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Econometrics
OLS regression

(%i1) `ratprint: false$`

First, load the "distrib" package.

(%i2) `load(distrib)$`

Now, randomly draw 10 values from the standard normal.

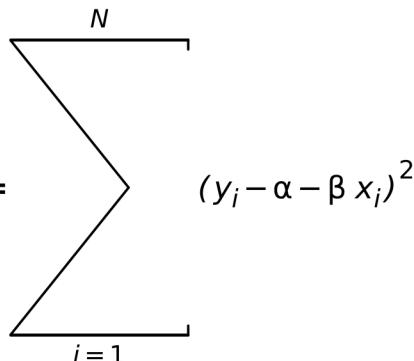
$y = \alpha + \beta x + u$

let the "true" parameter values be:
 $\alpha=2$, $\beta=3$

(%i6) `N:10$`
`x:random_normal(0,1,N)$`
`u:random_normal(0,1,N)$`
`y:2+3·x+u$`

Sum of Squared Errors (SSE):

(%i7) `sse(alpha,beta):= sum((y[i]-alpha-(beta·x[i]))^2,i,1,N);`



$$(y_i - \alpha - \beta x_i)^2$$

Minimize with respect to alpha and beta.

```
(%i13) sol:lbfgs(sse(alpha,beta),[alpha,beta],[2.01,2.99],0.0001,[-1,0])$  
alfa_hat:subst(sol[1],alpha)$  
beta_hat:subst(sol[2],beta)$
```

```
print("")$  
print(alpha," = ",alfa_hat)$  
print(beta," = ",beta_hat)$
```

$\alpha = 1.756831791098362$
 $\beta = 2.883730914684556$

First-order conditions imply that:

```
(%i21) dla(alpha,beta):="(diff(sse(alpha,beta),alpha))$  
dlb(alpha,beta):="(diff(sse(alpha,beta),beta))$
```

```
foc:float(solve([  
    dla(alpha,beta)=0,  
    dlb(alpha,beta)=0],[alpha,beta]))$
```

```
alfa_foc:subst(foc[1][1],alpha)$  
beta_foc:subst(foc[1][2],beta)$
```

```
print("")$  
print(alpha," = ",alfa_foc)$  
print(beta," = ",beta_foc)$
```

$\alpha = 1.756831791098364$
 $\beta = 2.883730914684557$

Check to see if second-order conditions are satisfied.

(%i25) /· set up the Hessian matrix ·/

```
daa(alpha,beta):="(diff(diff(sse(alpha,beta),alpha),alpha))$  
dab(alpha,beta):="(diff(diff(sse(alpha,beta),alpha),beta))$  
dbb(alpha,beta):="(diff(diff(sse(alpha,beta),beta),beta))$
```

```
H:matrix(  
    [daa(alfa_foc,beta_foc),dab(alfa_foc,beta_foc)],  
    [dab(alfa_foc,beta_foc),dbb(alfa_foc,beta_foc)])$
```

```
(%i38) print("")$  
    print("own-partials must be positive:")$  
    print("")$  
    print("d^2 sse"/"(d alpha)^2," = ",daa(alfa_foc,beta_foc))$  
    print("")$  
    print("d^2 sse"/"(d beta)^2," = ",dbb(alfa_foc,beta_foc))$  
    print("")$  
    print("")$  
    print("the Hessian matrix:")$  
    print("H = ",H)$  
    print("")$  
    print("determinant of Hessian must be positive")$  
    print("det(H) = ",determinant(H))$
```

own-partials must be positive:

$$\frac{d^2 \text{sse}}{(d \alpha)^2} = 20$$

$$\frac{d^2 \text{sse}}{(d \beta)^2} = 13.5906540538966$$

the Hessian matrix:

$$H = \begin{pmatrix} 20 & -2.572796488679064 \\ -2.572796488679064 & 13.5906540538966 \end{pmatrix}$$

determinant of Hessian must be positive

$$\det(H) = 265.1937993057728$$