Lecture 2A: Interest Rates

Measurement

present discounted value

→ a dollar now is worth more than a dollar one year from now (so we "discount the future")

$$PDV = \frac{\text{Future Value}}{(1+i)^n}$$

BOOK EXAMPLE

$$\$100 = \frac{\$133}{(1+0.10)^3}$$

→ note that \$20 million dollars paid out at \$1 million per year over 20 years is worth less as the interest rate rises

four types of credit market instruments

1. simple loan – credit issued on day one full loan and interest repaid at maturity
   money mkt instruments
2. fixed payment (fully amortized) loan
   same payment made on a regular basis until maturity (e.g. home + auto loans)

3. coupon bond – "coupon payments" made every year until maturity when face value (ie par value) is repaid

- coupon payments are interest payments
- US T-bonds & corporate bonds

4. discount bond – bought at a price below face value & face value paid at maturity

- no interest payments made
- US T-bills, US Savings bonds

Yield to Maturity

- How do we calculate the interest rate on each of these types of loans?

- simple loan

$$PDV = \frac{\text{Future Value}}{1+i} \qquad \$100 = \frac{\$110}{1+i}$$

$$i = \frac{\text{Future Value}}{PDV} - 1 \qquad i = \frac{\$110}{\$100} - 1 = 0.10$$

- fixed payment (fully amortized) loan

$$LV = FP\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^n}\right)$$

define: $\rho \equiv \frac{1}{1+i}$

$$\frac{LV}{FP} = \rho + \rho^2 + \rho^3 + \cdots + \rho^n$$

$$= \frac{1}{1-\rho}\left((1-\rho)\rho + (1-\rho)\rho^2 + (1-\rho)\rho^3 + \cdots + (1-\rho)\rho^n\right)$$

$$= \frac{1}{1-\rho}\left(\rho - \cancel{\rho^2} + \cancel{\rho^2} - \cancel{\rho^3} + \cancel{\rho^3} - \rho^4 + \cdots + \cancel{\rho^n} - \rho^{n+1}\right)$$

$$\frac{LV}{FP} = \frac{\rho}{1-\rho}\left(1 - \rho^n\right)$$

$$= \frac{\frac{1}{\cancel{1+i}}}{\frac{1+i}{\cancel{1+i}} - \frac{1}{\cancel{1+i}}}\left(1 - \frac{1}{(1+i)^n}\right) = \frac{1}{i}\left(1 - \frac{1}{(1+i)^n}\right)$$

if n and i large, then this term is almost <u>zero</u>

$$\boxed{i \approx \frac{FP}{LV}}$$

BOOK EXAMPLE

$$\$1000 = \$126\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots + \frac{1}{(1+i)^{25}}\right)$$

then they tell you to use a fancy calculator to obtain i

but if n and i are large, then

$$\frac{1}{(1+i)^n} \approx 0 \qquad \frac{1}{(1+0{,}12)^{25}} = 0{,}059$$

so for a quick approximation:

$$i \approx \frac{FP}{LV} \qquad i \approx \frac{126}{1000} = 12{,}6\%$$

- Coupon bond

$$P = C\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^n}\right) + \frac{F}{(1+i)^n}$$

$$P = \$100\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{10}}\right) + \frac{\$1000}{(1+i)^{10}}$$

You can use a method similar to the one above to show that

$$P = \frac{C}{i} + \frac{1}{(1+i)^n}\left(F - \frac{C}{i}\right)$$

this may be small, but

this is LARGE

~~so~~ NOT necessarily ZERO

in the textbook example, the yield to maturity is:

yield	price
7,13 %	1200
8,48 %	1100
10,00 %	1000
11,75 %	900
13,81 %	800

Note that the yield and price are ~~[illegible]~~ inversely related

"Coupon rate" $\equiv \frac{C}{P}$

Note that if $n = \infty$ (as in the case of a consol), then:

$$i = \frac{C}{P}$$

- discount bond REAL EASY

$$i = \frac{FV - P}{P} \qquad i = \frac{1000 - 900}{900} = 11.1\%$$

yield to maturity is the increase in price as a percentage of the initial price

The POINT to REMEMBER Bond prices and interest rates are inversely related. when interest rates rise, bond prices fall & vice versa.

$$\text{interest rate} \equiv \text{yield to maturity}$$

↑ most accurate measure of interest rates

$$\boxed{\text{current yield} = \frac{C}{P}}$$

coupon rate
same as yield to maturity on consol

Note as maturity horizon shortens the current yield the approximation betw/n current ~~yield~~ yield and the coupon rate becomes worse

$$\boxed{\text{yield on a discount basis} = \frac{FV - P}{FV} \cdot \frac{360}{\text{days to maturity}}}$$

PROBLEMS: 1. "year" as 360 days

2. use of percentage gain on face value (as opposed to purchase price)

as maturity lengthens the difference betw/n FV and P becomes larger (worsening the problem)

Rate of Return vs. Interest Rate

rate of return ≡ payments to owner plus change in purchase price as a percentage of the purchase price

because we include change in purchase price, the rate of return is not necessarily equal to the interest rate

$$\text{rate of return} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$= \text{current yield} + \text{rate of capital gain}$$

↳ %age change in purchase price

$$\text{rate of return} = i_c + g$$

so if interest rates rise, then the price falls $g < 0$ and (if you own the bond) you're screwed

For a given change in interest rate, longer bonds experience a wider percentage change in purchase price

Consequently price & return on long-term bonds are more volatile than those for short-term bonds → Interest Rate Risk

Real vs. Nominal Interest Rates

→ changes in the price level affect the true cost of borrowing

→ if your income keeps pace with the changes in the overall price level, then for a fixed given nominal interest rate borrowing becomes less expensive as the price level rises

$$r = i - \pi^e$$

real int rate = nominal interest rate − expected inflation rate

→ when real interest rate low there is more incentive to borrow + less to lend

→ similar distinction between real + nominal returns

→ expected inflation – how do you measure expectations of inflation? TIPS

Treasury Inflation Protection Securities

~~[illegible]~~

→ interest rate on an indexed (i.e. TIPS) bond provides a direct measure of the real interest rate, so subtracting its interest rate from that on non indexed bonds yields the expected rate of inflation

→ ~~[illegible]~~ According to Fed's H.15 for week ending 11 Feb 2011:

Nominal 10 yr – TIPS 10 yr = exp inflation

3,68% – 1,36% = 2,32%

BEHAVIOR OF INTEREST RATES

determinants of asset demand:

1. wealth
2. expected return
3. risk – the uncertainty associated w/ return
4. liquidity – how easily the asset can be converted into cash (relative to alternative assets)

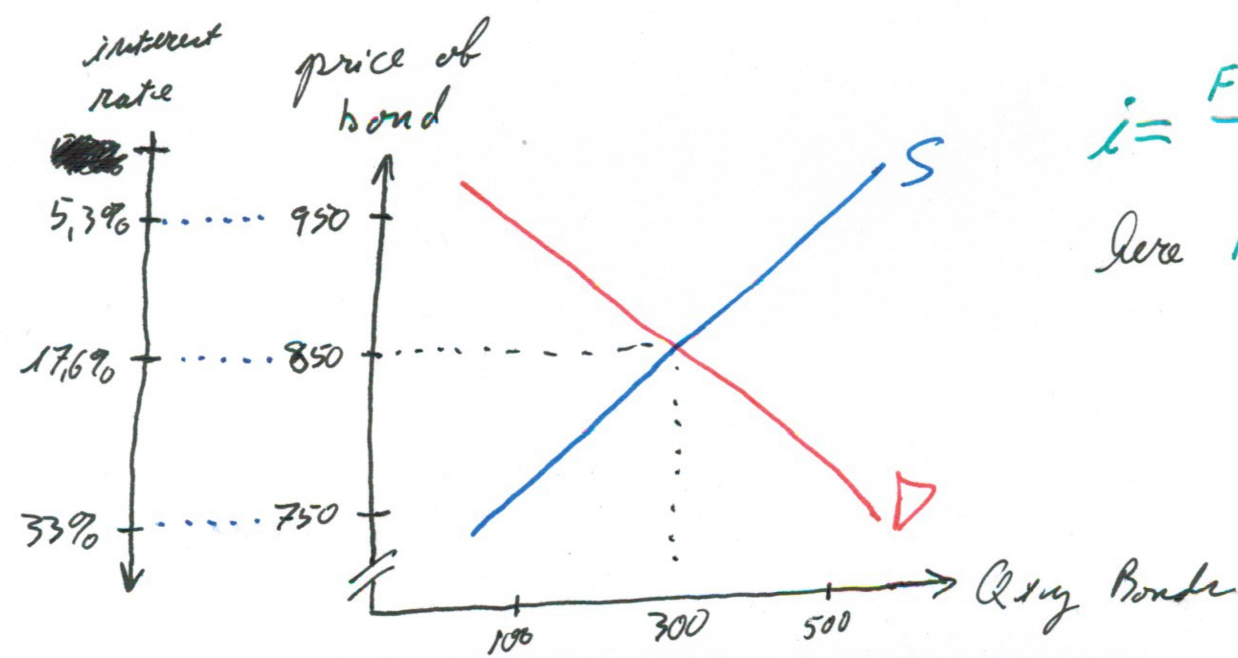
CETERIS PARIBUS:

→ As wealth rises, qty of asset demanded rises

→ As expected return rises, qty of asset demanded rises

→ As risk rises, qty of asset demanded falls

→ As liquidity rises, qty of asset demanded rises



$$i = \frac{FV - P}{P}$$

here $FV = 1000$

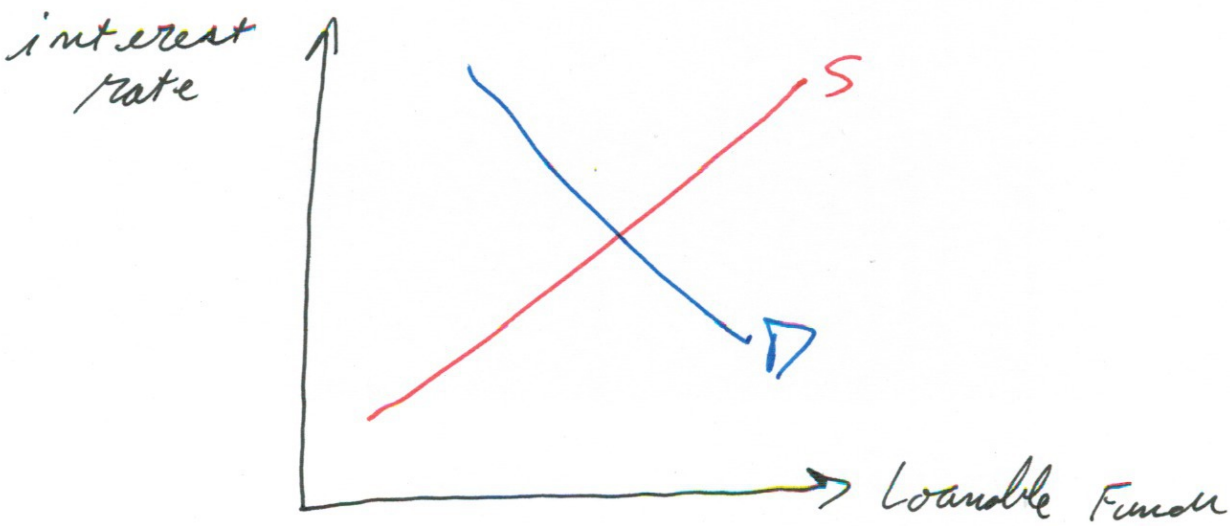
Asset Supply

→ all else equal, as the interest rate falls (i.e. higher price), the cost of borrowing falls, so firms issue bonds

ALTERNATIVE PRESENTATION

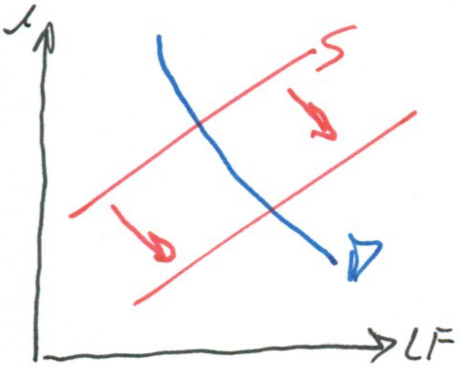
to get interest rate running in the conventional direction on the vertical axis ...

asset demand → supply of loanable funds
asset supply → demand for loanable funds



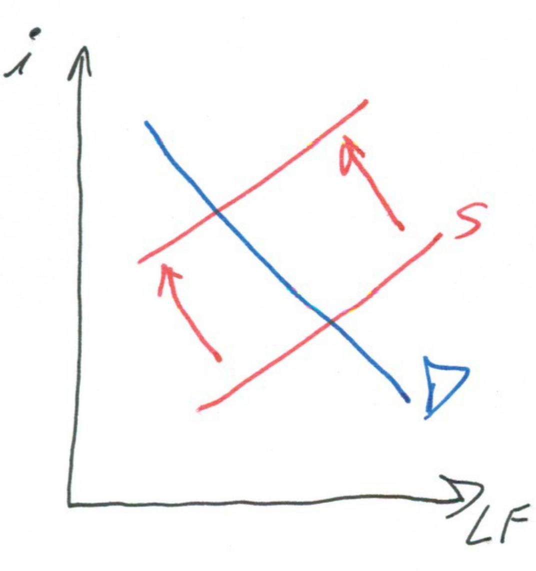
Shifts in { Asset Demand / Supply Loanable Funds

wealth — as wealth rises, qty of asset demanded rises ∴ qty of loanable funds supplied increases



during business cycle expansion there's an outward shift in supply of loanable funds

expected return — an increase in interest rates on ~~[illegible]~~ bond means a sharp drop in price, so if people expect higher interest rates, then they'll want to sell asset before others do (to beat the fall in price)



Note however that this is ~~[illegible]~~ almost a self-fulfilling prophecy

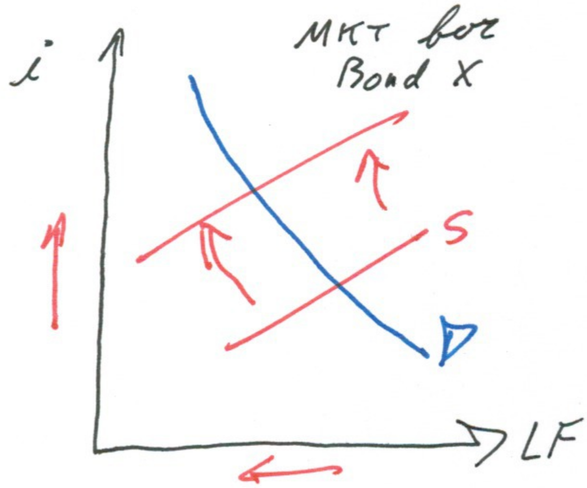
Also note that real interest rate falls when higher expected inflation, so lower expected real return would cause supply of loanable funds to shrink.

risk relative to other assets

→ an increase in riskiness of Bond X

→ a decrease in riskiness of Bond Y

both cause { demand for Bond X / supply LF to Bond X } to shrink



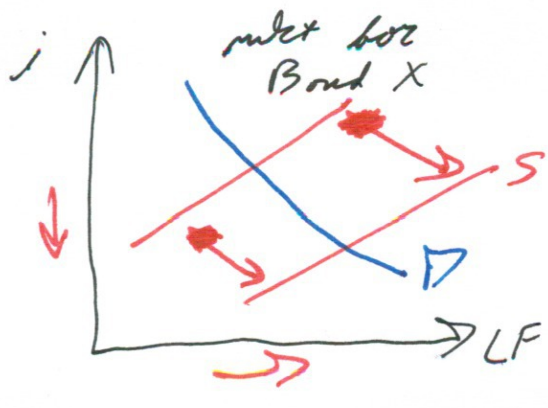
note that these factors would cause equilibrium interest rate (on Bond X) to rise

liquidity relative to other assets

→ an increase in liquidity of Bond X

→ a decrease in liquidity of Bond Y

both cause { demand for Bond X / supply LF to Bond X } to rise



equilibrium interest rate falls because now Bond X is more attractive

Shifts in { Asset Supply / Demand for Loanable Funds

→ expected profitability of investment ~~of~~ opportunities

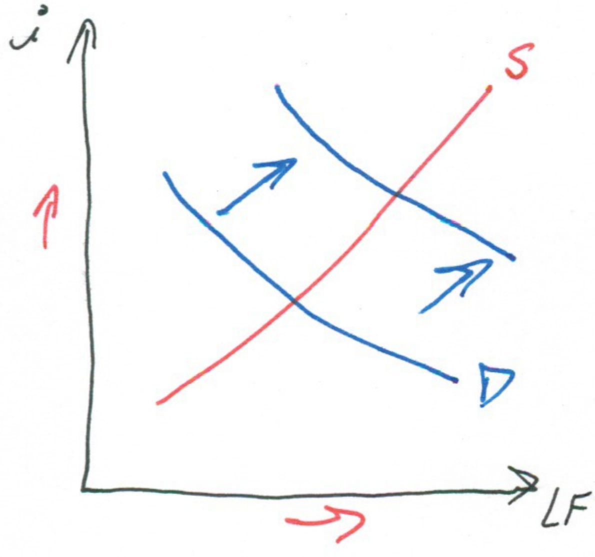
→ expected inflation

→ gov't ~~[illegible]~~ budget deficit

all else equal an increase in any one of these factors will cause:

→ increase in interest rate on Bond X

because demand for loanable funds rises



- profitability increase raises return on investment relative to cost of borrowing
- increase exp inflation reduces real cost of borrowing
- higher gov't deficits imply direct increase in demand for LF

Supply + Demand for Money
Liquidity Preference

We just examined equilibrium interest rate using supply + demand for bonds but alternative framework looks at equilibrium interest rate using supply + demand for money — theory of liquidity preference

ASSUME: people store wealth in money or bonds

total wealth $= B^S + M^S = B^D + M^D$

$$\text{bond supply} + \text{money supply} = \text{bond demand} + \text{money demand}$$

$$B^S - B^D = M^D - M^S$$

so if $B^S > B^D$ then $M^D > M^S$

if there's an excess supply of bonds then there's excess demand for money

An excess supply of bonds will cause bond prices to fall + interest rates to rise

Similarly, an excess demand for money will push up int rate advantage of

- loanable funds framework – easier to examine effect of change in expected inflation
- liquidity preference framework – easier to analyze the effect of change in income, price level or supply of money

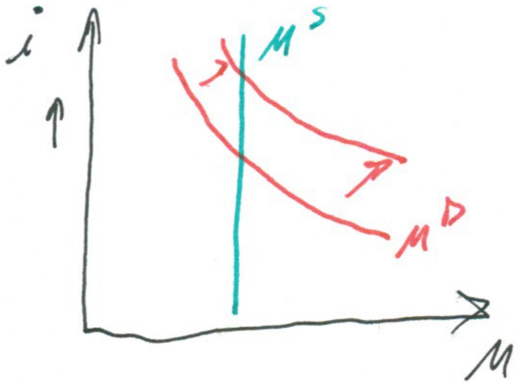
ASSUME: money earns no interest
bonds are the only interest-bearing asset

interest rate is the opp cost of holding money

shifts in demand for money
→ changes in income
→ changes in price level
shifts in supply of money
is a decision by central bank
Federal Reserve

affect equilibrium interest rate

income effect — as income rises people will want to purchase more goods & services, for which they will need to hold more money



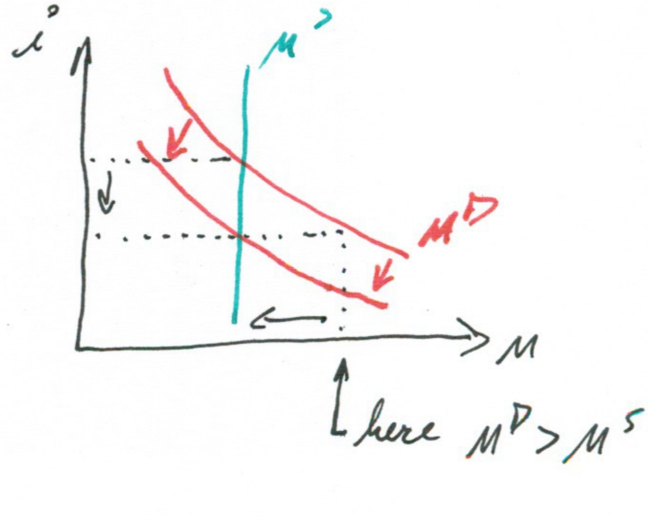
Note: Difference between INCOME & WEALTH

Income is a FLOW ∴ Increase Income → Increase i
Wealth is a STOCK ∴ Increase Wealth → Decrease i

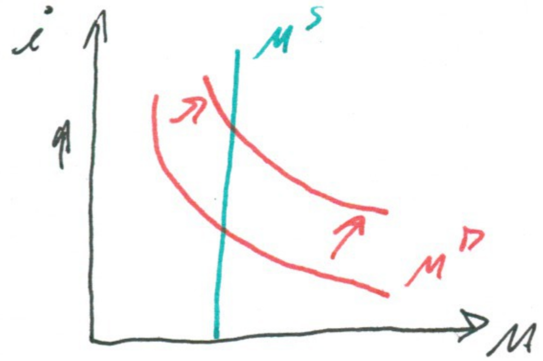
Increase in wealth increases the supply of loanable funds which pushes DOWN the equilibrium interest rate

$$B^S > B^D \Rightarrow M^D > M^S$$

so an increase in wealth would cause demand for money to contract



price level effect



you don't want to hold money for its own sake, you want to hold it for what it can buy

real money balance $\equiv \frac{M}{P}$

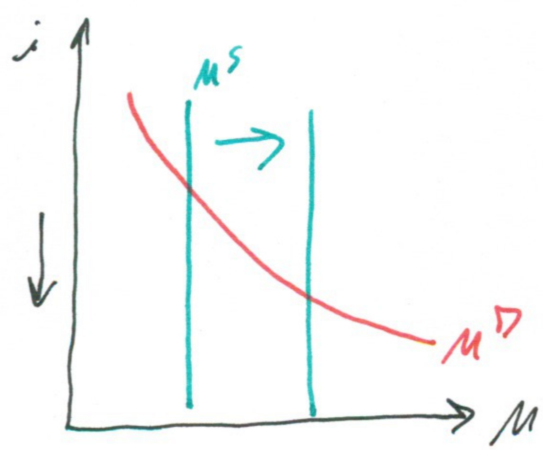
example: you want to hold enough money to buy two slices of pizza

$$\frac{M}{P} = \frac{\$4}{\$2/\text{slice}} = \frac{\$8}{\$4/\text{slice}} = 2 \text{ slices}$$

hold \$4 when price is \$2/slice

hold \$8 when price is \$4/slice

change in money supply



central bank (e.g. Fed Res) increase money supply pushes down the equilibrium int rate