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Economic Growth and
Economic Fluctuations

What's the difference between Marginal Cost and Average Cost?

"Marginal cost is *not* the cost of producing the "last" unit of output. The cost of producing the last unit of output is the same as the cost of producing the first or any other unit of output and is, in fact, the *average* cost of output. Marginal cost (in the finite sense) is the increase (or decrease) in cost resulting from the production of an extra increment of output, which is not the same thing as the "cost of the last unit." The decision to produce additional output entails the greater utilization of factor inputs. In most cases … this greater utilization will involve losses (or possibly gains) in input efficiency. When factor proportions and intensities are changed, the marginal productivities of the factors change because of the law of diminishing returns, therefore affecting the cost per unit of output."

– Eugene Silberberg, *The Structure of Economics* (1990)

Let's break Silberberg's definition of marginal cost into its component pieces. First, he ascribes changes in marginal cost to changes in marginal productivities of factor inputs. (By factor inputs, he means factors of production, like labor and capital).

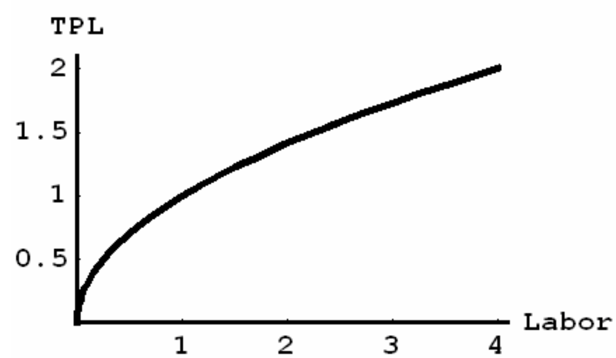
So what is the marginal product of labor and how is it affected by the law of diminishing returns?

Imagine a coal miner traveling deep underground to swing his pick at the coal face. The longer he swings his pick, the more coal he will produce, but it's exhausting work, so if his boss were to require him to work a double shift, the miner wouldn't double the amount of coal that he produces.

To be more specific, let's assume that the miner produces an amount of coal equal to the square root of the number of hours he works. The tonnage of coal that he produces is his "total product of labor (TPL)." So if he puts in zero hours, he produces zero tons of coal. If he puts in one hour he produces one ton. If he puts in two hours, he produces $\sqrt{2} = 1.41$ tons of coal, etc.

$$\text{tons of coal} = \sqrt{\text{hours}}$$

hours	coal	$\Delta\text{coal}/\Delta\text{hours}$
0	0	–
1	1	1
2	1.41	0.41
3	1.73	0.32
4	2	0.27



If the miner increases the number of hours that he spends mining from one hour to two hours, his output of coal will increase by 0.41 tons. Increasing the miner's hours from two to three hours only increases his output of coal by 0.32 tons however. Notice that **the additional coal he produces per additional hour that he works diminishes. This is the law of diminishing marginal returns.**

Notice also that the table lists **the ratio of the change in coal output to the change in the amount of hours worked.** That's the **slope of the total product function**, or the **"marginal product of labor."**

Plotting the miner's marginal product of labor against the amount of hours that he works shows the rate of output change at each amount of working hours.

$$\text{tons of coal} = \sqrt{\text{hours}}$$

hours	coal	Δcoal/Δhours
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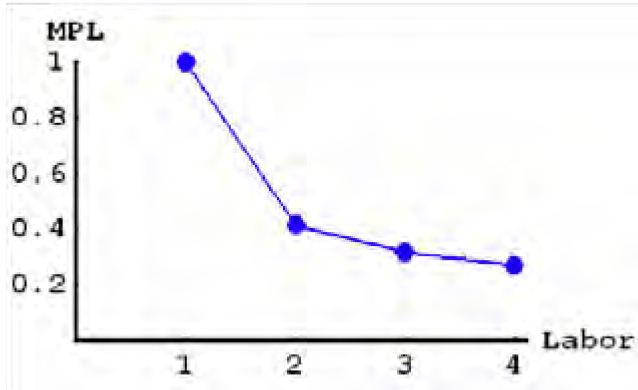


We don't need to measure the changes in the miner's coal output in one hour increments however. In fact, economists are usually more interested in a continuous rate of change. Those of you who have taken a course in calculus should know that the continuous rate of change in output is simply the first derivative of the total product function with respect to the number of hours worked.

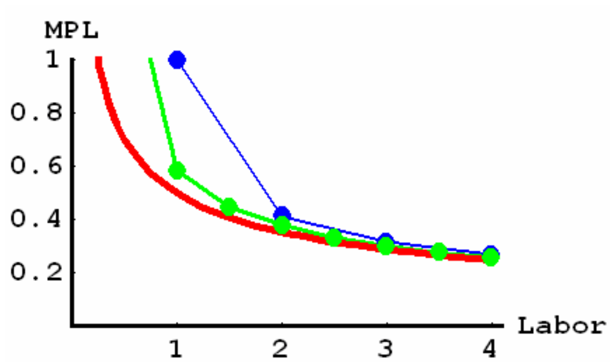
For those of you who have not taken a course in calculus, imagine that we can measure the miner's total output at each second in time. If we look at how much the miner's output increases from one second to the next and divide that change by one second, we'll have a good approximation of the first derivative.

For example, one second is $1/360$ of an hour or 0.002778 hours. If the miner works for exactly two hours, then he'll produce 1.414214 tons of coal. If he works for exactly two hours and one second, then he'll produce 1.415195 tons of coal. In other words, adding one second to a two hour workday increases his output of coal by 0.000982 tons. The marginal product of labor evaluated at two hours and one second is:

$$\frac{\sqrt{2.002778}-\sqrt{2}}{0.002778} = \frac{1.415195-1.414214}{0.002778} = \frac{0.000982}{0.002778} = 0.353431 \text{ tons per hour}$$

To see how successively smaller changes in units of time (by which output changes are measured) lead to closer and closer approximations to the first derivative, consider this graph of true marginal product (red), the change in output per half-hour change in work hours (green) and the change in output per one hour change in work hours (blue).

A table of the data points in the graph is given on the next page.



Now that you know what the "marginal product of labor" is, what do you think "marginal cost" is? It's the change in total cost per unit change in output, calculated for an infinitesimally small change in output.

Just as the marginal product of labor measures the slope of the total product of labor function, marginal cost measures the slope of the total cost function.

hours	coal	true MPL	Δcoal/Δhrs. Δhrs. = 0.5	Δcoal/Δhrs. Δhrs. = 1
0	0	infinite	–	–
0.5	0.71	0.71	1.41	–
1.0	1.00	0.50	0.59	1
1.5	1.22	0.41	0.45	–
2.0	1.41	0.35	0.38	0.41
2.5	1.58	0.32	0.33	–
3.0	1.73	0.29	0.30	0.32
3.5	1.87	0.27	0.28	–
4.0	2.00	0.25	0.26	0.27

Notice also what happens when we sum the last two columns of the table above (i.e. the columns of Δcoal/Δhrs.). The column listing a time interval of one hour sums to 2, which is exactly how many tons of coal are produced. The area under the marginal product curve equals total product because increasing the number of hours from zero to one yields one additional ton of coal per hour, increasing the number of hours from one to two yields 0.41 additional tons of coal per hour, etc.

The column listing a time interval of half an hour sums to 4, which when multiplied by 0.5 hours also equals 2 (tons of coal), so once again the area under the marginal product curve equals total product.

The column listing the true marginal product of labor sums to 3.1 plus infinity, which at first glance seems to contradict the results above, but keep in mind that the true marginal product is calculated using infinitesimally small intervals of time. So we'd have to multiply infinity plus 3.1 by the infinitesimally small intervals of time that we used to obtain the true marginal product to obtain 2 tons of coal. (In mathematical terms: we could integrate the true marginal product of labor from zero hours to four hours with respect to the number of hours the miner works to obtain 2 tons of coal).

♦ ♦ ♦

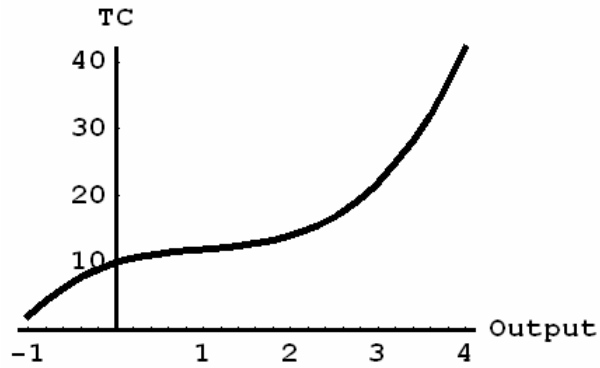
Now that we understand the law of diminishing returns and the concept of marginalism, let's reexamine Silberberg's quote. He says that to produce more output, a firm must hire more factors of production (like labor or capital) and/or use them more intensively, but such increased utilization reduces the efficiency of those factors of production (due to the law of diminishing returns) and raises the marginal cost of output.

For example, if the mining company only employs one miner and pays him a wage of $1 and that one miner is the only factor of production, then producing one ton of coal requires one hour of labor from the miner and costs a total of $1, but producing two tons requires four hours of labor and costs a total of $4. In this case, the marginal cost of increasing output from zero tons to one ton is $1 and the marginal cost of increasing output from one ton to two is: $4 – $1 = $3.

So let's examine a hypothetical firm's total, average and marginal costs by assuming that it faces a fixed cost of $10 and its variable cost is given by: $VC = X^3 - 3X^2 + 4X$, where X is the amount of output that it produces. Total cost is equal to fixed cost plus variable cost, so: $TC = X^3 - 3X^2 + 4X + 10$.

In the specification above, the firm's variable costs increase as the firm produces more output (and decrease as it produces less), therefore marginal cost reflects changes in variable cost. By definition, the firm's fixed costs do not change when it increases or decreases the amount of output it produces, therefore marginal cost does not reflect changes in fixed cost – because there are no changes in fixed cost.

In the graph at right, I have drawn a total cost curve running from negative one units of output to four units of output.



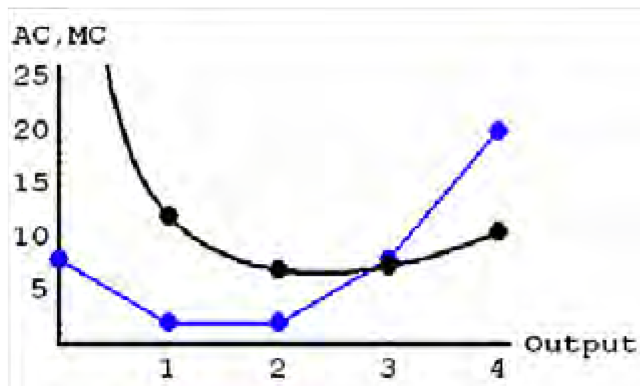
Now it should be obvious to you that a firm would not produce a negative output.

I drew the total cost function from negative one to better show the shape of the curve and because I'll use negative one to approximate the marginal cost at zero units of output.

The total cost curve depicted is everywhere increasing as output increases (i.e. is everywhere positively sloped), but it is not increasing at a constant rate. Initially total cost rises at a fairly rapid rate, but then the rate of increase slows, yielding a somewhat flat section. Finally, the rate of increase accelerates again. Since marginal cost is the rate of change in total cost (the slope of the total cost curve), the marginal cost curve will be U-shaped.

$$TC = X^3 - 3X^2 + 4X + 10$$

X	TC	VC	AC	$\Delta TC/\Delta X$ $\Delta X = 1$
–1	2	–8	–	–
0	10	0	infinite	8
1	12	2	12	2
2	14	4	7	2
3	22	12	$7\frac{1}{3}$	8
4	42	32	$10\frac{1}{2}$	20



If the firm faces a U-shaped marginal cost curve, then at low levels of output, it can increase the marginal productivity of its inputs by using them more intensively (a possibility I ruled out in the miner example), but at higher outputs, the firm confronts the law of diminishing returns and faces rising marginal cost.

The firm also faces a U-shaped average cost curve. The firm's average costs fall when it increases its production from zero to a moderate amount of output because its fixed cost is spread over a larger amount of output and (to a much lesser extent) because its average variable cost falls as inputs are used more efficiently (i.e. they yield a higher marginal product).

At high levels of output, the firm's average fixed cost approaches zero, but its variable costs rise rapidly due to the law of diminishing returns. At high levels of output, marginal cost exceeds both average variable cost and average cost, because the **averages spread the rising variable cost over the total amount of output, whereas marginal cost reflects changes in variable cost over small intervals.**

Finally, notice that the sum of the entries in the column containing the firm's marginal cost equals $32 – the variable cost. (Ignore the marginal cost of $8 that occurs when the firm produces zero units of output because it was calculated by increasing output from negative one to zero). It equals $32 because marginal cost examines changes in variable cost, so the sum of the marginal costs must equal variable cost at that level of output.