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Lucas' Imperfect Information Model

Why do wages and prices adjust slowly to shifts in aggregate demand?

Remember: If wages + prices adjusted immediately, then fiscal and monetary policy would have no effect on aggregate output.

BUT models like the "Sticky Wage Model" (which yield upward sloping AS curve) imply countercyclical real wages, whereas evidence suggests procyclical real wages

so we need a better model of aggregate supply

Today, we'll study RATIONAL EXPECTATIONS

Well... Before we can look at imperfect information, we first have to look at perfect information

individual prod. fn. $Q_i = L_i$

individual utility $U_i = C_i - \frac{1}{\gamma} L_i^{\gamma} \quad \gamma > 1$

individual consumption $C_i = \frac{P_i Q_i}{P}$ ← P is an index of prices

utility maximization:

$$\underset{L_i}{\text{Max}}\ U_i \qquad \frac{dU_i}{dL_i} = 0 \implies \underbrace{L_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma - 1}}}$$

Labor supply (and associated production) are increasing in the relative price of individual's product

taking logs: $l_i = \frac{1}{\gamma - 1}(p_i - p)$

lowercase for logarithms

demand for individual's good

$$q_i = y + z_i - \eta(p_i - p)$$

$\eta > 0$

demand shock

log of per producer demand for good

Remember that lower case variables denote logarithms

$y \equiv$ log of aggregate demand; it's assumed to equal the average of the q_i

$$y = \overline{q_i}$$

$p \equiv$ price index; it's assumed to be equal to the average of the p_i

$$p = \overline{p_i}$$

Note that demand for a good is higher when y (total production/total income) is higher and when its relative price $(p_i - p)$ is lower log of aggregate demand $\quad y = m - p$

Equilibrium

supply $\quad q_i = l_i = \frac{1}{\gamma - 1}(p_i - p)$

demand $\quad q_i = y + z_i - \eta(p_i - p)$

equilibrium

$$p_i = \frac{\gamma - 1}{1 + \eta(\gamma - 1)}(y + z_i) + p$$

but on average $\overline{p_i} = p$ and the average of z_i (the shocks) is zero, so

$$y = 0$$

$$m = p$$ ← Monetary Neutrality

An increase in m (log of money supply) raises all p_i raises the price level p

IMPERFECT INFORMATION

Suppose that individuals cannot observe the aggregate price level, so they cannot see how much the price of their good has risen relative to the aggregate price level. In other words, they have to estimate the relative price given their observation of their own price

$$r_i \equiv p_i - p \quad \text{unobserved}$$

Lucas assumes

1. Individual takes expectation

$$E[r_i \mid p_i] \equiv \text{expectation of relative price given observation of } p_i$$

2. Individual produces as if this estimate were certain

$$l_i = \frac{1}{\gamma - 1} E[r_i \mid p_i]$$

RATIONAL EXPECTATIONS

But how does individual compute $E[r_i | p_i]$? Assume:

$$m \sim N(E[m], vm)$$

$$r_i \sim N(0, vr)$$

which implies that p and r_i are normal and independent.

Since p and r_i are normal and independent, p_i is distributed normally and its variance is the sum of the variances of p and r_i:

$$p_i \sim N\left(E[p] + E[r_i],\ vp + vr\right)$$

where vp is variance of p and vr is variance of r_i

Romer shows that:

$$E[p] = E[m] \qquad vp = \frac{vm}{(1+b)^2}$$

$$E[r_i] = 0 \qquad vr = \frac{vr}{(r+b)^2}$$

Foreshadowing. Don't worry about this just yet.

But how does individual estimate $E[r_i | p_i]$? Since r_i and p_i are jointly normally distributed:

$$E[r_i | p_i] = \alpha + \beta p_i$$

Recall that:

$$r_i \equiv p_i - p$$

therefore:

$$E[r_i | p_i] = \underbrace{\frac{Vr}{Vr + Vp}}(p_i - E[p])$$

fraction of the overall variance of p_i due to variance of r_i (rel. price)

So labor supply of individual:

$$l_i = \frac{1}{\gamma - 1} \cdot \frac{Vr}{Vr + Vp}(p_i - E[p])$$

Define: $b \equiv \frac{1}{\gamma - 1} \cdot \frac{v_z}{v_z + v_p}$

so that $l_i = b(p_i - E[p])$

Recall that $q_i = l_i$ so aggregating generates Aggregate Supply function

$$\Sigma q_i = y = b(p - E[p])$$

expected value of price index

aggregate output

true value of price index

So surprise increases in the price level cause $y > 0$ aggregate output to exceed the natural rate

Remember: $y \equiv$ log of output

natural rate of y is zero

the log of output

Equilibrium

supply $\quad y = b(p - E[p])$

demand $\quad y = m - p \leftarrow$ same as in perfect ~~inform~~ information model

equilibrium

$$p = \frac{1}{1+b}\left(m + b\,E[p]\right)$$

$$y = \frac{b}{1+b}\left(m - E[p]\right)$$

After m is determined, the equilibrium price equation must hold therefore it is expected to hold and

$$E[p] = \frac{1}{1+b}\left(E[m] + b\,E[p]\right)$$

which implies that

$$E[p] = E[m]$$

monetary surprises	$m - E[m]$
expected value of log of money supply	$E[m]$

Since $E[p] = E[m]$

$$\boxed{\begin{aligned} p &= E[m] + \frac{1}{1+b}\left(m - E[m]\right) \\ y &= \frac{b}{1+b}\left(m - E[m]\right) \end{aligned}}$$

Remember that the aggregate demand equation is $y = m - p$, so the observed component of aggregate ~~dem~~ demand $E[m]$ only affects the price level p

BUT monetary surprises have "real effects" (i.e. they affect the value of aggregate output)

Romer shows that

$$b = \frac{1}{\gamma - 1}\left(\frac{V_z}{V_z + V_m \dfrac{(n+b)^2}{(1+b)^2}}\right)$$

Implications

Suppose that money supply follows a random walk with drift

$$m_t = m_{t-1} + \underset{\uparrow \text{ drift}}{c} + \underset{\uparrow \text{ white noise}}{u_t}$$

then $E[m_t] = m_{t-1} + c$

and $p_t = m_{t-1} + c + \dfrac{1}{1+b} u_t$

$$y_t = \frac{b}{1+b} u_t$$

Since we're using logarithms

inflation rate $\equiv (p_t - p_{t-1}) \equiv \pi_t$

$$\pi_t = (m_{t-1} - m_{t-2}) + \frac{1}{1-b}(u_t - u_{t-1})$$

$$\boxed{\begin{aligned} \pi_t &= c + \frac{1}{1+b}\left(b\,u_{t-1} + \underline{\underline{u_t}}\right) \\ y_t &= \frac{b}{1+b}\,\underline{\underline{u_t}} \end{aligned}}$$

So shocks u_t affect both the inflation rate and aggregate output

Positive inflation surprises increase aggregate output (Phillips curve)

There is no exploitable output-inflation tradeoff however

If monetary policymakers attempted to increase the ~~le~~ level of output by increasing the the rate of money supply growth (i.e. increase c), then aggregate output would temporarily rise BUT once the public observes the change, then monetary surprises would once again average to zero leaving aggregate output unchanged

Lucas Critique If policymakers attempt to take advantage of statistical relationships, the public will (sooner or later) "catch on" and cause those statistical relationships to break down