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original: 29 April 2014

Quantitative Analysis
estimating parameters of Cauchy distribution

First, load the "distrib" package.

```
(%i1) load(distrib)$
```

Now, randomly draw 50 values from the standard Cauchy.

```
(%i2) N:50$
      x:random_cauchy(0,1,N)$
```

Set up the log-likelihood function.

```
(%i4) loglik(mu,gamma):= -N*log(%pi*gamma) -
          sum( log(1+(((x[i]-mu)/gamma)^2)),i,1,N);
```

(%o4) $\mathrm{loglik}(\mu,\Gamma):=(-N)\log(\pi\Gamma)-\sum_{i=1}^{N}\log\left(1+\left(\frac{x_i-\mu}{\Gamma}\right)^2\right)$

Maximize it with respect to mu and gamma.

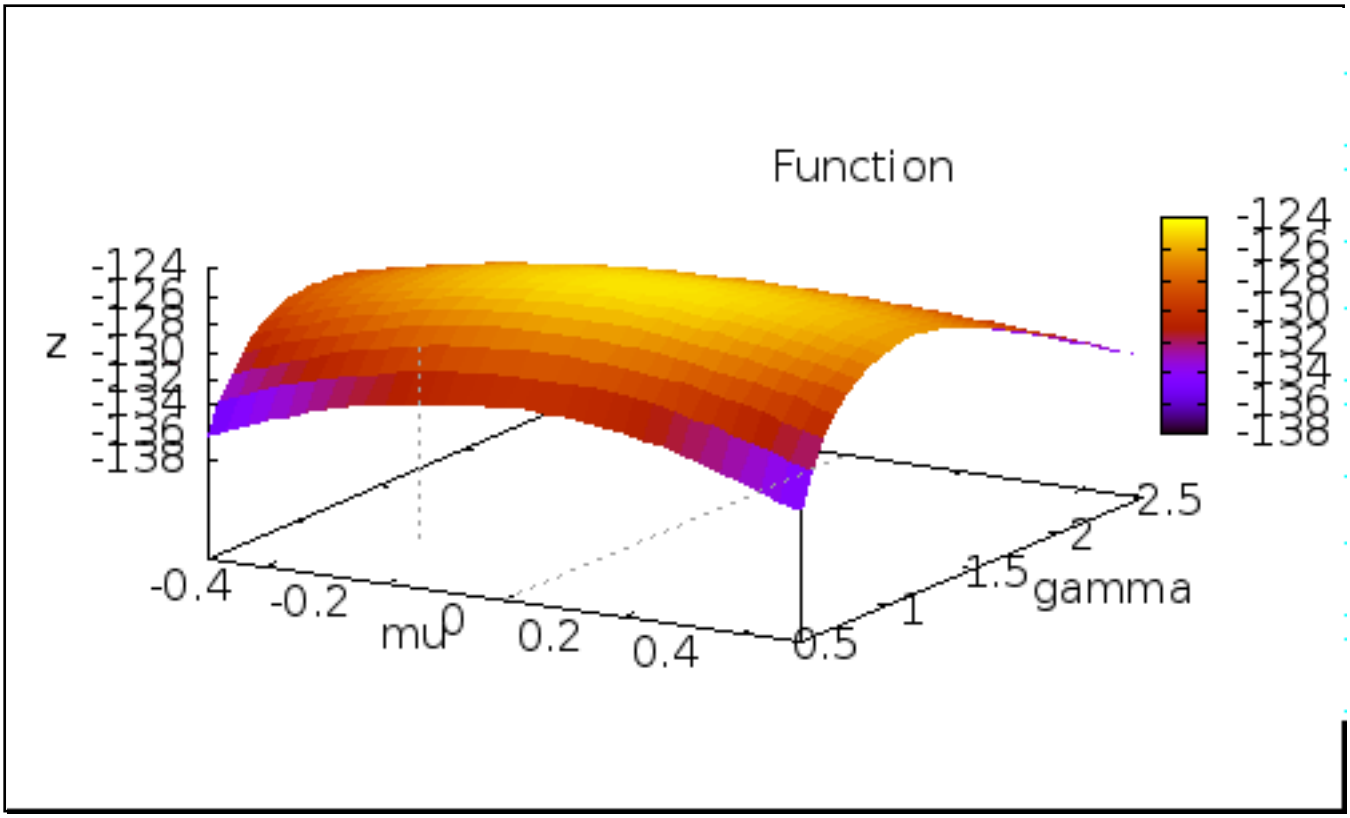
```
(%i5) sol:lbfgs(-loglik(mu,gamma),'[mu,gamma],[0.01,0.99],0.0001,[-1,0])$
      mu_max:subst(sol[1],mu)$
      gamma_max:subst(sol[2],gamma)$

      print("")$
      print(mu," = ",mu_max)$
      print(gamma," = ",gamma_max)$
      print("")$
```

μ = 0.080902253273153
Γ = 1.036534260262574

To see the maximum, plot the log likelihood function in "mu-gamma" space.

```
(%i12) wxplot3d(loglik(mu,gamma),[mu,-0.5,0.5],[gamma,0.5,2.5])$
(%t12)
```



Check to see if second-order conditions are satisfied.

```
(%i13) /* set up the Hessian matrix */
       dxx(mu,gamma):=''(diff(diff(loglik(mu,gamma),mu),mu))$
       dyy(mu,gamma):=''(diff(diff(loglik(mu,gamma),gamma),gamma))$
       dxy(mu,gamma):=''(diff(diff(loglik(mu,gamma),mu),gamma))$
       H:matrix(
           [dxx(mu_max,gamma_max),dxy(mu_max,gamma_max)],
           [dxy(mu_max,gamma_max),dyy(mu_max,gamma_max)])$
```

```
(%i17) print("")$
       print("own-partials must be negative:")$
       print("")$
       print("d^2 loglik(mu,gamma)"/"(d mu)^2"," = ",dxx(mu_max,gamma_max))$
       print("")$
       print("d^2 loglik(mu,gamma)"/"(d gamma)^2"," = ",dyy(mu_max,gamma_max)
       print("")$
```

own-partials must be negative:

$$\frac{d^2\ loglik(mu,gamma)}{(d\ mu)^2} = -22.2581034728271$$

$$\frac{d^2\ loglik(mu,gamma)}{(d\ gamma)^2} = -24.27935715351495$$

```
(%i24) print("")$
       print("the cross-partial:")$
       print("")$
       print("d^2 loglik(mu,gamma)"/"d mu d gamma"," = ",dxy(mu_max,gamma_max
       print("")$
```

the cross-partial:

$$\frac{d^2\ loglik(mu,gamma)}{d\ mu\ d\ gamma} = 2.664077718337032$$

```
(%i29) print("")$
       print("the Hessian matrix:")$
       print("H = ",H)$
       print("")$
       print("determinant of Hessian must be positive")$
       print("det(H) = ",determinant(H))$
       print("")$
```

the Hessian matrix:

$$H = \begin{bmatrix} -22.2581034728271 & 2.664077718337032 \\ 2.664077718337032 & -24.27935715351495 \end{bmatrix}$$

determinant of Hessian must be positive

det(H) = 533.3151336873207

```
(%i36) info:-1*invert(H)$

       print("")$
       print("the information matrix:")$
       print("-1*(H^-1) = ",info)$
       print("")$
       print(mu,": ",mu_max,"   se:",sqrt(info[1,1]))$
       print(gamma,": ",gamma_max,"   var:",info[2,2])$
       print("")$
```

the information matrix:

-1(H^-1) =* $\begin{bmatrix} 0.045525348185131 & 0.0049953161837312 \\ 0.0049953161837312 & 0.04173536820329 \end{bmatrix}$

μ *:* 0.080902253273153 *se:* 0.21336669886637

Γ *:* 1.036534260262574 *var:* 0.04173536820329