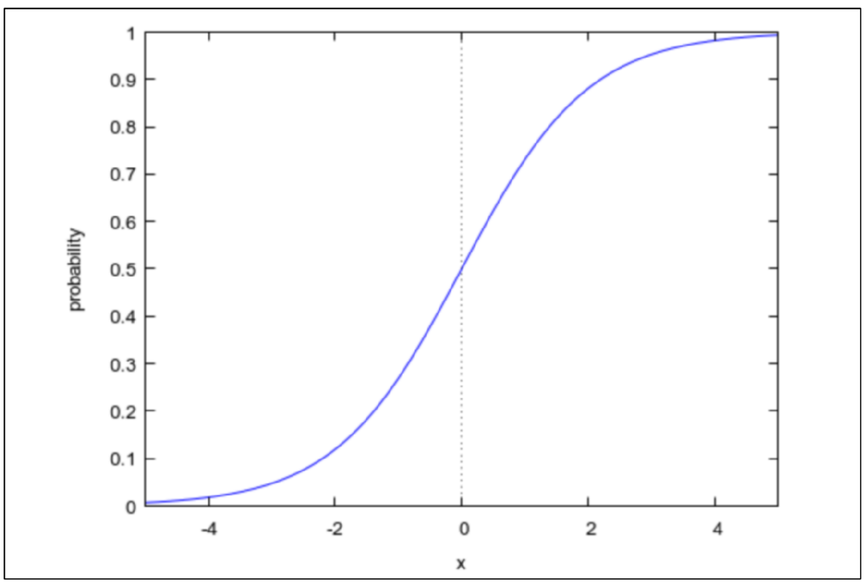
(%i2)
```
L(x):=1/(1+exp(−x))$
l(x):=''(diff(L(x),x))$
```

(%i7)
```
print("")$
print("We will focus on the logistic distribution:")$
print("Prob(Y=1|x) = L(x) = ",L(x))$
print("")$
wxplot2d(L(x),[x,−5,5],[ylabel,"probability"])$
```

We will focus on the logistic distribution:

$$Prob(Y=1|x) = L(x) = \frac{1}{\%e^{-x}+1}$$

(%t7)

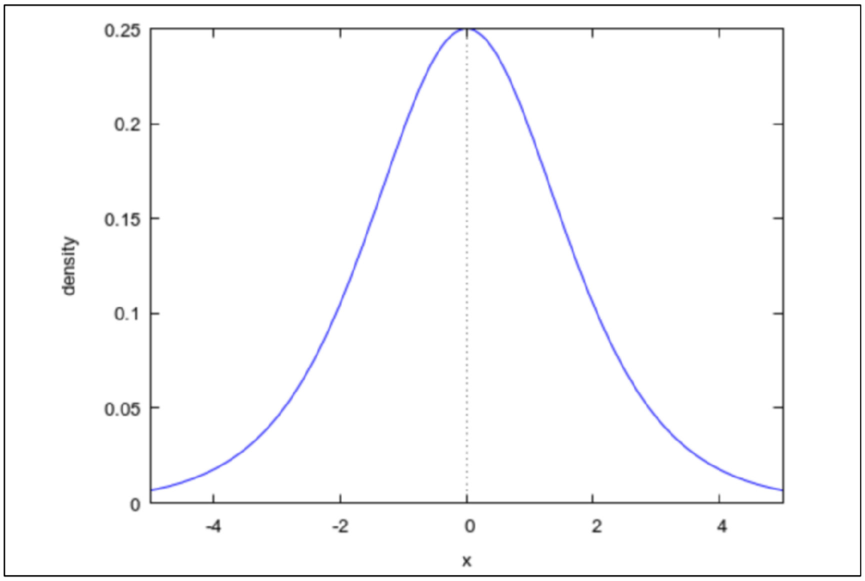


```
(%i12) print("")$
       print("which has probability density:")$
       print("l(x) = L(x)·(1−L(x)) = ",l(x))$
       print("")$
       wxplot2d(l(x),[x,−5,5],[ylabel,"density"])$
```

which has probability density:

l(x) = L(x)(1−L(x)) =* $\frac{\%e^{-x}}{(\%e^{-x}+1)^2}$

(%t12)



now we need some data, so we'll make the probability a decreasing function of xx

```
(%i15) nn:9$
       yy:[1,1,1,1,1,0,0,0,0]$
       xx:[1,1,1,2,3,3,4,4,5]$
```

when y[i] = 1, the log-likelihood function is an increasing function of L(alpha+beta*xx[i])
when y[i] = 0, the log-likelihood function is an decreasing function of L(alpha+beta*xx[i])

define: ss[i] == alpha+beta*xx[i]

loglik = sum{ y[i]*log(L(ss[i])) + (1-y[i])*log(1-L(ss[i])) }

Because the logistic distribution is symmetric:
1 - L(x) = L(-x)
we can write:
loglik = sum{ y[i]*log(L(ss[i])) + (1-y[i])*log(L(-ss[i])) }

Now, recall that y[i] equals zero or one. So we add "ss" if y[i]=1 and we subtract "ss" if y[i]=0.
loglik = sum{ log(L((2*y[i]-1)*ss[i])}

or in terms of the regression model:
loglik = sum{ log(L((2*yy[i]-1)*(alpha+beta*xx[i])))}

(%i16) loglik(alpha,beta):=sum(log(L((2·yy[i]−1)·(alpha+beta·xx[i]))),i,1,nn);

(%o16) $\mathrm{loglik}(\alpha,\beta):=\sum_{i=1}^{nn}\log\left(L\left(\left(2\,yy_i-1\right)\left(\alpha+\beta\,xx_i\right)\right)\right)$

maximize it

```
(%i22) sol:lbfgs(−loglik(alpha,beta),'[alpha,beta],[2.01,2.99],0.0001,[−1,0])$
       alfa_hat:subst(sol[1],alpha)$
       beta_hat:subst(sol[2],beta)$

       print("")$
       print(alpha," = ",alfa_hat)$
       print(beta," = ",beta_hat)$
```

α = 30.82207992910421
β = −10.27407803563996

now plot to show that the probability is a decreasing function of x

```
(%i24) print("")$
       wxplot2d(L(alfa_hat+beta_hat·x),[x,2,4],[ylabel,"probability"])$
```

(%t24)