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Macro HW 4, problem 1 -- marginal product of labor

```
(%i1) round_digits(value,digits) :=
         float( round( 10^(digits) * value ) / 10^(digits)) $
```

1. In its introduction to the Solow Model without technological progress, Lecture 5 contains a derivation of the marginal product of capital.

   a. If the production function is given by: Y = (K^a)*(L^(1– a)) , what is the marginal product of labor?

```
(%i2) Y(K,L,a)   :=   (K^a)*(L^(1-a)) $
      MPL(K,L,a) := ''(diff( Y(K,L,a) ,L) ) $
      print("")$
      print("When the production function is given by:  Y = ", Y(K,L,a) )$
      print("the marginal product of labor is: MPL = ", MPL(K,L,a) )$
```

When the production function is given by: $Y = K^{a} L^{1 - a}$

the marginal product of labor is: MPL = $\frac{(1 - a) K^{a}}{L^{a}}$

   b. Assuming that a = 0.5 and that K = 1 , calculate the marginal product of labor from one unit of labor input to five units.

```
(%i7) print("")$
      print("when a=0.5 and K=1, then: MPL = ",
         MPL(K,L,a) ," = ", MPL(1,L,0.5))$
      print("when L=1, MPL=", round_digits( MPL(1,1,0.5) ,2) )$
      print("when L=2, MPL=", round_digits( MPL(1,2,0.5) ,2) )$
      print("when L=3, MPL=", round_digits( MPL(1,3,0.5) ,2) )$
      print("when L=4, MPL=", round_digits( MPL(1,4,0.5) ,2) )$
      print("when L=5, MPL=", round_digits( MPL(1,5,0.5) ,2) )$
```

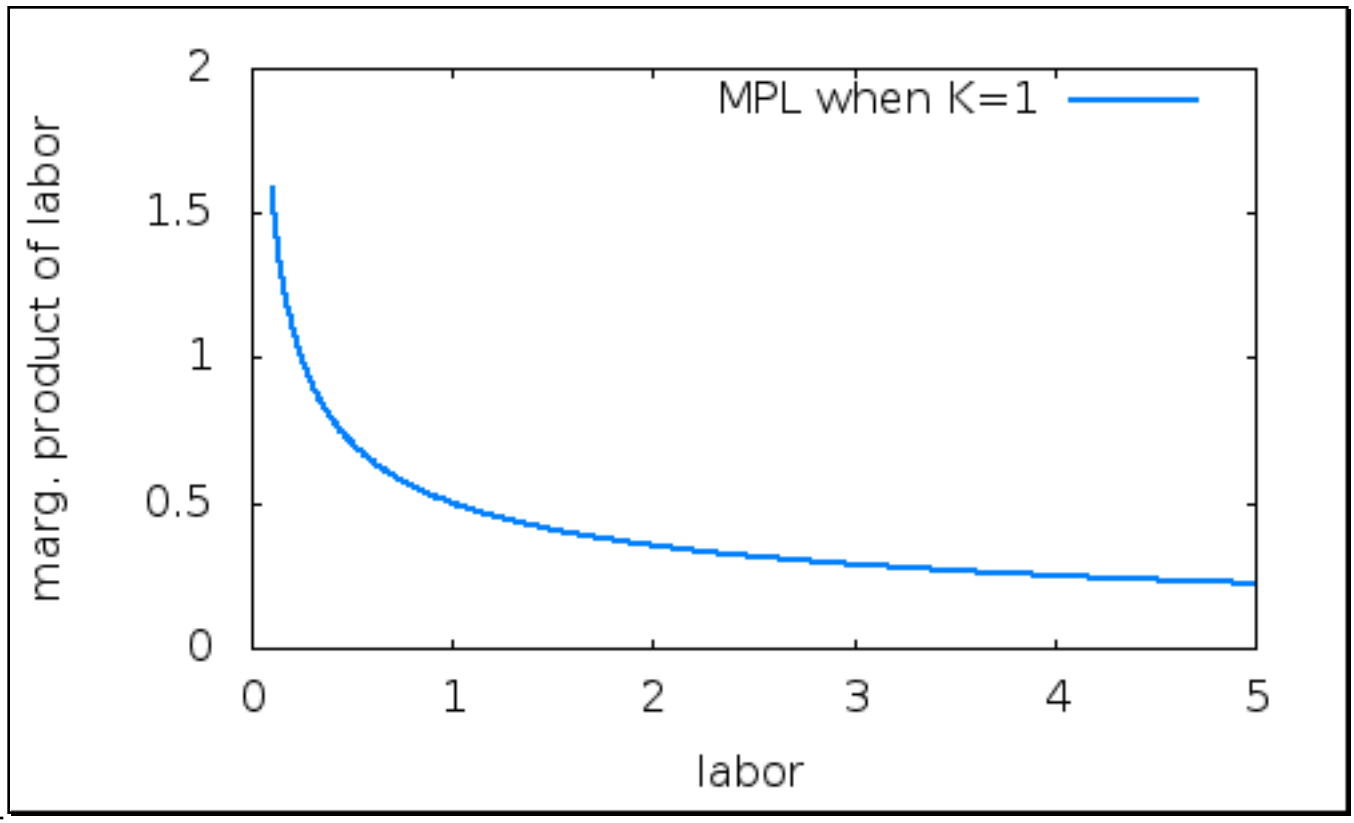
when a=0.5 and K=1, then: MPL = $\frac{(1 - a) K^{a}}{L^{a}} = \frac{0.5}{L^{0.5}}$

when L=1, MPL= 0.5
when L=2, MPL= 0.35
when L=3, MPL= 0.29
when L=4, MPL= 0.25
when L=5, MPL= 0.22

c. On a graph, plot the marginal product of labor using the values you just calculated.

```
(%i14) print("")$
       wxplot2d( [MPL(1,L,0.5)] , [L,0.1,5] ,
        [x,0,5] , [y,0,2],
        [style,[lines,2]],
        [legend,"MPL when K=1"] ,
        [xlabel,"labor"] , [ylabel,"marg. product of labor"])$
```

(%t15)



d. Assuming that a = 0.5 and that K = 2 , calculate the marginal product of labor from one unit of labor input to five units.

```
(%i16) print("")$
       print("when a=0.5 and K=2, then: MPL = ",
          MPL(K,L,a) ," = ", MPL(2,L,0.5) ) $
       print("when L=1, MPL=", round_digits( MPL(2,1,0.5) ,2) )$
       print("when L=2, MPL=", round_digits( MPL(2,2,0.5) ,2) )$
       print("when L=3, MPL=", round_digits( MPL(2,3,0.5) ,2) )$
       print("when L=4, MPL=", round_digits( MPL(2,4,0.5) ,2) )$
       print("when L=5, MPL=", round_digits( MPL(2,5,0.5) ,2) )$
```

$$\text{when } a=0.5 \text{ and } K=2, \text{ then: } MPL = \frac{(1-a)\,K^{a}}{L^{a}} = \frac{0.70710678118655}{L^{0.5}}$$

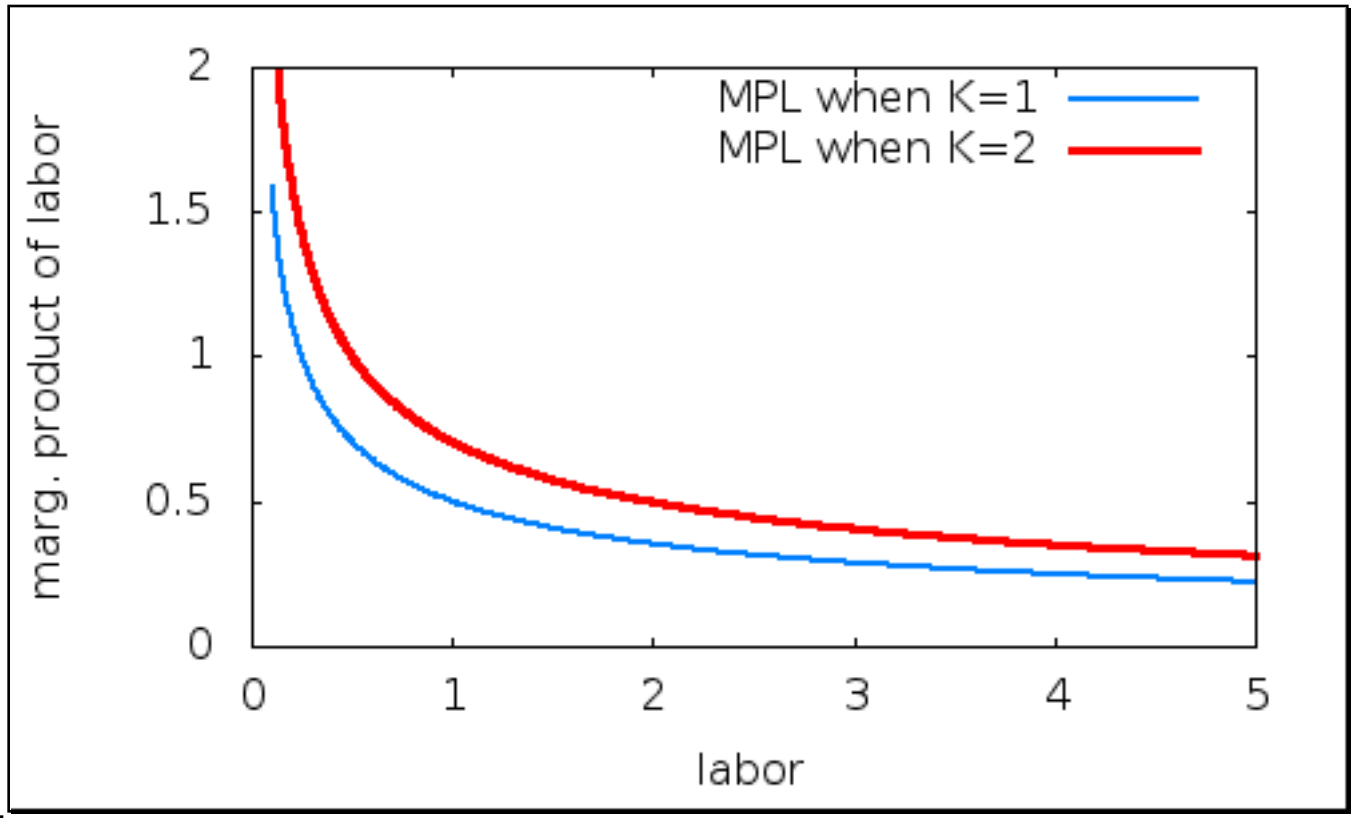
when L=1, MPL= 0.71
when L=2, MPL= 0.5
when L=3, MPL= 0.41
when L=4, MPL= 0.35
when L=5, MPL= 0.32

e. On the same graph, plot the marginal product of labor using the values you just calculated.

```
(%i23) print("")$
       wxplot2d( [MPL(1,L,0.5),MPL(2,L,0.5)] , [L,0.1,5] ,
        [x,0,5] , [y,0,2],
        [style,[lines,2],[lines,3]],
        [legend,"MPL when K=1","MPL when K=2"] ,
        [xlabel,"labor"] , [ylabel,"marg. product of labor"])$
```

```
plot2d: some values were clipped.
```

(%t24)



f. What happens to the marginal product of labor when the economy's stock of capital increases?