MACRO GRAD

12. III. 2009

p-1

RAMSEY MODEL

assumptions

→ large number of identical firms

$$Y = F(K, AL) \qquad g \equiv \frac{\dot{A}}{A}$$

→ large number of identical HHs and each member of a HH supplies one unit labor, so that

$$\frac{L}{H} \equiv \text{labor per HH} = \text{each HH's labor supply}$$

$$n \equiv \frac{\dot{L}}{L}$$

→ each HH has initial capital ~~[illegible]~~

$$\frac{K(0)}{H} \equiv \text{initial capital per HH} = \text{initial capital of each HH}$$

→ each HH's utility

$$U = \int_{t=0}^{\infty} e^{-\rho t} u(c) \frac{L}{H} dt$$

ρ is discount rate
high ρ implies high value of current consumption

$$u(c) = \frac{c^{1-\theta}}{1-\theta} \qquad \theta > 0$$

$$\beta \equiv \rho - n - (1-\theta)g > 0$$

utility fn: constant relative risk aversion

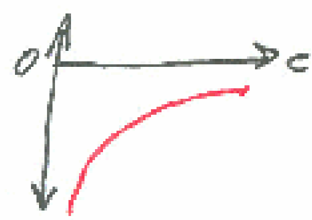
$$\text{coeff. of rel. risk aversion} \equiv \frac{-c \cdot u''}{u'} = \frac{-c \cdot -\theta \cdot c^{-\theta-1}}{c^{-\theta}} = \theta$$

θ determines household's willingness to shift consumption between different periods

the smaller is θ, the more slowly marginal utility falls as consumption rises

→ if $\theta \simeq 0$ then utility linear in consumption

→ if $\theta = 2$, then $u(c) = \frac{-1}{c}$



→ if $\theta = 1$, then $u(c) = \ln c$

the assumption that

$$\beta \equiv \rho - n - (1-\theta)g > 0$$

ensures that lifetime utility does not diverge

My pet peeve: The textbook states that ~~[illegible]~~ $r = f'(k)$ is the real interest rate

That is <u>WRONG</u>

Firms hire capital up to the point where

$$r = p\,MPK$$

That's the rental rate of capital ~~[illegible]~~ (expressed in <u>dollar</u> <u>terms</u>)

The <u>rate</u> <u>of</u> <u>return</u> equals the <u>real</u> <u>interest</u> <u>rate</u> under conditions of perfect competition

$$r = \frac{P_{OUTPUT}\,MPK}{P_{CAPITAL}}$$

That's the real interest rate (expressed as a percentage)

So we will assume that

$$\frac{\partial \text{output}}{\partial \text{capital}} = f'$$

so that $r \equiv$ real interest rate

$$r = f'(k)$$

→ real wage per unit of eff labor

$$w = f(k) - k f'(k)$$

→ worker's labor income: Aw

Household Budget constraint

→ ~~real~~ real interest rate may vary over time, so define

$$R(t) \equiv \int_{\tau=0}^{t} r(\tau)\, d\tau$$

→ PDV of one unit of output at t in terms of output at time 0 is:

$$e^{-R(t)}$$

→ consumption must be less than or equal to income

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} A(t) w(t) \frac{L(t)}{H} dt$$

→ define consumption and capital in terms of eff labor

$$\tilde{c} \equiv \frac{C}{A} \qquad \tilde{k} \equiv \frac{K}{AL}$$

→ define initial variables as $X_0 \equiv X(0)$

→ dropping the time notation:

$$\int_{t=0}^{\infty} e^{-R} \tilde{c} \frac{AL}{H} dt \leq \tilde{k}_0 \frac{A_0 L_0}{H} + \int_{t=0}^{\infty} e^{-R} \frac{wAL}{H} dt$$

$$A(t) = A(0) e^{gt} \quad // \quad L(t) = L(0) e^{nt}$$

$$A(t) L(t) = A(0) L(0) e^{(g+n)t}$$

→ dividing both sides by $A_0 L_0 / H$

$$\int_{t=0}^{\infty} e^{-R} \tilde{c} e^{(n+g)t} dt \leq \tilde{k}_0 + \int_{t=0}^{\infty} e^{-R} w e^{(n+g)t} dt$$

→ Rearranging terms:

$$\tilde{k}_0 + \int_{t=0}^{\infty} e^{-R}(w-\tilde{c})\, e^{(n+g)t}\, dt \geq 0$$

OPTIMIZATION

→ Recall that household's utility:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c)\, \frac{L}{H}\, dt$$

and

$$u(c) = \frac{c^{1-\theta}}{1-\theta} = \frac{A^{1-\theta}\, \tilde{c}^{\,1-\theta}}{1-\theta}$$

$$= A_0^{1-\theta}\, e^{(1-\theta)gt}\, \frac{\tilde{c}^{\,1-\theta}}{1-\theta}$$

→ Define:

$$B \equiv A_0^{1-\theta}\, \frac{L_0}{H} \qquad \beta \equiv \rho - n - (1-\theta)g > 0$$

$$U = B \int_{t=0}^{\infty} e^{-\beta t}\, \frac{\tilde{c}^{\,1-\theta}}{1-\theta}\, dt$$

→ Households maximise utility subject to budget constraint

$$\underset{c}{\text{Max}} \quad \mathcal{L} = B\int_{t=0}^{\infty} e^{-\beta t} \frac{\tilde{c}^{1-\theta}}{1-\theta} dt$$

$$+ \lambda \left\{ k_0 + \int_{t=0}^{\infty} e^{-R}(w - \tilde{c}) e^{(n+g)t} dt \right\}$$

(HHs satisfy budget constraint with equality because marginal utility of consumption is always positive).

→ Households pick $\tilde{c}$ at each point in time to maximise utility

$$\frac{d\mathcal{L}}{d\tilde{c}} = 0 \Rightarrow \underbrace{B e^{-\beta t} \tilde{c}^{-\theta} = \lambda e^{-R} e^{(n+g)t}}_{\text{must be equal for every } t}$$

→ taking logs of both sides and taking derivative of both sides w/ ~~resp~~ respect to time

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{r - n - g - \beta}{\theta}$$

← because $R(t) \equiv \int_{\tau=0}^{t} r(\tau) d\tau$

$$\frac{dR(t)}{dt} = r(t)$$

since $\beta \equiv \rho - n - (1-\theta)g$

$$\star \boxed{\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{r(t)-\rho}{\theta} - g}$$

growth rate of consumption per unit of effective labor

since $\hat{c} \equiv \frac{c}{A}$ $\quad \frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{\dot{c}}{c} - \frac{\dot{A}}{A}$

growth rate of consumption per worker

$$= \frac{\dot{c}}{c} - g \Rightarrow \boxed{\frac{\dot{c}}{c} = \frac{\dot{\tilde{c}}}{\tilde{c}} + g}$$

$$\star \boxed{\frac{\dot{c}}{c} = \frac{r(t)-\rho}{\theta}}$$

growth rate of consumption per worker

Consumption per worker grows if the real return exceeds the rate at which household discounts future consumption

That tells us about the growth rate of consumption, but we can't consume if we don't produce. We need capital to produce

$$\dot{\tilde{k}} = [f(\tilde{k}) - \tilde{c}] - (n+g)\tilde{k}$$

Output not consumed is saved (and invested)

Analogous to Solow Model

Now let's examine the dynamics of $\dot{\tilde{c}}$ and $\dot{\tilde{k}}$

$r(t) = f'(\tilde{k}(t))$

so $\dot{\tilde{c}} > 0$ if

$f'(\tilde{k}) > \rho + \theta g$

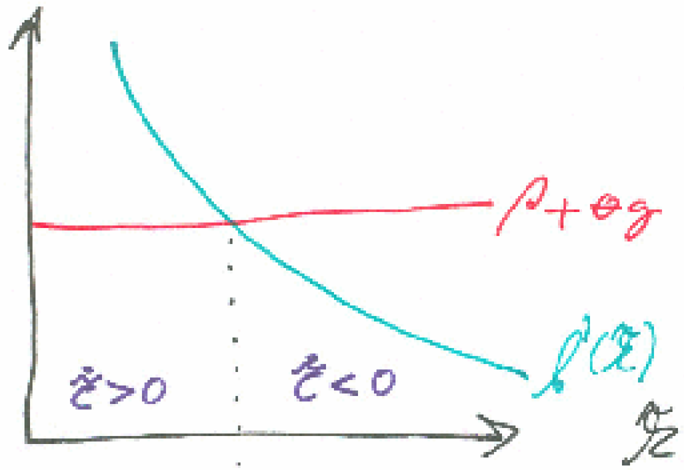
in Solow, steady state when $s f(\tilde{k}) = (n+g)\tilde{k}$

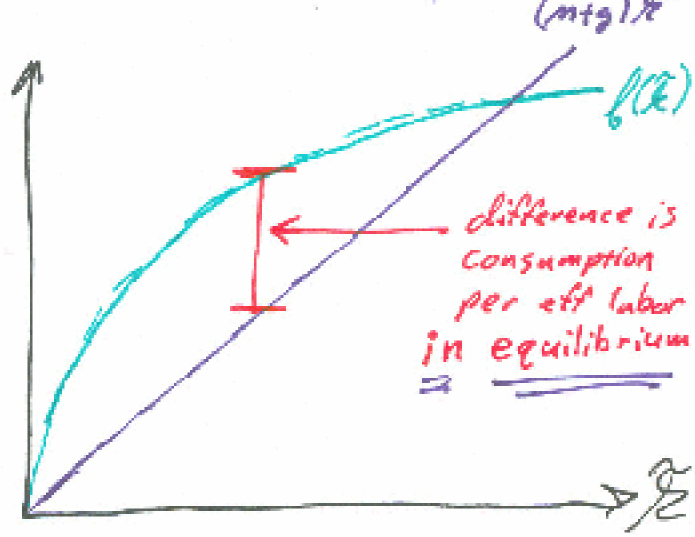
here, SS when

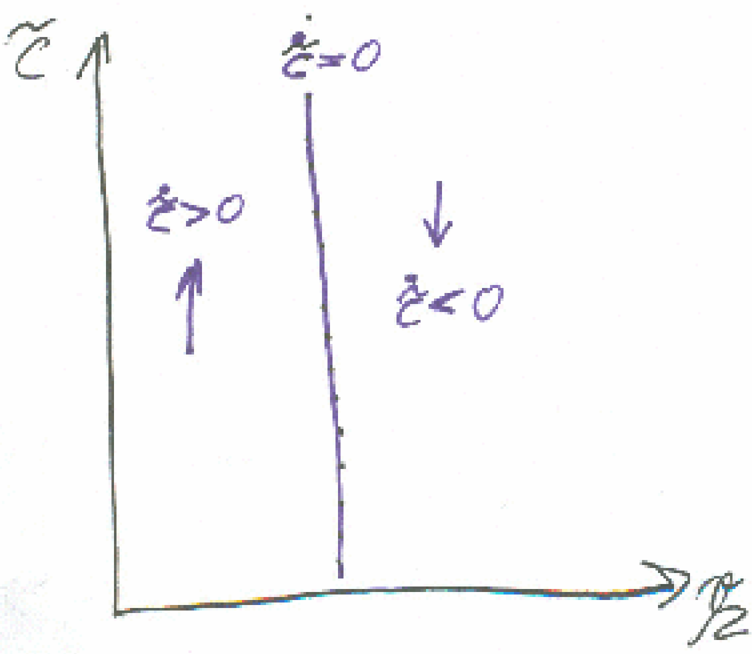
$f(\tilde{k}) - \tilde{c} = (n+g)\tilde{k}$

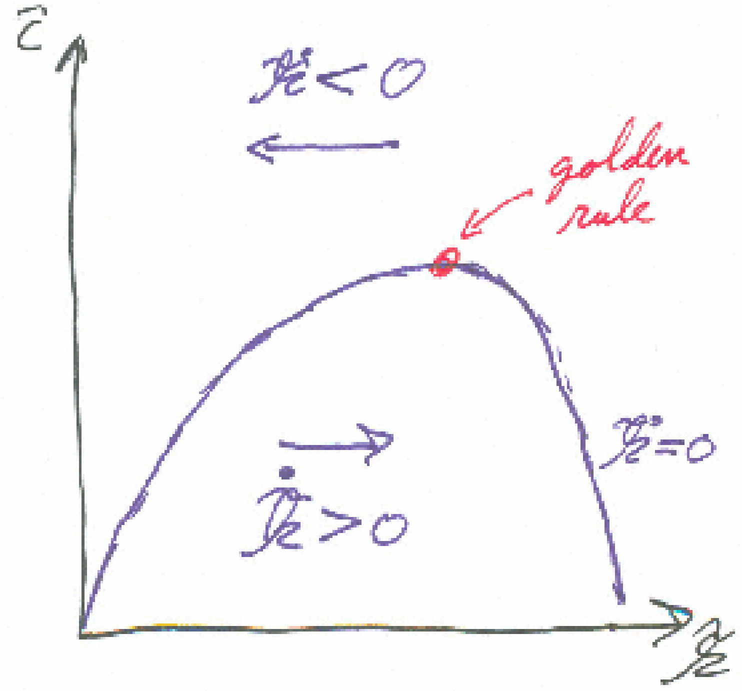
so here $\dot{\tilde{k}} > 0$ if

$f(\tilde{k}) - \tilde{c} > (n+g)\tilde{k}$

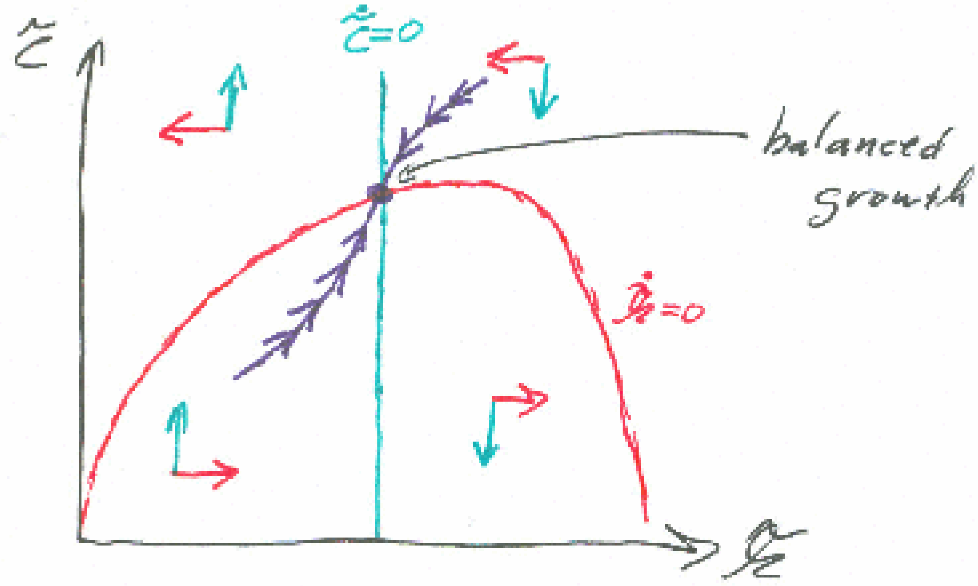








Put the two together



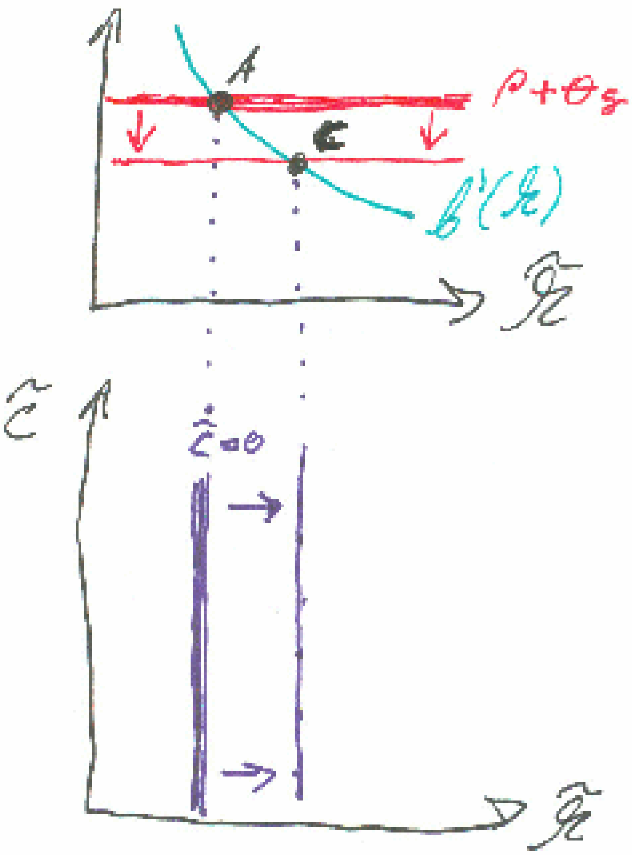
Satisfaction of the budget constraint + $\tilde{k} \geq 0$ requires that initial consumption be chosen so that $\tilde{c}$ + $\tilde{k}$ move along saddle path to equilibrium

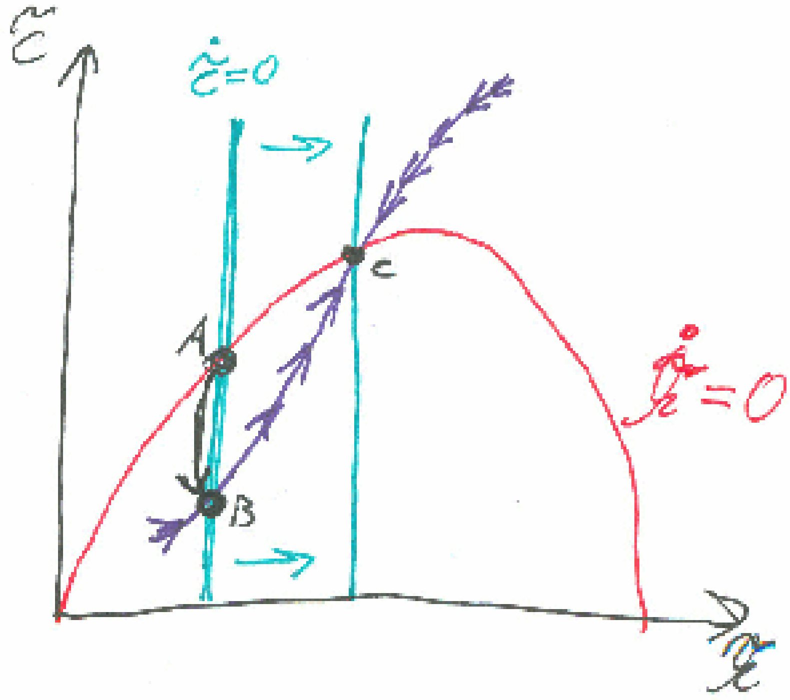
balanced growth of $\tilde{c}$ + $\tilde{k}$ at eqlbm point

Permanent Decrease ~~[illegible]~~ in Discount Rate

is Ramsey Model's analog to an increase in saving rate in Solow Model

⇒ A decrease in the discount rate means that households discount future consumption at a lower rate, i.e. they're more patient about consumption





→ So when there is a <u>permanent</u> decrease in the discount rate, households immediately reduce their consumption (i.e. they save more) which enables them to accumulate capital, their consumption then grows as the economy converges to a new equilibrium

→ Romer also shows that a permanent decrease in the discount rate causes a temporary increase in the growth rates of capital per worker/eff. labor and output per worker/eff. labor

→ The Solow Model exhibits the same temporary increase in growth rates

The only difference is that the fraction of output saved is <u>NOT</u> constant (as it is in the Solow Model)

Permanent vs. Temporary Increase in Gov't Spending

Assume: gov't purchases have no productive value

Assume: gov't purchases do not affect household utility

Assume: lump-sum taxes

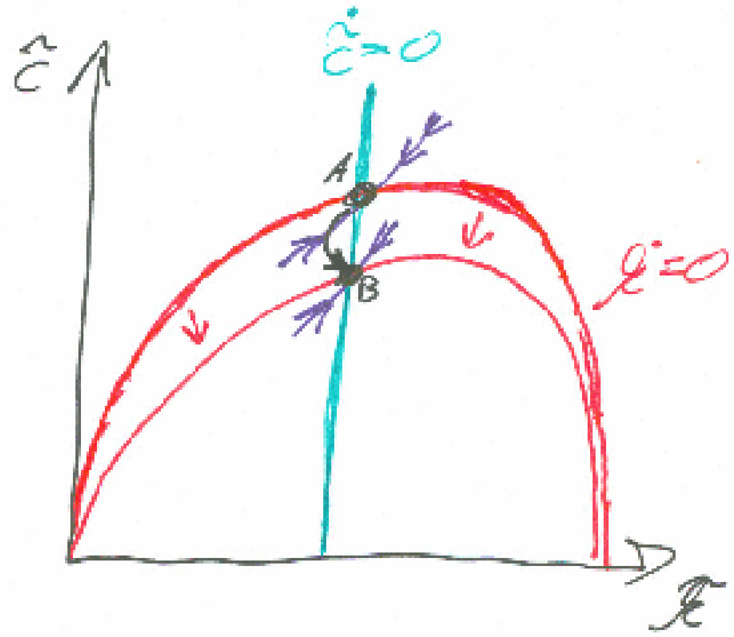
Case here: balanced budget

$$\dot{\tilde{k}} = f(\tilde{k}) - \tilde{c} - \tilde{G} - (n+g)\tilde{k}$$

↑ gov't purchases per eff. labor

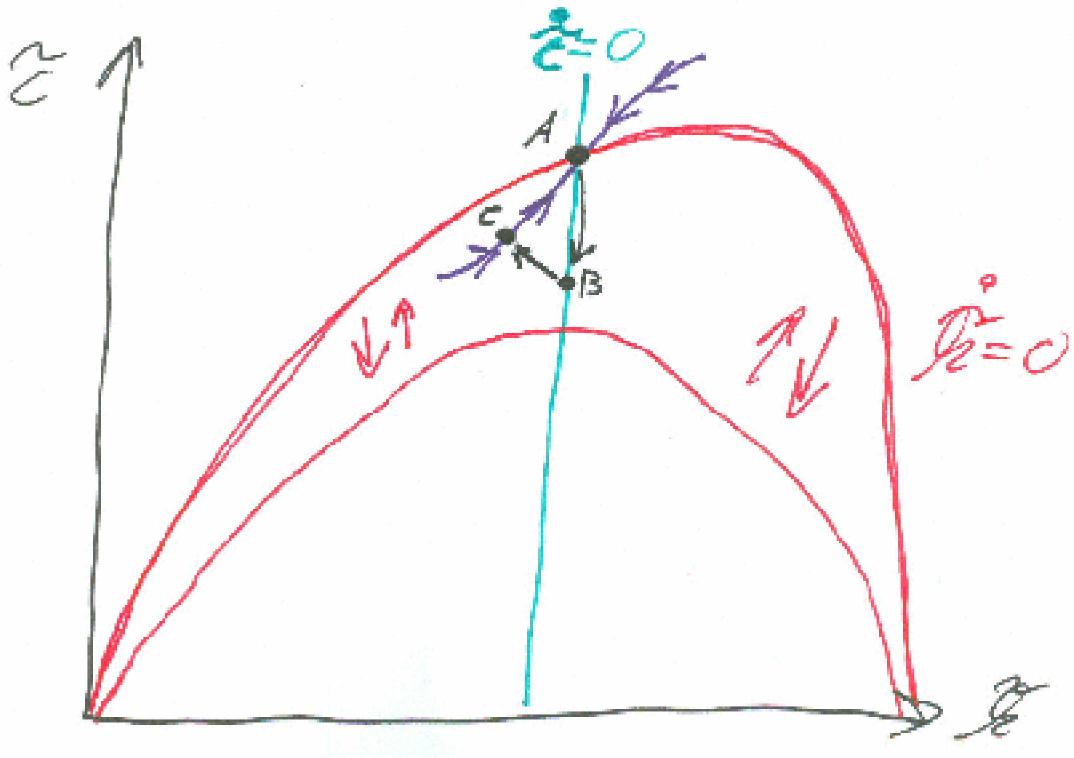
A PERMANENT increase in $\tilde{G}$ shifts $\dot{\tilde{k}}=0$ locus downward because higher gov't spending reduces amount of goods households can purchase in equilibrium

Immediate ↓ $\tilde{c}$ to new equilibrium level



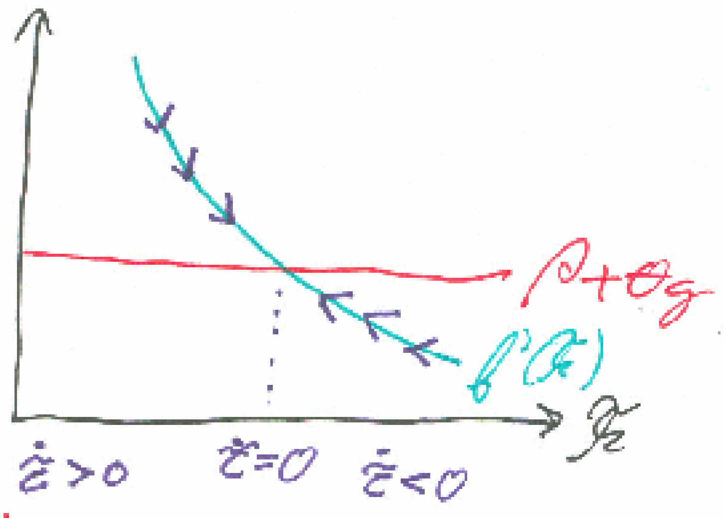
A TEMPORARY increase in $\tilde{G}$ temporarily shifts the $\dot{\tilde{k}} = 0$ locus downward, but households foresee the return to a lower level of $\tilde{G}$ so they smooth their consumption over time, i.e. they initially reduce their consumption, but then they allow their holdings of $\tilde{k}$ to fall (dissaving)

As ~~they~~ the period of high $\tilde{G}$ comes to an end, they're on the saddle path back to the old equilibrium



Romer points to Barro's (1987) test of the theory that a temporary increase in $\tilde{G}$ causes households to smooth their consumption over time by examining UK real interest rates during wars (which are expected to be temporary)

To accept greater future consumption in exchange for less current consumption households must be compensated with higher real interest rate



in this region $\tilde{c}$ growing over time

in this region $\tilde{c}$ falling over time

Barro (1987) found that UK real interest rates were positively correlated with military spending as a percentage of GDP

→ In the long run the government must balance its budget, so even if it finances an increase in $\bar{G}$ with sales of bonds (budget deficit) households will foresee tax increase down the road

RICARDIAN EQUIVALENCE

→ only the path of government purchases (and not the path of taxes) affects the economy

→ but not much evidence in favor of Ricardian equivalence