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Econometrics
shape of the likelihood function

First, load the "distrib" package.

```
(%i1) load(distrib)$
```

Now, randomly draw 100 values from the standard normal.

```
(%i8) N:100$
x:random_normal(0,1,N)$

mnx:(1/N)·sum(x[i],i,1,N)$
sdx:sqrt((1/N)·sum((x[i]−mnx)^2,i,1,N))$

print("")$
print("mean of x: ",mnx)$
print("std. dev.: ",sdx)$
```

mean of x: −0.03336473769806096
std. dev.: 0.9894328810974044

Set up the log-likelihood function.

```
(%i9) loglik(mu,sigma):= −N·log(sigma) − (N/2)·log(2·%pi) − (1/2)·sum(((x[i]−mu)/sigma)^2,i,1
```

(%o9) $$\mathrm{loglik}(\mu,\sigma):=(-N)\log(\sigma)-\frac{N}{2}\log(2\,\pi)+\left(-\frac{1}{2}\right)\sum_{i=1}^{N}\left(\frac{x_i-\mu}{\sigma}\right)^2$$

Deriviatives must be taken with respect to sigma^2, so define:

gamma == sigma^2

and take derivatives with respect to gamma.

```
(%i10) loglik(mu,gamma):= −(N/2)·log(gamma) − (N/2)·log(2·%pi) − (1/(2·gamma))·sum((x[i]−m
```

(%o10) $$\mathrm{loglik}(\mu,\gamma):=\left(-\frac{N}{2}\right)\log(\gamma)-\frac{N}{2}\log(2\,\pi)+\left(-\frac{1}{2\,\gamma}\right)\sum_{i=1}^{N}(x_i-\mu)^2$$

Maximize it with respect to mu and gamma.

```
(%i16) sol:lbfgs(−loglik(mu,gamma),'[mu,gamma],[0.01,0.99],0.0001,[−1,0])$
mu_max:subst(sol[1],mu)$
gamma_max:subst(sol[2],gamma)$

print("")$
print(mu," = ",mu_max)$
print(sigma^2," = ",gamma_max)$
```

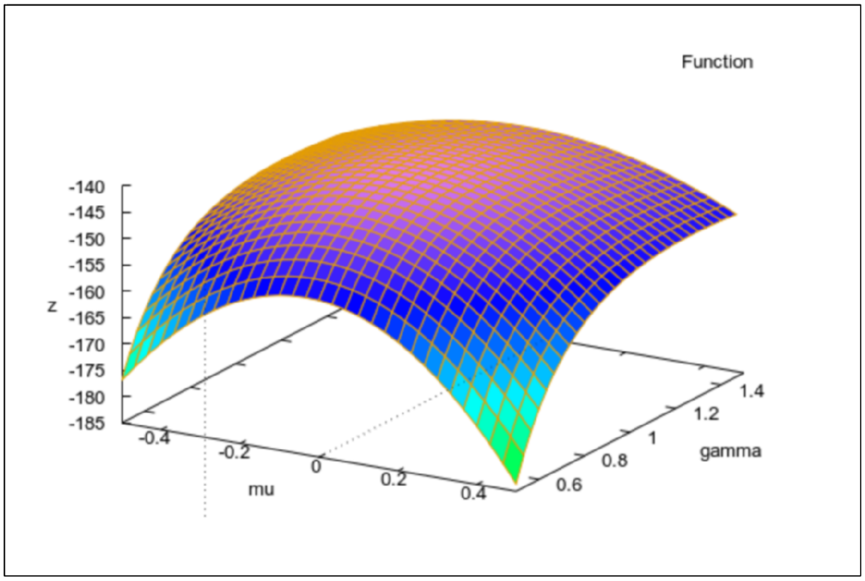
$\mu \;=\; -0.03336475655253121$

$\sigma^2 \;=\; 0.978977378206844$

To see the maximum, plot the log likelihood function in "mu-gamma" space.

```
(%i17) wxplot3d(loglik(mu,gamma),[mu,−0.5,0.5],[gamma,0.5,1.5])$
```

(%t17)



Check to see if second-order conditions are satisfied.

```
(%i21) /· set up the Hessian matrix ·/
dxx(mu,gamma):=''(diff(diff(loglik(mu,gamma),mu),mu))$
dyy(mu,gamma):=''(diff(diff(loglik(mu,gamma),gamma),gamma))$
dxy(mu,gamma):=''(diff(diff(loglik(mu,gamma),mu),gamma))$
H:matrix(
   [dxx(mu_max,gamma_max),dxy(mu_max,gamma_max)],
   [dxy(mu_max,gamma_max),dyy(mu_max,gamma_max)])$
```

(%i39)
```
print("")$
print("own−partials must be negative:")$
print("")$
print("d^2 loglik(mu,gamma)"/"(d mu)^2"," = ",dxx(mu_max,gamma_max))$
print("")$
print("d^2 loglik(mu,gamma)"/"(d gamma)^2"," = ",dyy(mu_max,gamma_max))$
print("")$
print("")$
print("the cross−partial:")$
print("")$
print("d^2 loglik(mu,gamma)"/"d mu d gamma"," = ",dxy(mu_max,gamma_max))$
print("")$
print("")$
print("the Hessian matrix:")$
print("H = ",H)$
print("")$
print("determinant of Hessian must be positive")$
print("det(H) = ",determinant(H))$
```

own−partials must be negative:

$$\frac{\text{d^2 loglik(mu,gamma)}}{\text{(d mu)^2}} = -102.1474062895776$$

$$\frac{\text{d^2 loglik(mu,gamma)}}{\text{(d gamma)^2}} = -52.17046817327451$$

the cross−partial:

$$\frac{\text{d^2 loglik(mu,gamma)}}{\text{d mu d gamma}} = -1.967292917628464\ 10^{-6}$$

the Hessian matrix:

$$H = \begin{pmatrix} -102.1474062895776 & -1.967292917628464\ 10^{-6} \\ -1.967292917628464\ 10^{-6} & -52.17046817327451 \end{pmatrix}$$

determinant of Hessian must be positive

det(H) = 5329.078008812947

```
(%i46) info:−1·invert(H)$

print("")$
print("the information matrix:")$
print("−1·(H^−1) = ",info)$
print("")$
print(mu,": ",mu_max,"   se:",sqrt(info[1,1]))$
print(sigma^2,": ",gamma_max,"   se:",sqrt(info[2,2]))$
```

the information matrix:

$$-1*(H\hat{}-1) = \begin{pmatrix} 0.009789773782068448 & -3.691619665493841\ 10^{-10} \\ -3.691619665493841\ 10^{-10} & 0.01916793226157539 \end{pmatrix}$$

μ *:* -0.03336475655253121 *se:* 0.0989432856846206

σ^2 *:* 0.978977378206844 *se:* 0.1384483017648659