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Midterm Review

As discussed in class, I will ask you to solve several problems on the midterm exam. The best way to prepare is to do the homework exercises in my "Calculus Tricks." Compare those notes with the chapters on differentiation in Sydsæter and Hammond (chaps. 6 and 7). Next, look at the optimization problems that we discussed in class and compare them with Sydsæter and Hammond's chapter on optimization (chap. 8). Finally, I have prepared a sample problem for you.

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Sample Problem – Consider the case of a firm that acts as a monopsonist in the local labor market. As the sole employer in town, it can push down wages and increase profit by reducing the employment level. The firm's goal is to maximize profit:

$$\Pi(L) = p \cdot Q(L) - w(L) \cdot L$$

with respect to labor, L. For simplicity, assume the production function exhibits diminishing marginal returns and is given by:

$$Q(L) = 100 \cdot \sqrt{L}$$

assume that the quantity of labor supplied is an increasing function of the wage rate and the inverse of that relationship is given by:

$$w(L) = L \cdot \sqrt{L}$$

and assume that the firm sells its output in a perfectly competitive market (so that the market price of output is exogenous to the firm) and that the price is fixed at:

$$p = 5$$

1. Derive the marginal benefit of increasing employment, L.
2. Derive the marginal cost of increasing employment, L.
3. What is the necessary condition for maximizing $\Pi(L)$ with respect to L?
4. What is the sufficient condition for maximizing $\Pi(L)$?
5. What is the value of L that maximizes $\Pi(L)$?
   (a) what is the value of $w(L)$?
   (b) what is the value of $\Pi(L)$?

Now, assume that the government imposes a minimum wage:

$$w_{min} = 65$$

and answer the same set of questions.

6. Derive the marginal benefit of increasing employment, L.
7. Derive the marginal cost of increasing employment, L.
8. What is the necessary condition for maximizing $\Pi(L)$ with respect to L?
9. What is the sufficient condition for maximizing $\Pi(L)$?
10. What is the value of L that maximizes $\Pi(L)$?
    (a) what is the value of $\Pi(L)$?
11. What happened to total employment in the town after the minimum wage was imposed?
12. What happened to the firm's profit after the minimum wage was imposed?