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Econometrics
OLS regression

```
(%i1) ratprint: false$
```

First, load the "distrib" package.

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(%i2) load(distrib)$
```

Now, randomly draw 10 values from the standard normal.

y = alpha + beta*x + u

let the "true" parameter values be:
alpha=2 , beta=3

```
(%i6) N:10$
x:random_normal(0,1,N)$
u:random_normal(0,1,N)$
y:2+3·x+u$
```

Sum of Squared Errors (SSE):

```
(%i7) sse(alpha,beta):= sum((y[i]−alpha−(beta·x[i]))^2,i,1,N);
```

(%o7) $\mathrm{sse}(\alpha,\beta) := \sum_{i=1}^{N} (y_i - \alpha - \beta\, x_i)^2$

Minimize with respect to alpha and beta.

```
(%i13) sol:lbfgs(sse(alpha,beta),'[alpha,beta],[2.01,2.99],0.0001,[−1,0])$
       alfa_hat:subst(sol[1],alpha)$
       beta_hat:subst(sol[2],beta)$

       print("")$
       print(alpha," = ",alfa_hat)$
       print(beta," = ",beta_hat)$
```

α = 1.756831791098362
β = 2.883730914684556

First-order conditions imply that:

```
(%i21) dla(alpha,beta):=''(diff(sse(alpha,beta),alpha))$
       dlb(alpha,beta):=''(diff(sse(alpha,beta),beta))$

       foc:float(solve([
                 dla(alpha,beta)=0,
                 dlb(alpha,beta)=0],[alpha,beta]))$

       alfa_foc:subst(foc[1][1],alpha)$
       beta_foc:subst(foc[1][2],beta)$

       print("")$
       print(alpha," = ",alfa_foc)$
       print(beta," = ",beta_foc)$
```

α = 1.756831791098364
β = 2.883730914684557

Check to see if second-order conditions are satisfied.

```
(%i25) /· set up the Hessian matrix ·/

       daa(alpha,beta):=''(diff(diff(sse(alpha,beta),alpha),alpha))$
       dab(alpha,beta):=''(diff(diff(sse(alpha,beta),alpha),beta))$
       dbb(alpha,beta):=''(diff(diff(sse(alpha,beta),beta),beta))$

       H:matrix(
          [daa(alfa_foc,beta_foc),dab(alfa_foc,beta_foc)],
          [dab(alfa_foc,beta_foc),dbb(alfa_foc,beta_foc)])$
```

```
(%i38) print("")$
       print("own−partials must be positive:")$
       print("")$
       print("d^2 sse"/"(d alpha)^2"," = ",daa(alfa_foc,beta_foc))$
       print("")$
       print("d^2 sse"/"(d beta)^2"," = ",dbb(alfa_foc,beta_foc))$
       print("")$
       print("")$
       print("the Hessian matrix:")$
       print("H = ",H)$
       print("")$
       print("determinant of Hessian must be positive")$
       print("det(H) = ",determinant(H))$
```

own−partials must be positive:

$$\frac{d\text{^}2\ sse}{(d\ alpha)\text{^}2} = 20$$

$$\frac{d\text{^}2\ sse}{(d\ beta)\text{^}2} = 13.5906540538966$$

the Hessian matrix:

$$H = \begin{pmatrix} 20 & -2.572796488679064 \\ -2.572796488679064 & 13.5906540538966 \end{pmatrix}$$

determinant of Hessian must be positive

det(H) = 265.1937993057728