Lecture 5

Economic Growth: the Solow Model

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Economic Growth and Economic Fluctuations

Why Study Economic Growth?

Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.

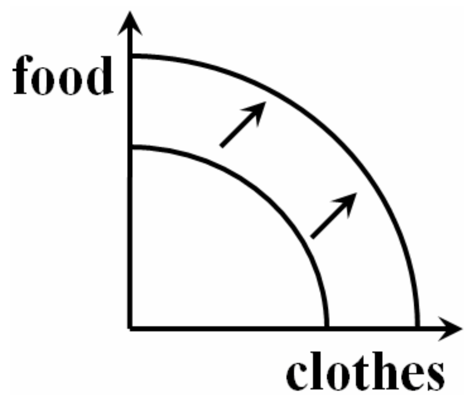
– Robert E. Lucas, Jr.

What is Economic Growth?

- **Before the Industrial Revolution in Great Britain, every society in the world was agrarian.**
- **Then technical change and capital accumulation increased British society's ability to produce textiles and agricultural products**
- **and a rapid and sustained increase in real output per capita began**
- **As a result, more could be produced with fewer resources**
  - new products
  - more output and
  - wider choice

- **Economic growth shifts the society's production possibility frontier up and to the right**

- **Economic growth allows each member of society to produce and consume more of all goods**



Plan of this Lecture

- **In the previous lecture, we learned some basic measures of how national income is distributed among factors of production and how national income is allocated among the goods produced.**
- **In this lecture, we'll examine the model of economic growth developed by Robert M. Solow in the 1950s.**

- **The Solow Model was one of the first attempts to describe how:**
  - **saving,**
  - **population growth and**
  - **technological progress**
- **affect the growth of output per worker over time – i.e. we're looking at LONG RUN economic growth.**

- **We'll use Solow's model to examine:**
  - **why the standards of living vary so widely among countries and**
  - **how economic policy can be used to influence standards of living**
  - **How much of an economy's output should be consumed today and how much should be saved for the future?**

Demand-Side Assumptions

- **To simplify the discussion, we'll examine a closed economy without a government. In other words:**
  - **there is no international trade** (so net exports equal zero) **and**
  - **government purchases equal zero**
- **so output is divided among consumption and investment:**

$$Y = C + I$$

- **Ultimately, we want to examine living standards, so we want to focus on per capita output, consumption and investment.**
- **It's easier to examine per worker variables in this model however.**

- **Nonetheless, per worker variables will yield fairly good approximations of living standards, so if we denote the labor force by L, we can define:**
  - **output per worker** as: $y \equiv Y/L$
  - **consumption per worker** as: $c \equiv C/L$ and
  - **investment per worker** as: $i \equiv I/L$

$$\boxed{\textbf{so:}\quad y = c + i}$$

Demand-Side Assumptions

- **Consumption per worker is the amount of output that is not invested**

$$c = y - i$$

- **The Solow Model assumes that consumption and investment** (per worker) **are proportional to income:**

$$c = (1 - s) \cdot y \qquad \textbf{where: } i = s \cdot y$$

- **So that the saving rate – denoted by the letter s – is constant.**

- **In other words, every year a fraction $(1 - s)$ of income is consumed and a fraction s of income is saved.**

Supply-Side Assumptions

- **We'll also assume that:**
  - **output is produced using capital,** K**, and labor,** L
  - **there is no technological progress** (we'll drop this assumption later)
  - **the production function exhibits constant returns to scale (CRS)**

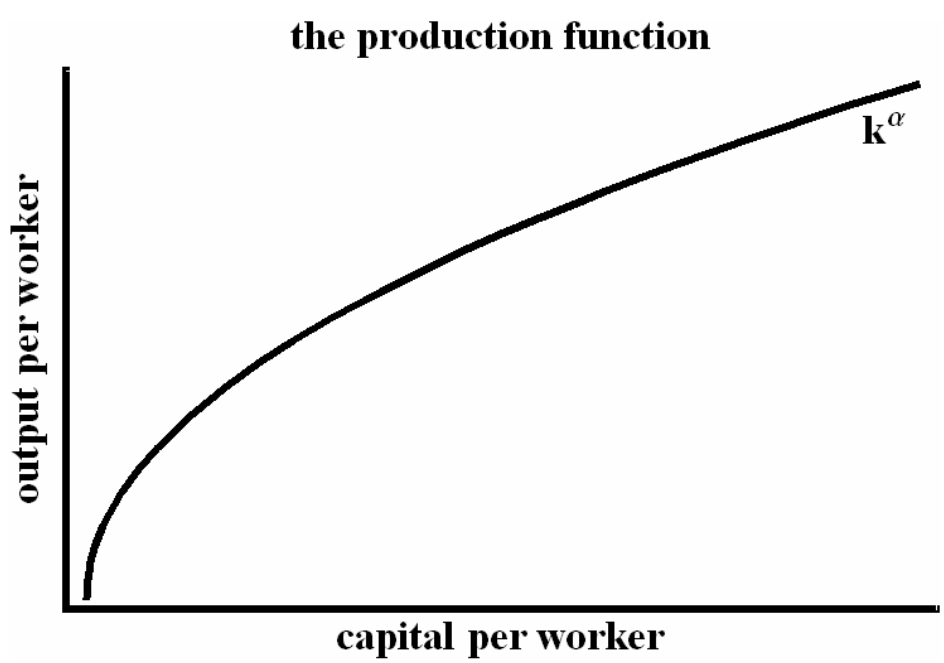
$$Y = K^{\alpha} \cdot L^{1-\alpha} \qquad \text{where: } 0 < \alpha < 1$$

- **Once again, we want to focus on per worker variables, so define:**
  - **output per worker** as: $y \equiv Y/L$ and
  - **capital per worker** as: $k \equiv K/L$

- **One convenient feature of the assumption of CRS is that we can define output per worker entirely in terms of capital per worker:**

$$\left.\begin{aligned} \frac{Y}{L} &= K^{\alpha} \cdot \frac{L^{1-\alpha}}{L} \\ &= K^{\alpha} \cdot L^{-\alpha} \end{aligned}\right\} \Rightarrow \quad y = k^{\alpha}$$

Production per Worker



Supply-Side Assumptions

- **Another convenient feature of the assumption of CRS concerns the Marginal Product of Capital (MPK)** – the derivative of output per worker with respect to capital**:**

$$MPK \equiv \frac{dY}{dK}$$

- **the Marginal Product of Capital equals the derivative of output per worker with respect to capital per worker:**

$$\frac{dY}{dK} = \alpha \cdot K^{\alpha-1} \cdot L^{1-\alpha} \qquad y = k^{\alpha}$$

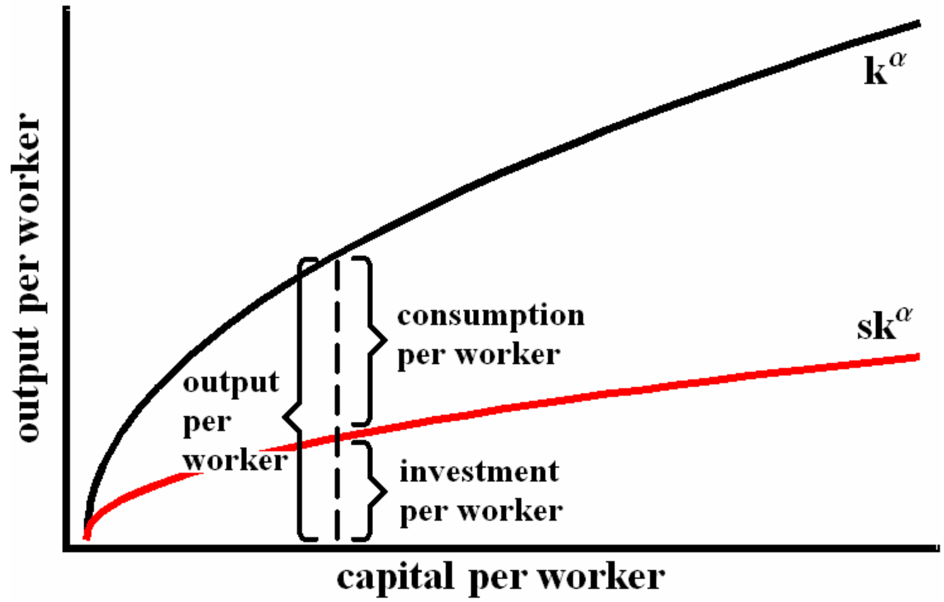
$$\frac{dY}{dK} = \alpha \cdot \frac{K^{\alpha-1}}{L^{\alpha-1}} = \alpha \cdot k^{\alpha-1} \qquad \frac{dy}{dk} = \alpha \cdot k^{\alpha-1}$$

- **What this tells us is that increases in the capital stock per worker increase output per worker, but each successive increase in the capital stock yield ever smaller increases in output per worker – because output per worker exhibits diminishing marginal returns.**

Accumulation of Capital

- **The underlying theory behind the Solow Model:**
  - **countries with higher levels of capital per worker**
  - **have higher levels of output per worker.**
- **Think about that a second.**
- **If Solow's theory is correct, then all we have to do to increase output per worker – and lift billions of people out of poverty – is increase the amount of capital that they have to work with.**
- **So what determines the level of capital per worker in a country?**
- **The Solow Model assumes that:**
  - **investment increases the capital stock, but**
  - **a constant fraction of the capital stock depreciates each year**
- **Our definition of investment,** $i = s \cdot y$ **implies that annual investment in capital is a fraction,** s**, of the total output per year, i.e.** $I = s \cdot Y$
- **Let** δ **denote the fraction of the capital stock that depreciates in a year.**
- **Therefore:** $\dot{K} = sY - \delta K$ **where:** $\dot{K} = \frac{dK}{dt}$

Tradeoff between Consumption and Investment



- **The saving rate** s **determines the allocation of output per worker between consumption and investment.**

Evolution of Capital per Worker

- **Now that we now how the total capital stock evolves from year to year, finding out how the capital stock per worker evolves from one year to the next is straightforward.**
- Recall from the Calculus Tricks that the percentage change in a ratio is equal to the percentage change in the numerator minus the percentage change in the denominator.
- **So we can find the evolution capital per worker over time:**

$$\frac{\dot{k}}{k} = \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{k} = \frac{K}{L}\cdot\left(\frac{sY - \delta K}{K} - \frac{\dot{L}}{L}\right)$$

$$= \frac{sY - \delta K}{L} - k\cdot\frac{\dot{L}}{L}$$

$$\text{define: } n \equiv \frac{\dot{L}}{L} \qquad \mathbf{\dot{k} = sk^{\alpha} - (\delta + n)\cdot k}$$

- Note that: n is the constant **exogenous** annual growth rate of the labor force
- the model assumes that there are no cyclical fluctuations in employment

Key Equation of the Solow Model

$$\dot{k} = sk^{\alpha} - (\delta + n) \cdot k$$

- **Growth of the capital stock per worker over time,** $\dot{k}$
  - **is an increasing function of investment, i.e.** sk^{α}
  - **a decreasing function of the depreciation rate** and
  - **a decreasing function of the growth rate of the labor force**

- **Although we've used math to obtain this result, the result should also be intuitive:**
  - **The capital stock per worker increases at higher saving rates because at higher saving rates more output is being devoted to accumulating capital.**
  - **By definition, depreciation decreases the capital stock, so faster rates of depreciation reduce the capital stock per worker.**
  - **Faster rates of growth of the labor force will also lead to lower levels of capital per worker, because the total capital stock must be spread over a larger labor force.**

Evolution of Capital per Worker

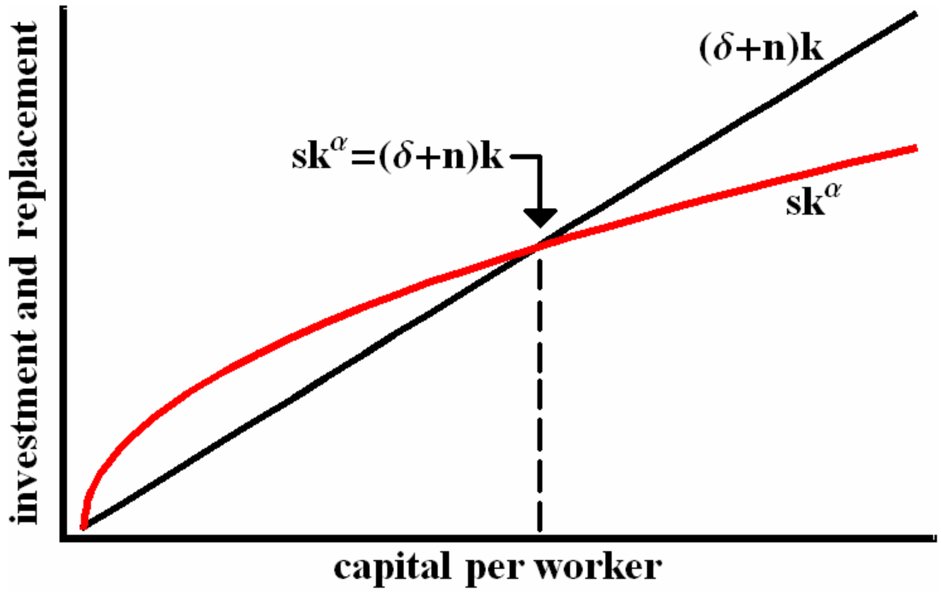
Whether capital per worker is growing, falling or remaining constant over time, depends on whether investment in new capital per worker exceeds, falls short of or is equal to the replacement requirement: $(n + \delta) \cdot k$**.**

- **if** $sk^{\alpha} > (n + \delta) \cdot k$**, then capital per worker increases over time**
  - in this case, **investment in new capital per worker exceeds the replacement requirement** and
  - **output per worker is growing over time**

- **if** $sk^{\alpha} < (n + \delta) \cdot k$**, then capital per worker decreases over time**
  - in this case, **investment in new capital per worker falls short of the replacement requirement** and
  - **output per worker is falling over time**

- **if** $sk^{\alpha} = (n + \delta) \cdot k$**, then capital per worker remains constant over time**
  - in this case, **investment in new capital per worker equals the replacement requirement** and
  - **output per worker is constant over time**
  - **this is called the steady state** (since the level of capital per worker is "steady")

Steady State



- **The capital per worker must converge to the steady state**
- **Once capital per worker converges to the steady state level it remains at that level,** unless the saving rate, depreciation rate or growth rate of the labor force changes.

Convergence to the Steady State

- **As an example of convergence consider an economy that initially starts at a level of capital per worker that is below the steady state level.**
- **If initial** $k = 1$ **and if** $\alpha = 0.5 \quad y = k^{0.5} \quad s = 0.08 \quad \delta = 0.02 \quad$ and $\quad n = 0.02$

year	k	y	$c = (1-s)\cdot y$	$i = s \cdot k^{\alpha}$	$(\delta + n)\cdot k$	Δk
0	1.000	1.000	0.920	0.080	0.040	0.0400
1	1.040	1.020	0.938	0.082	0.042	0.0400
2	1.080	1.039	0.956	0.083	0.043	0.0399
3	1.120	1.058	0.974	0.085	0.045	0.0399
4	1.160	1.077	0.991	0.086	0.046	0.0398
5	1.200	1.095	1.008	0.088	0.048	0.0396
…						
10	1.396	1.182	1.087	0.095	0.056	0.0387
…						
25	1.945	1.394	1.283	0.112	0.078	0.0338
…						
100	3.484	1.867	1.717	0.149	0.139	0.0100
…						
∞	4.000	2.000	1.840	0.160	0.160	0.0000

- After 35 years, this economy's level of output per worker will have converged halfway to its steady state level – i.e. $y = 1.5$ at 35 years.

Steady State

- **Once the economy has converged to its steady state, the level of capital per worker stops growing** (or falling as the case may be)**, i.e.** $\dot{k} = 0$
- **At the steady state:** $sk^{\alpha} = (\delta + n) \cdot k$**. If we solve this equation for** k**, we find the steady state level of capital per worker:**

$$k_{SS} = \left(\frac{s}{n + \delta} \right)^{\frac{1}{1-\alpha}}$$

- **which implies that the steady state level of output per worker is:**

$$y_{SS} = \left(\frac{s}{n + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- **In the long-run, the steady-state level of output per worker is constant and depends only on:**
  - **the saving rate**
  - **the labor force growth rate and**
  - **the rate at which capital depreciates**

Economic Growth

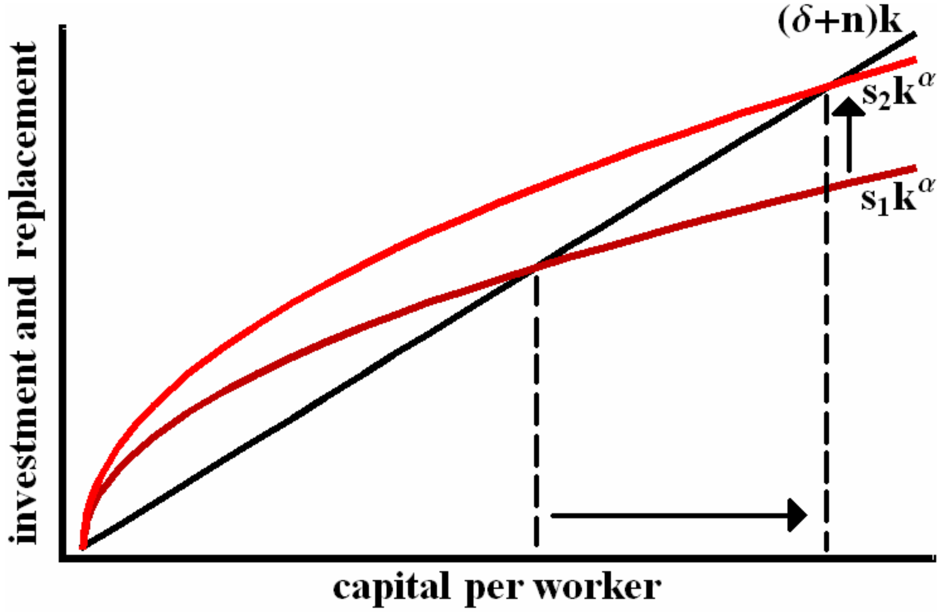
- **The interesting thing about this model of economic growth is that there is no growth once the economy reaches steady state.**
- **But what do politicians say?**
  - Politicians say tax cuts will be great for economic growth
  - Politicians say protecting open space will be great for economic growth
  - Politicians say building a new stadium will be great for economic growth
- **Are they lying?**

- **Public policy cannot affect the steady state growth rate.**
- **But public policy can affect the steady state level of output per worker, which will affect living standards.**

- **If policymakers found a way to increase the saving rate the economy will converge to a higher steady state level of output per worker.**

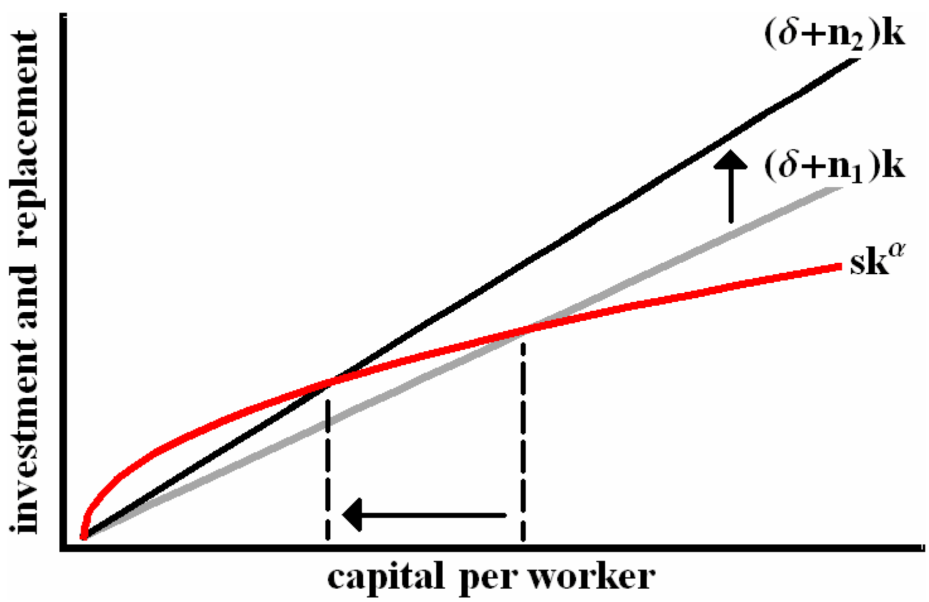
- **Conversely, if policymakers pursued a policy that increased the labor force growth rate, then the economy would converge to a lower steady state level of output per worker.**

Increasing the Saving Rate



- **Many economists favor low corporate tax rates as a way to encourage saving, in the hope that lower rates will stimulate savings/investment.**
- **At a higher saving rate, the economy will converge to a higher steady state level of output per worker.**

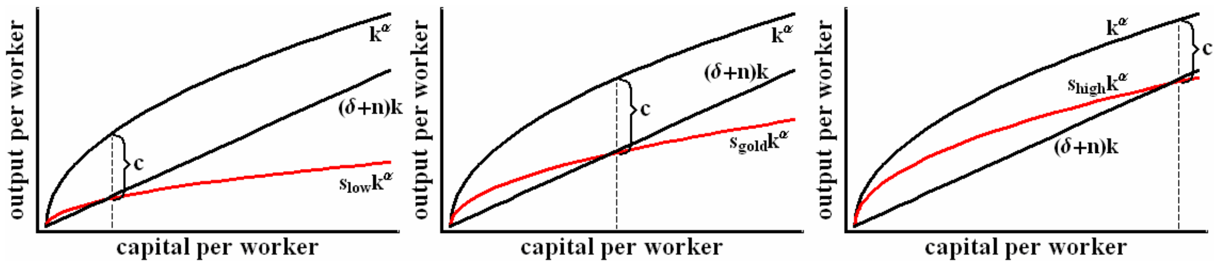
Increasing the Labor Force Growth Rate



- **One reason living standards are low in some countries is because they have high rates of population growth** (high rates of labor force growth)**.**
- **At a higher labor force growth rate, the economy will converge to a lower steady state level of output per worker.**

the Golden Rule

- **So far, it would appear that the goal of public policy should be to reach a higher steady state level of output per worker**
- **In practice, that should be the goal – our saving rate is too low – but …**
- **If a benevolent policymaker could choose the saving rate – which would enable him/her to choose the steady state level of output per worker, then which steady steady state should he/she choose?**
  - Extreme example #1: You wouldn't want a saving rate of 1%
  - Extreme example #2: You wouldn't want a saving rate of 99%



- **If the policymaker followed the Golden Rule of "Do unto others …" then he/she would want to choose the steady state with the highest level of consumption per worker.** This case is depicted in the middle panel.

the Golden Rule

- Using our Calculus Tricks, we can find the saving rate which maximizes consumption across steady states in the same way that we found a firm's maximum profit:
  - take the derivative of consumption with respect to the saving rate
  - and set it equal to zero
- This time it's easier because we can use the chain rule:

$$\underset{S}{\text{Max}} \quad c_{SS}(s) = k_{SS}(s)^{\alpha} - (n+\delta)\cdot k_{SS}(s)$$

$$c'_{SS}(s) = \left(\alpha \cdot k_{SS}^{\alpha-1} - (n+\delta)\right)\cdot k'_{SS}(s) = 0 \quad \Rightarrow \quad \alpha \cdot k_{GOLD}^{\alpha-1} = (n+\delta)$$

- In other words, the Golden Rule steady state level of capital per worker corresponds to level of capital per worker which equates the Marginal Product of Capital to $(n+\delta)$.
- **The Golden Rule level is a CHOICE.**
- **The economy does NOT converge to the Golden Rule level on its own!**

Technological Progress

- **You may have noticed that if the steady state level of output per worker is constant, then:**
  - **as the economy approaches steady state**
  - **growth of output per worker is zero and therefore**
  - **growth of income per worker is zero**

- **Is this realistic? No.**

- **We can introduce more realism into the model if we introduce technological progress into the model.**
- **If we define a variable A to denote the efficiency of labor**
  - **which reflects society's knowledge about production methods or**
  - **which reflects improvements in the health, education of skills of the labor force**
- **then as the available technology improves, the efficiency of labor rises.**

Technological Progress

- **So redefine the production function as:**

$$Y = K^{\alpha} \cdot (AL)^{1-\alpha} \qquad \text{where: } 0 < \alpha < 1$$

- **Once again, we want to focus on per worker variables, but now we have to focus on labor in efficiency units, so define:**
  - **output per unit of effective labor** as: $\tilde{y} \equiv Y/AL$ and
  - **capital per unit of effective labor** as: $\tilde{k} \equiv K/AL$

- **Since the production function still assumes constant returns to scale, so we can define output per unit of effective labor in terms of capital per unit of effective labor:**

$$\left.\begin{aligned} \frac{Y}{AL} &= K^{\alpha} \cdot \frac{(AL)^{1-\alpha}}{AL} \\ &= K^{\alpha} \cdot (AL)^{-\alpha} \end{aligned}\right\} \Rightarrow \quad \tilde{y} = \tilde{k}^{\alpha}$$

Evolution of Capital per unit of Effective Labor

- **Using the Calculus Tricks once again, we can find the evolution capital per unit of effective labor over time:**

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \left(\frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{\tilde{k}} = \frac{K}{AL}\cdot\left(\frac{sY-\delta K}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right)$$

$$\text{define}: \ g \equiv \frac{\dot{A}}{A} \qquad\qquad = \frac{sY-\delta K}{AL} - \tilde{k}\cdot\left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right)$$

$$\text{define}: \ n \equiv \frac{\dot{L}}{L} \qquad\qquad \dot{\tilde{\mathbf{k}}} = \mathbf{s}\tilde{\mathbf{k}}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{g} + \mathbf{n})\cdot\tilde{\mathbf{k}}$$

- Note that: g is the **exogenous** annual growth rate of technological progress
  n is the **exogenous** annual growth rate of the labor force
- We're assuming that technology grows at a constant rate and that there are no cyclical fluctuations in the level of technology.
- We're still assuming that the labor force grows at a constant rate and that there are no cyclical fluctuations in employment.

Key Equation of the Solow Model with Technological Progress

$$\dot{\tilde{\mathbf{k}}} = \mathbf{s}\tilde{\mathbf{k}}^{\boldsymbol{\alpha}} - (\boldsymbol{\delta} + \mathbf{g} + \mathbf{n})\cdot\tilde{\mathbf{k}}$$

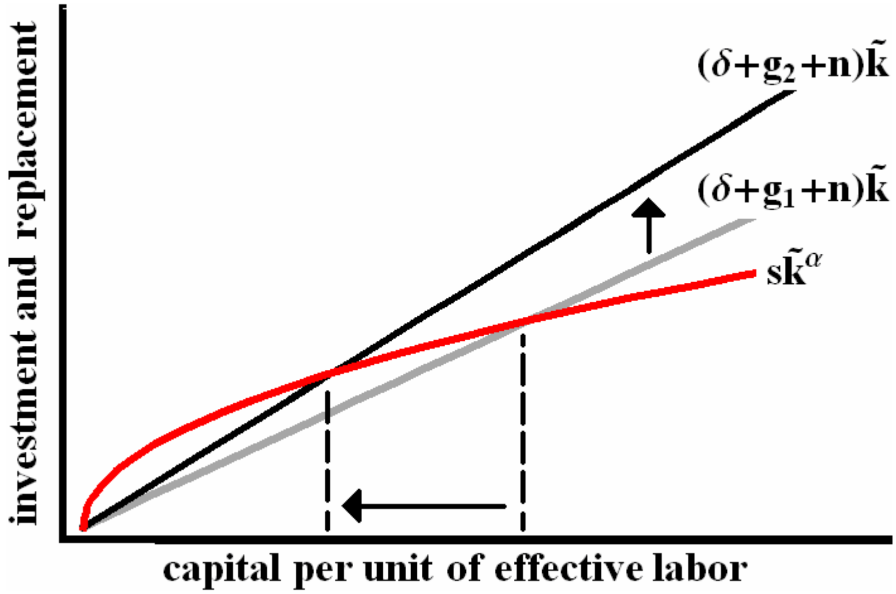
- **This "Key Equation" has the same interpretation as the previous one but this time:**
  - **growth of the capital stock per unit of effective labor over time,** $\dot{\tilde{k}}$**,**
  - **is a decreasing function of the growth rate of technological progress,** g**.**
- **And this time the steady state level of the capital stock per unit of effective labor, will occur when:**

$$\dot{\tilde{k}} = 0 \Rightarrow s\tilde{k}^{\alpha} = (\delta + g + n)\cdot\tilde{k}$$

- **Solve this equation for** $\tilde{k}$**, we can find obtain the steady state levels of capital and output per unit of effective labor:**

$$\tilde{k}_{SS} = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \quad \textbf{and} \quad \tilde{y}_{SS} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Increasing the Rate of Technological Progress



- If policymakers were able to find a way to increase the rate of technological progress, the steady state level of capital per unit of effective labor would fall, but …
- … this would be a **GOOD THING**

the Rate of Technological Progress

- A faster rate of technological progress would lower the steady state level of capital per unit of effective labor, but …
- … this would be a **GOOD THING**
- The rate of growth of output per unit of effective labor is:

$$\frac{\dot{\tilde{y}}}{\tilde{y}} = \frac{\dot{Y}}{Y} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} = \frac{\dot{Y}}{Y} - g - n$$

- The rate of growth of output per worker is:

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{Y}}{Y} - n$$

- In steady state, the growth rate of output per unit of effective labor is zero, but this implies that the rate of growth of output per worker is equal to the rate of growth of technological progress.

$$\frac{\dot{\tilde{y}}}{\tilde{y}} = 0 \Rightarrow \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} \quad \text{or more simply:} \quad \frac{\dot{y}}{y} = g$$

- So a faster rate of growth of technological progress implies a rapidly rising standard of living for the residents of that economy

Homework #5

1. In its introduction to the Solow Model without technological progress, Lecture 5 contains a derivation of the marginal product of capital.

a. If the production function is given by: $Y = K^{\alpha} L^{1-\alpha}$, what is the marginal product of <u>labor</u>?

b. Assuming that $\alpha = 0.5$ and that $K = 1$, calculate the marginal product of labor from one unit of labor input to five units. Hint: Use a calculator!

c. On a graph, plot the marginal product of labor using the values you just calculated.

d. Assuming that $\alpha = 0.5$ and that $K = 2$, calculate the marginal product of labor from one unit of labor input to five units.

e. On the <u>same</u> graph, plot the marginal product of labor using the values you just calculated.

f. What happens to the marginal product of labor when the economy's stock of capital increases?

2. In Lecture 3, you learned that a firm hires labor up to the point where the wage equals the price times the marginal product of labor (MPL), i.e. $w = p \cdot MPL$, where labor is supplied at wage rate, w, and the labor demand is given by $p \cdot MPL$. Since we're now discussing economy-wide aggregates, it's convenient to normalize the price level to $p = 1$.

a. If we assume that the wage rate, w, is constant at a given point in time, then how will the quantity of labor that the economy demands respond to a sudden increase in the capital stock?

b. If we assume that the quantity of labor supplied, L, is constant at a given point in time, then how will the wage rate respond to a sudden increase in the capital stock?

c. Which assumption does the Solow Model make?

3. In 2003, Pres. George W. Bush convinced Congress to reduce the maximum tax rate that shareholders pay on dividends from 38.6 percent to 15 percent. In lobbying for this measure, he argued that cutting the tax would encourage people to invest more – i.e. increase the economy's saving rate.

Opponents of the policy argued that cutting the tax on dividends was a giveaway to Pres. Bush's rich friends and that it would not benefit workers.

Answer the following questions using the Solow Model without technological progress. Throughout the problem, assume that the U.S. economy was in steady state when Pres. Bush announced his dividend tax plan. Until part **e.**, assume that Pres. Bush's tax policy would increase the saving rate.

a. Under what condition would Pres. Bush's tax policy increase consumption per worker? Under what condition would it decrease consumption per worker?

b. How would the marginal product of labor differ between the initial steady state and the one to which the economy will converge to after reduction of the tax on dividends?

c. How would Pres. Bush's tax policy affect wage income, $w \cdot L$?

d. Given your answers to the previous three questions, was Pres. Bush's tax policy a giveaway to the rich without any benefit for workers?

e. Now assume that Pres. Bush's tax policy would <u>not</u> increase the saving rate. Under this assumption, was the tax policy giveaway to the rich without any benefit for workers?

4. Assume that $\alpha = 0.3$, that output grows at 3.0 percent annually, that the annual depreciation rate is 4.0 percent and that the capital-output ratio is 2.5, i.e. $K/Y = 2.5$. Finally, assume that the economy is in steady state and that the economy is described by the Solow Model with technological progress, i.e. $Y = K^{\alpha}(AL)^{1-\alpha}$. Recall that in this model, $\tilde{y} = \tilde{k}^{\alpha}$.

a. What must be the saving rate in the initial steady state? Hint: Use the steady state relationship: $s\tilde{k}^{\alpha} = (\delta + g + n) \cdot \tilde{k}$

b. What is the marginal product of capital in the initial steady state?

c. Suppose that public policy raises the saving rate so that the economy reaches the Golden Rule level of capital per unit of effective labor. What will the marginal product of capital be at the Golden Rule steady state? Compare the marginal product of capital at the Golden Rule steady state to the marginal product of capital in the initial steady state. Explain.

d. What will the capital-output ratio be at the Golden Rule steady state? Hint: There's a very simple relationship between the marginal product of capital and the capital-output ratio.

e. What must the saving rate be to reach the Golden Rule steady state?

Lecture 6

Economic Growth: Human Capital

Eric Doviak

Economic Growth and Economic Fluctuations

Why Study Another Growth Model?

- **Because the Solow Model doesn't work.**
- **Recall from Lecture 3 that in the Long-Run:**
  - **the firm hires labor up to the point where the wage equals the price times the marginal product of labor (MPL):** $w = p \cdot MPL$
  - **the firm hires capital up to the point where the rental rate on capital equals the price times the marginal product of capital (MPK):** $r = p \cdot MPK$
- **Next recall from Lecture 5, the production function that incorporates technological progress:**

$$Y = K^{\alpha} \cdot (AL)^{1-\alpha}$$

- **The MPL of this production function is:**

$$MPL = (1-\alpha) \cdot K^{\alpha} \cdot A^{1-\alpha} \cdot L^{-\alpha}$$

- **The MPK of this production function is:**

$$MPK = \alpha \cdot K^{\alpha-1} \cdot (AL)^{1-\alpha}$$

- **Finally, recall from Lecture 4, that if economic profit is zero, then:**

$$p \cdot Y = r \cdot K + w \cdot L$$

Why Study Another Growth Model?

- **Now bring all three of those conditions together:**

$$p \cdot Y = r \cdot K + w \cdot L$$
$$p \cdot Y = p \cdot MPK \cdot K + p \cdot MPL \cdot L$$
$$Y = \alpha \cdot K^{\alpha-1} \cdot (AL)^{1-\alpha} \cdot K + (1-\alpha) \cdot K^{\alpha} \cdot A^{1-\alpha} \cdot L^{-\alpha} \cdot L$$
$$Y = \alpha \cdot Y + (1-\alpha) \cdot Y$$

- **This implies that:**
  - **Capital's share of national income is equal to** α **and**
  - **Labor's share of national income is equal to** $(1-\alpha)$
- **In practice, we know that capital's share of national income is roughly constant across countries and approximately one-third, i.e.** $\alpha \approx 1/3$
- **Next, consider two countries. According to the Penn World Tables, in 2000, real GDP per worker was $64,437 in the U.S. and $1479 in Nigeria.**
- **Now if differences in capital per worker explain differences in output per worker and if** $\alpha \approx 1/3$**, then capital per worker is about 83,077 times higher in the U.S. than it is in Nigeria.**
- **Does that look right to you? … I didn't think so.**

What's wrong with the Solow Model?

- **The Solow Model that we examined in the previous lecture assumes that output is produced using:**
  - **physical capital** (i.e. machinery, buildings, etc.)
  - **human labor**
- In an extension of that model, we also incorporated technological progress
- **If we were to use the Solow Model to examine levels of output across countries, then we would have to assume that there are no difference in human labor across countries** (i.e. that human labor is homogenous)
- **Is that assumption realistic? No.**
- **If you ever work with a poorly educated person, you'll notice that they're much less productive than you are.**
- **When a complication arises in the task that they are performing, they don't know what to do and often make bad decisions.**
- **Models of economic growth that incorporate "Human Capital" attempt to examine differences in education levels.**

What is Human Capital?

- **The Solow Model that we examined in the previous lecture assumes that the Marginal Product of Labor is positive.**
- **But what would be the marginal product of a person without any child-rearing or education at all?** (i.e. someone who was "raised by wolves").
- **It would be virtually zero.**
- **In this sense, all of the returns to human labor must reflect returns to human capital.**

- **If we assume that there is some minimum level of human capital that human beings acquire more or less automatically** (e.g. the ability to walk and talk, etc.)**, then:**
  - **we can interpret this minimum level as the input of "raw labor"**
  - **and separately examine differences in output levels that occur because some societies have higher average levels of human capital than others**

the Mankiw-Romer-Weil Model

- **To analyze the effects of human capital accumulation on growth of output per worker, we'll examine a model developed by N. Gregory Mankiw, David Romer and David N. Weil in 1992.**
- **Their model is very similar to the Solow Model developed in the previous lecture.**

- **Once again, we'll examine a closed economy without a government.**
  - **there's no international trade** (i.e. net exports equal zero) **and**
  - **government purchases equal zero**
- **so output is divided among consumption and investment:**

$$Y = C + I$$

- **This time however, investment is divided into investment in physical capital and investment in human capital:**

$$I = I_K + I_H$$

Demand-Side Assumptions

- In contrast to the approach taken in the previous lecture, we'll incorporate technology into the model immediately and focus on variables defined in terms of units of effective labor
- After all, excluding technology simply means assuming that $A = 1$ and $g = 0$

- **Denoting the effective labor force by** AL**, we can define:**
  - **output per unit of effective labor as:** $y \equiv Y/AL$
  - **consumption per unit of effective labor as:** $c \equiv C/AL$
  - **investment in physical capital per unit of eff. labor as:** $i_K \equiv I_K/AL$
  - **investment in human capital per unit of eff. labor as:** $i_H \equiv I_H/AL$

- **Consumption and investment in both physical and human capital** (per unit of effective labor) **are proportional to income:**

$$c = (1 - s_K - s_H) \cdot y \qquad \textbf{where: } i_K = s_K \cdot y \textbf{ and } i_H = s_H \cdot y$$

- **so that the saving rates – denoted by** s_K **and** s_H **– are constant.**

Supply-Side Assumptions

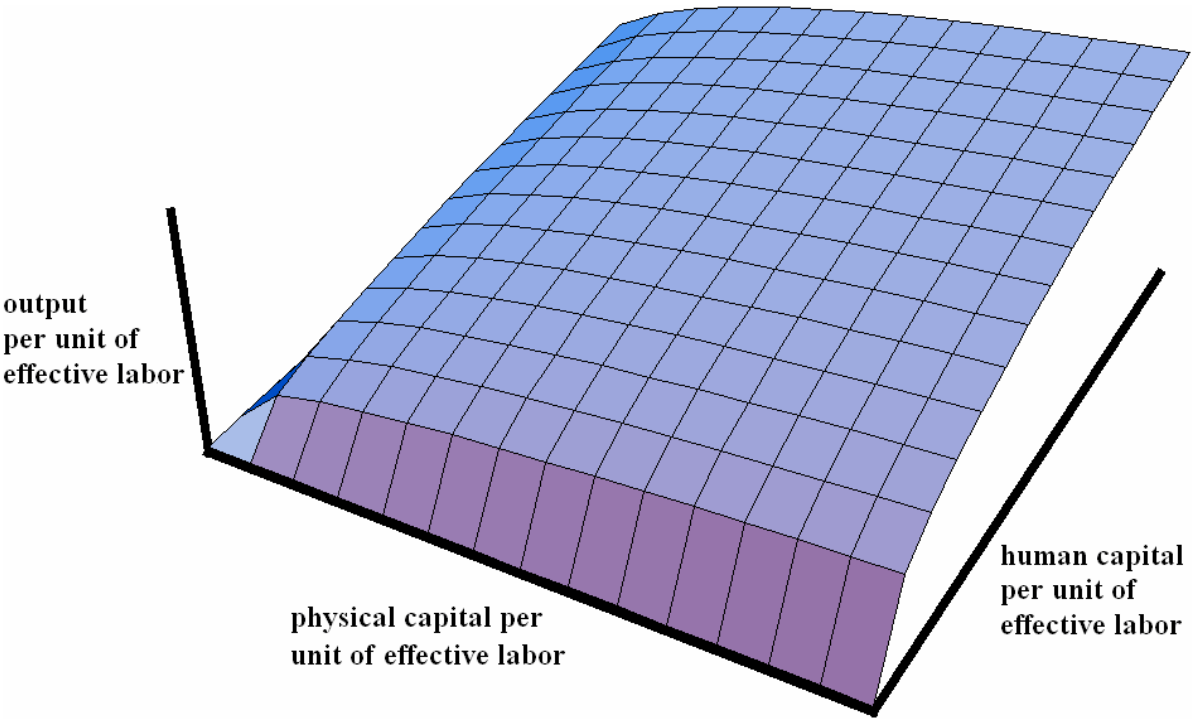
- **We'll also assume that:**
  - **output is produced using physical capital,** K**, human capital,** H**, and effective labor,** AL
  - **as the available technology improves, labor efficiency,** A**, rises**
  - **the production function exhibits constant returns to scale (CRS)**

$$Y = K^{\alpha} \cdot H^{\beta} \cdot (AL)^{1-\alpha-\beta} \qquad \text{where: } \begin{array}{c} 0 < \alpha < 1 \qquad 0 < \beta < 1 \\ 0 < \alpha + \beta < 1 \end{array}$$

- **Focusing on variables defined in terms of labor efficiency units, define:**
  - **output per unit of effective labor as:** $y \equiv Y/AL$
  - **physical capital per unit of effective labor as:** $k \equiv K/AL$
  - **human capital per unit of effective labor as:** $h \equiv H/AL$

$$\left.\begin{aligned} \frac{Y}{AL} &= K^{\alpha} \cdot H^{\beta} \cdot \frac{(AL)^{1-\alpha-\beta}}{AL} \\ &= K^{\alpha} \cdot (AL)^{-\alpha} \cdot H^{\beta} \cdot (AL)^{-\beta} \end{aligned}\right\} \Rightarrow \quad y = k^{\alpha} \cdot h^{\beta}$$

the Production Function



Supply-Side Assumptions

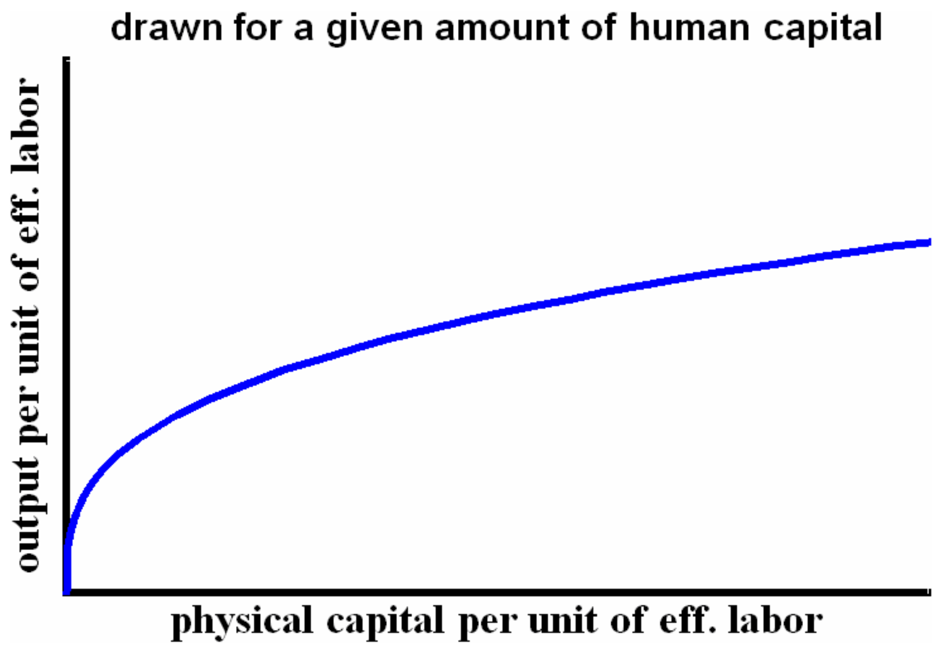
- **The Marginal Product of Capital (MPK) and the Marginal Product of Human Capital (MPH) again have convenient properties:**

$$\frac{dY}{dK} = \alpha \cdot K^{\alpha-1} \cdot H^{\beta} \cdot L^{1-\alpha-\beta} \qquad \frac{dY}{dH} = \beta \cdot K^{\alpha} \cdot H^{\beta-1} \cdot L^{1-\alpha-\beta}$$

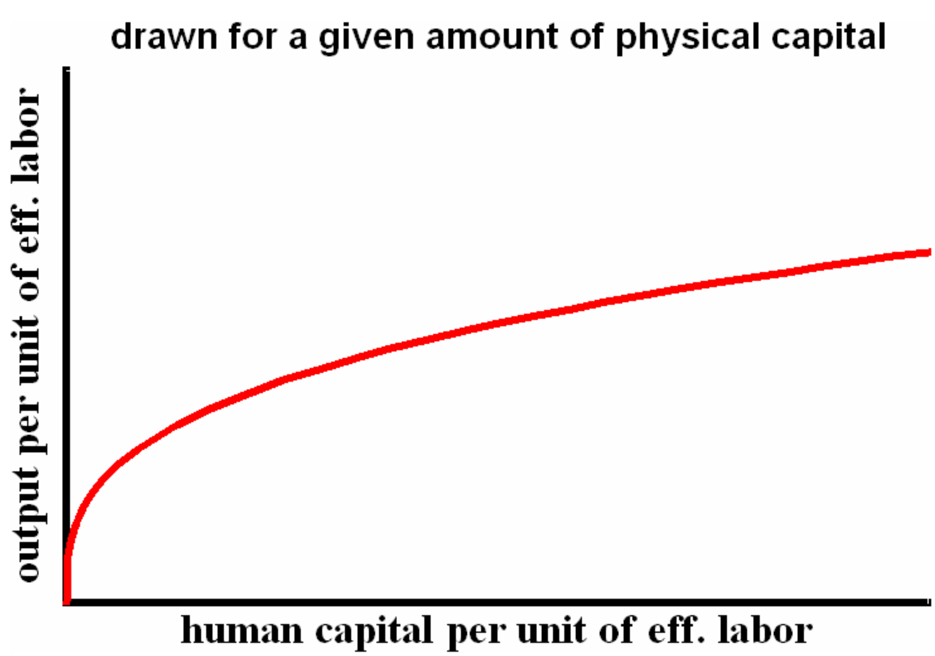
$$\frac{dy}{dk} = \quad MPK = \alpha \cdot k^{\alpha-1} \cdot h^{\beta} \qquad \frac{dy}{dh} = \quad MPH = \beta \cdot k^{\alpha} \cdot h^{\beta-1}$$

- **Increases in the physical capital stock per unit of effective labor increase output per unit of effective labor, but**
  - **each successive increase in the physical capital stock yields ever smaller increases in output per unit of effective labor**
  - **because output per unit of effective labor exhibits diminishing marginal returns to physical capital.**
- **Increases in the human capital stock per unit of effective labor increase output per unit of effective labor, but**
  - **each successive increase in the human capital stock yields ever smaller increases in output per unit of effective labor**
  - **because output per unit of effective labor exhibits diminishing marginal returns to human capital.**

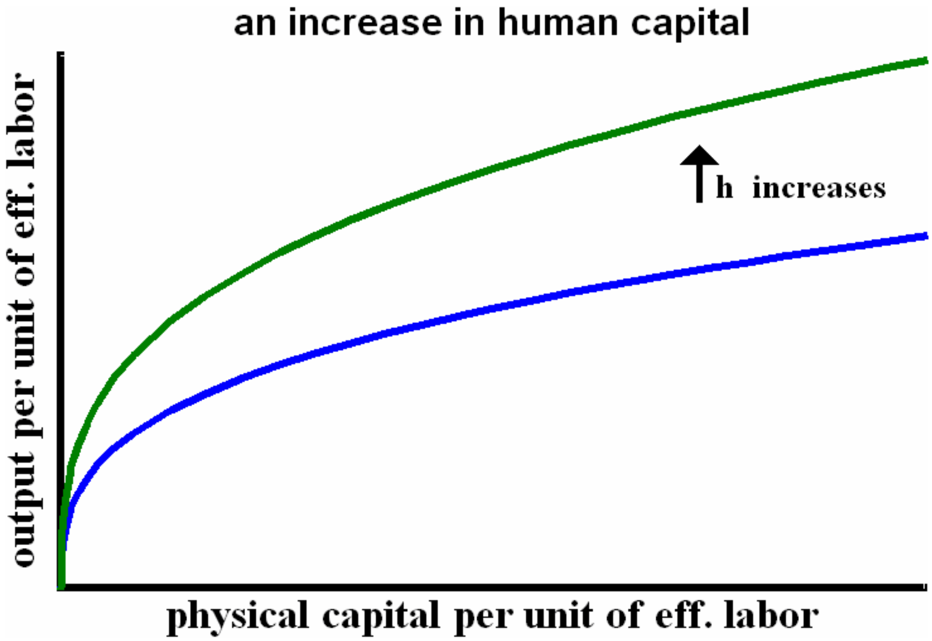
Cross Section: Output and Physical Capital



Cross Section: Output and Human Capital

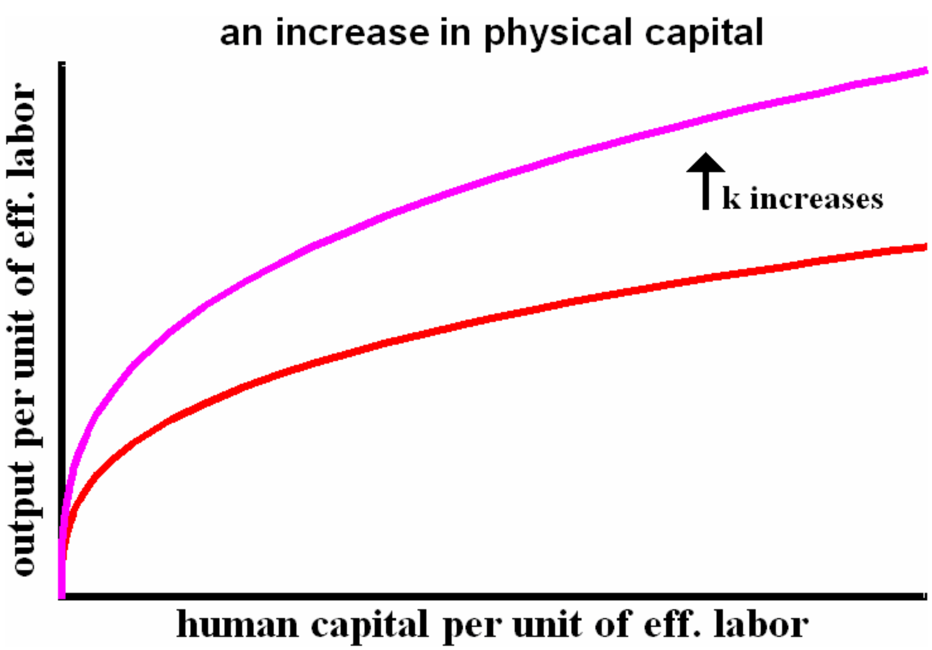


a Shift: Output and Physical Capital



- **An increase in the stock of human capital** (per unit of effective labor) **enable the economy to produce more output** (per unit of effective labor) **at every level of physical capital** (per unit of effective labor)

a Shift: Output and Human Capital



- **An increase in the stock of physical capital** (per unit of effective labor) **enable the economy to produce more output** (per unit of effective labor) **at every level of human capital** (per unit of effective labor)

Physical and Human Capital

- **The underlying theory behind the Mankiw-Romer-Weil Model:**
  - **countries with higher levels of:**
    - **physical capital per unit of effective labor and**
    - **human capital per unit of effective labor**
  - **have higher levels of output per worker.**
- **If their theory is correct, then all we have to do to increase output per worker – and lift billions of people out of poverty – is:**
  - **increase the amount of physical capital that they have to work with**
  - **provide them with more schooling – to increase their levels of human capital**
- **So what determines the levels of physical and human capital per unit of effective labor in a country?**
- **Mankiw, Romer and Weil assume that:**
  - **investment in physical capital increases the physical capital stock**
  - **a fraction of the physical capital stock depreciates each year**
  - **investment in human capital increases the human capital stock**
  - **a fraction of the human capital stock depreciates each year** (i.e. every year you forget some of what you learned earlier in life)

Physical and Human Capital

- $i_K = s_K \cdot y$ **implies that annual investment in physical capital is a fraction,** s_K**, of the total output per year, i.e.** $I_K = s_K \cdot Y$
- **Let** δ_K **denote the annual depreciation rate of the physical capital stock**
- **Therefore:** $\dot{K} = sY - \delta_K K$ **where:** $\dot{K} = \frac{dK}{dt}$

♦ ♦ ♦

- $i_H = s_H \cdot y$ **implies that annual investment in human capital is a fraction,** s_H**, of the total output per year, i.e.** $I_H = s_H \cdot Y$
- **Let** δ_H **denote the annual depreciation rate of the human capital stock**
- **Therefore:** $\dot{H} = s_H Y - \delta_H H$ **where:** $\dot{H} = \frac{dH}{dt}$

♦ ♦ ♦

- **It is very, very convenient to assume that physical and human capital depreciate at the same rate:** $\delta = \delta_K = \delta_H$

Evolution of Physical Capital

- **Now that we now how the total physical capital stock evolves from year to year, finding out how the physical capital stock per unit of effective labor evolves from one year to the next is straightforward.**
- Recalling our Calculus Tricks …
- **we can find the evolution physical capital stock per unit of effective labor over time:**

$$\frac{\dot{k}}{k} = \left(\frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{k} = \frac{K}{AL} \cdot \left(\frac{s_K Y - \delta K}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right)$$

$$\text{define}: \ g \equiv \frac{\dot{A}}{A} \qquad = \frac{s_K Y - \delta K}{AL} - k \cdot \left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right)$$

$$\text{define}: \ n \equiv \frac{\dot{L}}{L} \qquad \boldsymbol{\dot{k} = s_K k^{\alpha} h^{\beta} - (\delta + g + n) \cdot k}$$

- Note that: g is the **exogenous** annual growth rate of technological progress
  n is the **exogenous** annual growth rate of the labor force

Evolution of Human Capital

- **Similarly, now that we now how the total human capital stock evolves from year to year, finding out how the human capital stock per unit of effective labor evolves from one year to the next is straightforward.**
- Recalling our Calculus Tricks …
- **we can find the evolution human capital stock per unit of effective labor over time:**

$$\frac{\dot{h}}{h} = \left(\frac{\dot{H}}{H} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right) \Rightarrow \dot{h} = \frac{H}{AL} \cdot \left(\frac{s_H Y - \delta H}{H} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}\right)$$

$$\text{define}: \ g \equiv \frac{\dot{A}}{A} \qquad = \frac{s_H Y - \delta H}{AL} - h \cdot \left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right)$$

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- Note that: g is the **exogenous** annual growth rate of technological progress
  n is the **exogenous** annual growth rate of the labor force

The Two Key Equations of the Model

$$\dot{k} = s_K k^{\alpha} h^{\beta} - (\delta + g + n) \cdot k$$

- **Growth of the physical capital stock per unit of effective labor, $\dot{k}$**
  - **is an increasing function of physical capital investment, i.e. $s_K k^{\alpha} h^{\beta}$**
  - **a decreasing function of the depreciation rate**
  - **a decreasing function of the growth rate of technological progress**
  - **a decreasing function of the growth rate of the labor force**

♦ ♦ ♦

$$\dot{h} = s_H k^{\alpha} h^{\beta} - (\delta + g + n) \cdot h$$

- **Growth of the human capital stock per unit of effective labor, $\dot{h}$**
  - **is an increasing function of human capital investment, i.e. $s_H k^{\alpha} h^{\beta}$**
  - **a decreasing function of the depreciation rate**
  - **a decreasing function of the growth rate of technological progress**
  - **a decreasing function of the growth rate of the labor force**

Intuition – Physical Capital

- **These results should also be intuitive.**
- **Focusing first on the evolution of the physical capital stock:**
  - The physical capital stock per unit of effective labor increases at higher physical capital saving rates because at higher physical capital saving rates more output is being devoted to accumulating physical capital.
  - By definition, depreciation decreases the physical capital stock, so faster rates of depreciation reduce the physical capital stock per unit of effective labor.
  - **Faster rates of growth of the technological progress will also lead to lower levels of physical capital per unit of effective labor:**
    - **because the total physical capital stock must be spread over a larger <u>effective</u> labor force.**
    - **The effective labor force is defined as labor augmented by technology**
      - **If we increase technology and hold the labor force and the physical capital stock constant,**
      - **then the ratio of physical capital to effective labor must fall.**
  - Faster rates of growth of the labor force will also lead to lower levels of physical capital per unit of effective labor, because the total physical capital stock must be spread over a larger effective labor force.

Intuition – Human Capital

- **Focusing now on the evolution of the human capital stock:**
  - The human capital stock per unit of effective labor increases at higher human capital saving rates because at higher human capital saving rates more output is being devoted to accumulating human capital.
  - Depreciation decreases the human capital stock. **For example, your own personal stock of human capital depreciates as you forget what you learned.** So faster rates of depreciation reduce the human capital stock per unit of effective labor.
  - **Faster rates of growth of the technological progress will also lead to lower levels of human capital per unit of effective labor:**
    - **because the total human capital stock must be spread over a larger <u>effective</u> labor force.**
    - **The effective labor force is defined as labor augmented by technology**
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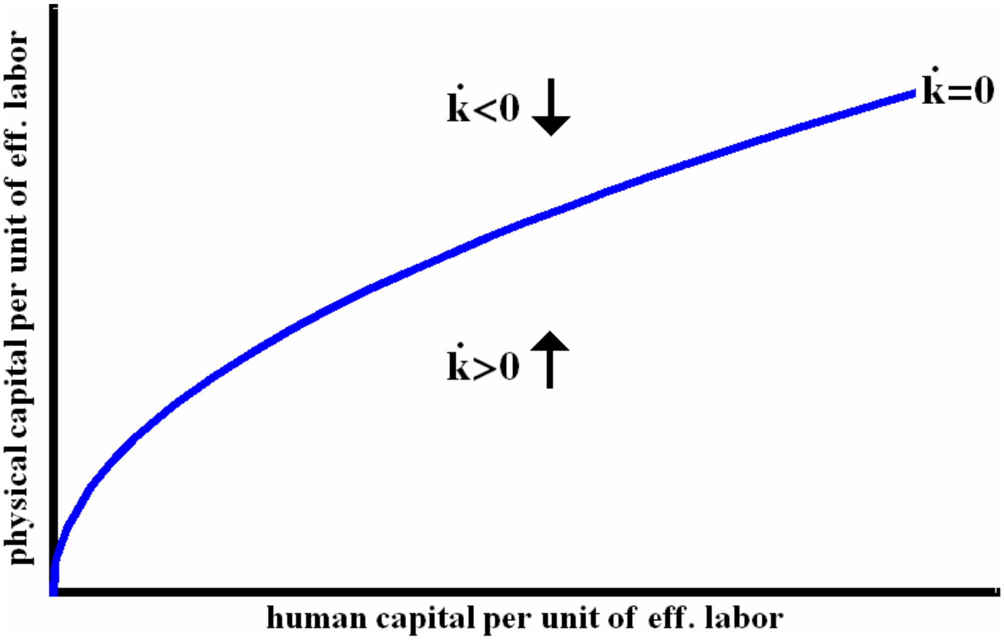
  - Faster rates of growth of the labor force will also lead to lower levels of human capital per unit of effective labor, because the total human capital stock must be spread over a larger effective labor force.

Evolution of Physical Capital

Whether physical capital per unit of effective labor is growing, falling or remaining constant over time, depends on whether investment in new physical capital per unit of effective labor exceeds, falls short of or is equal to the replacement requirement.

- **if** $s_K k^{\alpha} h^{\beta} > (\delta + g + n) \cdot k$**, then:** $\dot{k} > 0$
  - **physical capital per unit of effective labor increases over time**
  - and **investment in new physical capital per unit of effective labor exceeds the replacement requirement**
- **if** $s_K k^{\alpha} h^{\beta} < (\delta + g + n) \cdot k$**, then:** $\dot{k} < 0$
  - **physical capital per unit of effective labor decreases over time**
  - and **investment in new physical capital per unit of effective labor falls short of the replacement requirement**
- **if** $s_K k^{\alpha} h^{\beta} = (\delta + g + n) \cdot k$**, then:** $\dot{k} = 0$
  - **physical capital per unit of effective labor is constant over time**
  - and **investment in new physical capital per unit of effective labor exceeds the replacement requirement**
- **Notice that there are many combinations of** h **and** k **for which** $s_K k^{\alpha} h^{\beta} = (\delta + g + n) \cdot k$

Evolution of Physical Capital



- **The blue line represents the combinations of** h **and** k **for which** $s_K k^{\alpha} h^{\beta} = (\delta + g + n) \cdot k$

Evolution of Human Capital

Whether human capital per unit of effective labor is growing, falling or remaining constant over time, depends on whether investment in new physical capital per unit of effective labor exceeds, falls short of or is equal to the replacement requirement.

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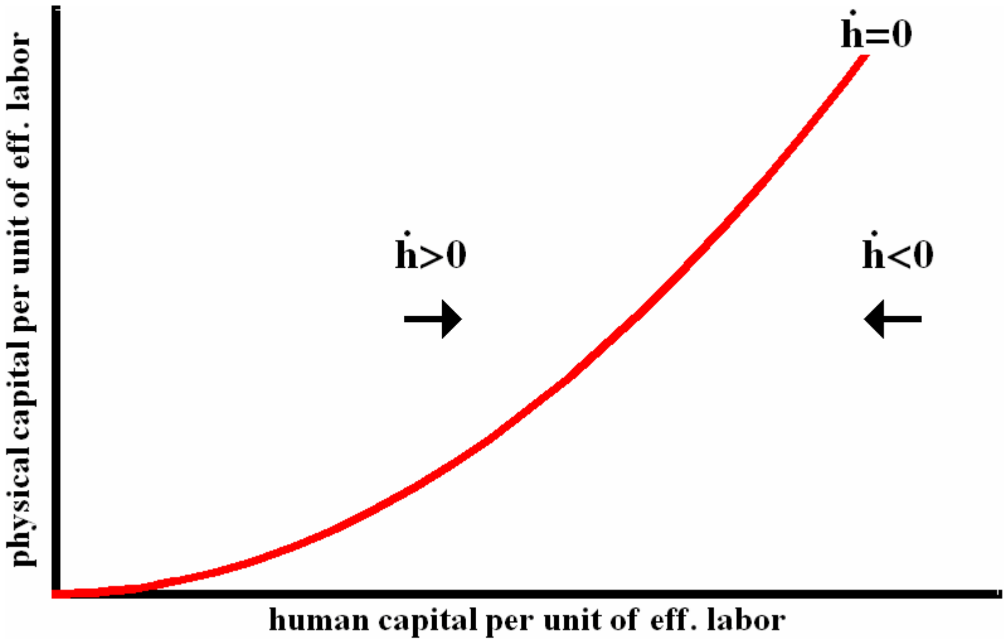
  - **human capital per unit of effective labor is constant over time**
  - and **investment in new human capital per unit of effective labor exceeds the replacement requirement**
- **Notice that there are many combinations of** h **and** k **for which** $s_H k^{\alpha} h^{\beta} = (\delta + g + n) \cdot h$

Evolution of Human Capital



- **The red line represents the combinations of** h **and** k **for which** $s_H k^{\alpha} h^{\beta} = (\delta + g + n) \cdot h$

Steady State

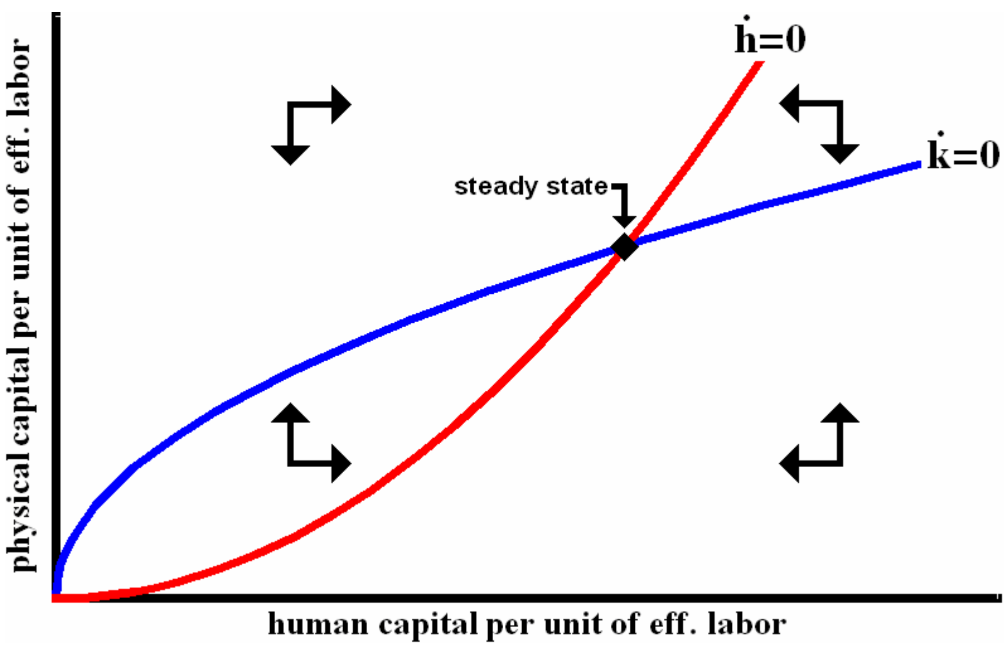
- **In the Mankiw-Romer-Weil Model, the economy must converge to a steady state where:**
  - **physical capital per unit of effective labor is constant over time**
  - **human capital per unit of effective labor is constant over time**
- **For example, imagine a country devastated by war and emigration:**
  - **its physical capital stock was destroyed by bombing campaigns**
  - **its human capital stock was depleted by the emigration of its best and brightest to America**
- **Is the country now doomed to perpetual poverty? No.**
- **If the economy devotes a large share of its** (substantially reduced) **output to investment in new physical and human capital, then:**
  - **over time it will replace its lost physical capital stock**
  - **over time it will replace its lost human capital stock**
  - **over time it will converge to a higher steady state level of output per unit of effective labor**
- **This is an incredibly optimistic model!**
- **All a country needs is high saving rates!**

Steady State

- Now, imagine a very rich country:
  - **it has so many factories and machines that its physical capital stock** (per unit of effective labor) **is the highest in the world**
  - **it boasts the best universities in the world and its human capital stock** (per unit of effective labor) **is also the highest in the world**
- **Will this country always be the richest in the world? Not necessarily.**
- **If the residents of this country suddenly become decadent and consume all of their output and stop investing new physical and human capital, then over time:**
  - **its physical capital stock** (per unit of effective labor) **will diminish**
  - **its human capital stock** (per unit of effective labor) **will diminish**
  - **over time it will converge to a lower steady state level of output per unit of effective labor**

"Lazy hands make a man poor, but diligent hands bring wealth."
– Proverbs 10:4

Steady State



Steady State

- **Regardless of the economy's initial levels of:**
  - **physical capital per unit of effective labor and**
  - **human capital per unit of effective labor**
- **it will converge to a steady state level of output per unit of effective labor which is determined by:**
  - **its physical capital saving rate,** s_K
  - **its human capital saving rate,** s_H
  - **its labor force growth rate,** n
  - **its rate of technological progress,** g and
  - **the rate at which its physical and human capital depreciates,** δ
- **The steady state physical capital stock per unit of eff. labor is:**

$$k_{SS} = \left(\frac{s_K^{1-\beta} \cdot s_H^{\beta}}{\delta + n + g} \right)^{\frac{1}{1-\alpha-\beta}}$$

- **The steady state human capital stock per unit of eff. labor is:**

$$h_{SS} = \left(\frac{s_K^{1-\alpha} \cdot s_H^{\alpha}}{\delta + n + g} \right)^{\frac{1}{1-\alpha-\beta}}$$

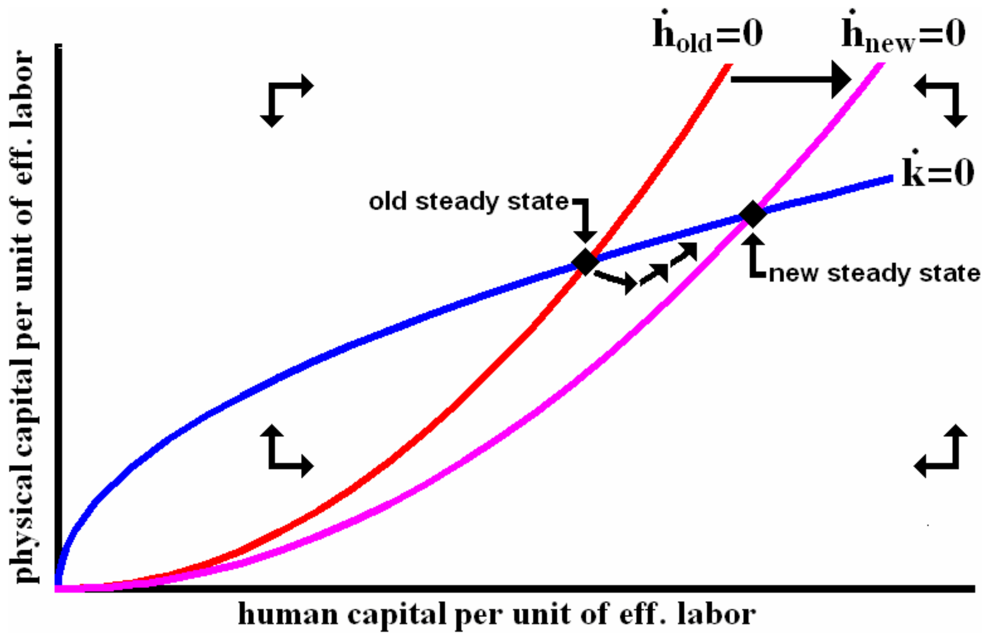
Steady State

- **The steady state level of output per unit of effective labor is:**

$$y_{SS} = \left(\frac{s_K^{\alpha} \cdot s_H^{\beta}}{(\delta + n + g)^{\alpha+\beta}} \right)^{\frac{1}{1-\alpha-\beta}}$$

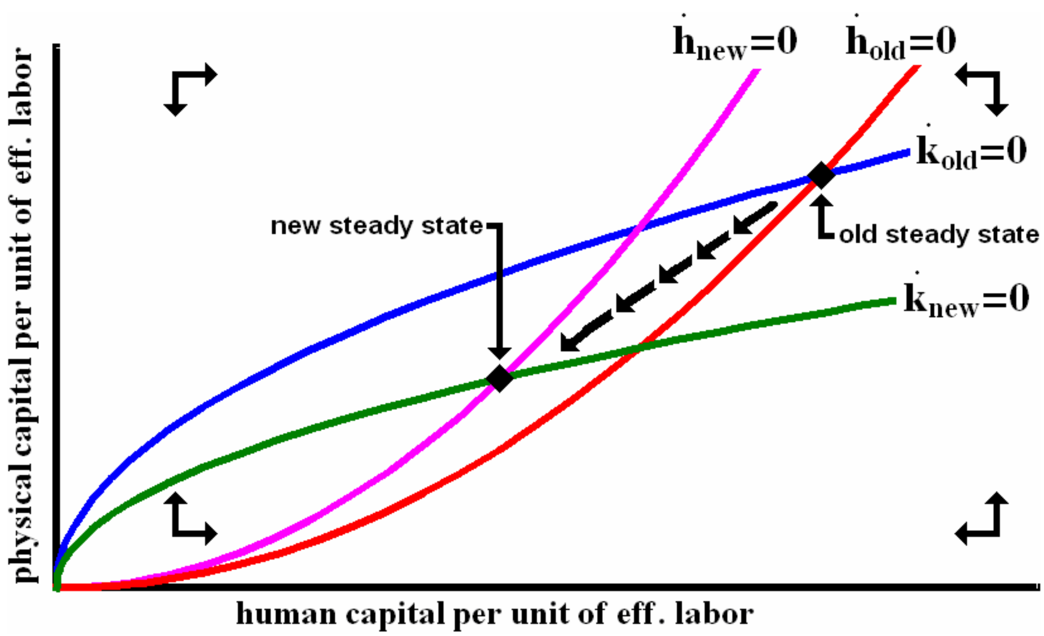
- **This equation tells us that the steady state level of output per unit of effective labor:**
  - **is higher when the economy has a higher physical capital saving rate,** s_K
  - **is higher when the economy has a higher human capital saving rate,** s_H
  - **is lower when the rate at which economy's physical and human capital depreciates,** δ**, is higher**
  - **is lower when the economy has a higher labor force growth rate,** n
  - **is lower when the economy has a higher rate of technological progress,** g

Increasing the Human Capital Saving Rate



- If the economy's human capital saving rate increased, then
  - the economy would converge to higher steady state levels of physical and human capital per unit of effective labor
  - which would increase output per unit of effective labor.

Increasing the Labor Force Growth Rate



- If the economy's labor force growth rate increased, then the economy would converge to a lower steady state level of output per unit of effective labor.

So what do we want?

- **If we want our economy's living standards to be higher, then we want:**
  - **a higher physical capital saving rate,** s_K
  - **a higher human capital saving rate,** s_H
  - **a lower rate of physical and human capital depreciation,** δ
  - **a lower labor force growth rate,** n
  - **a HIGHER rate of technological progress,** g
- All of these should be intuitive, although the last "want" – a higher rate of technological progress – can be confusing.
- After all, doesn't a higher rate of technological progress reduce the steady state level of output per unit of effective labor? Yes, but …
  - A person doesn't consume output per unit of effective labor
  - A person consumes output per worker
- Recall from Lecture 5 that when we incorporate technological progress into the model the steady state growth rate of output per worker is equal to the rate of growth of technological progress.
- **A faster rate of growth of technological progress implies a rapidly rising standard of living for the residents of that economy**

So how can we increase the growth rate of technological progress?

- **The answer to that question is worth an instant Nobel Prize.**
- **Economists have developed other models that attempt to answer that question.** Below is a summary of some of a few theories.

Research and Development Models

- **Some models of R&D predict that the long-run growth rate of output per worker is an increasing function of the growth rate of the labor force**
- **But that's a little odd.**
- **On average, the growth rate of output per worker is <u>not</u> higher in countries with faster population growth.**
- **As a model of worldwide economic growth however, such models are more plausible. If the variable** A **in our models:**
  - **represents knowledge that can be used anywhere in the world and**
  - **if the growth rate of that knowledge,** g**, depends on the growth rate of the labor force**
- **then the larger the world population is the more people there are to make discoveries that advance the rate of technological progress.**

So how can we increase the growth rate of technological progress?

"Learning by Doing" in AK Models

- **In AK models, the source of technological progress:**
  - **does <u>not</u> depend on a Research and Development sector, but rather**
  - **depends on how much new knowledge is generated by everyday economic activity**
- **The underlying theory behind these models is that:**
  - **learning occurs as new capital is produced, so**
  - **producing new capital has benefits that are not captured by the conventional return on capital investment,** r
- **Increased capital therefore raises output through:**
  - **its direct contribution to output**
  - **by indirectly contributing to the development of new ideas**
- **There's no steady state in these models. Instead the long-run growth <u>rate</u> of output per worker is proportionate to the saving rate.**
- **The implication of such models is that the government should intervene to subsidize the accumulation of new capital.**

So how can we increase the growth rate of technological progress?

International Trade and Foreign Direct Investment

- **Trade enables a less developed trade partner to learn from the more developed trade partner how to implement the managerial** (and other) **practices best suited to using new technologies.**
- **Also in the absence of trade, domestic producers may seek government protection from competition – licensing requirements, etc.**
- **When a country opens to trade however:**
  - **international competition forces domestic producers to cease their** (socially wasteful) **protective activities and**
  - **allocate resources towards becoming more productive by adopting new technologies.**
- **Recipients of FDI acquire knowledge of foreign managerial practices, which they can compare with their own to find more efficient methods of production**
- **Trade and FDI cannot affect the growth <u>rate</u> of technological progress**
- **It enables less developed countries to import a whole <u>level</u> of technology**

So how can we increase the growth rate of technological progress?

political structure

- **A country's political structure affects the rate at which new technologies are adopted.**
- **If there's a high risk that the government will infringe upon the returns to technology adoption by expropriating industrial capacity, businesses will be less likely to undertake an investment in new technology.**
- **Similarly, if the government:**
  - **redistributes tax revenues to a minority of the elite rather than**
  - **allocating tax revenues to public goods that are necessary for business development** (such as roads, communications, sewage, etc.)
- **then businesses will be less likely to undertake an investment in new technology**

So how can we increase the growth rate of technological progress?

political structure

- **If the adoption of new technology is costly, but use of that new technology greatly reduces the cost of producing a good, then the entry of firms using the new technology will lower the market price.**
- **Producers who continue to use the older, less productive technology may find it more profitable to lobby government to block the use of the new technology rather than adopting it.**
- **Such lobbying benefits the users of the old technology at the expense of the majority of society.**
- **In theory, a democratic government should protect property rights and act in the interest of the majority of the society and not in the interest of an elite minority or a vested interest.**

Homework #6

1. According to a study released in 1997 by the National Center for Health Statistics, a woman's educational level is the best predictor of how many children she will have. The study found a direct relationship between years of education and birth rates, with the highest birth rates among women with the lowest educational attainment.

Assume that this finding is true for women all over the world and comment on why the UN Millennium Project – a body commissioned by the UN Secretary-General to advise on development strategies – recently recommended that the UN should:

"Focus on women's and girls' health (including reproductive health) and education outcomes, access to economic and political opportunities, right to control assets, and freedom from violence."

- **a.** In your answer, discuss how empowering women meets two of the conclusions of the Mankiw-Romer-Weil Model about the ways to improve living standards within a country.
- **b.** Illustrate your answer with a diagram of how empowering women increases steady state income per worker.
- **c.** In your answer, discuss two ways that empowering women can increase the level of technology (i.e. the level of labor efficiency) within the economy.

2. The 1983 Economic Report of the President contained the following statement: "Devoting a larger share of national output to investment would help restore rapid productivity growth and rising living standards." Do you agree with this claim? Explain.

What factors affect a country's level of economic development?

	GDP per cap. at PPP USD	Human Dev. Index*	saving as % of GDP	annual pop. growth rate	net secondary school enrollment	Gender-Empower. Measure**	Imports as % of GDP	Exports as % of GDP	Gini Index***
	2002	2002	1990-2000	1975-2002	1990-91		2002	2002	
Norway	36,600	95.6%	33.8%	0.4%	88%	0.908	27%	41%	25.8%
Australia	28,260	94.6%	22.5%	1.3%	79%	0.806	22%	20%	35.2%
Sweden	26,050	94.6%	24.6%	0.3%	85%	0.854	37%	43%	25.0%
Canada	29,480	94.3%	26.4%	1.1%	89%	0.787	39%	44%	33.1%
Netherlands	29,100	94.2%	27.3%	0.6%	84%	0.817	56%	62%	32.6%
United States	35,750	93.9%	19.8%	1.0%	85%	0.769	14%	10%	40.8%
Japan	26,940	93.8%	33.8%	0.5%	97%	0.531	10%	11%	24.9%
Ireland	36,360	93.6%	30.4%	0.8%	80%	0.710	83%	98%	35.9%
United Kingdom	26,150	93.6%	17.4%	0.2%	81%	0.698	28%	26%	36.0%
Switzerland	30,010	93.6%	30.7%	0.5%	80%	0.771	38%	44%	33.1%
Finland	26,190	93.5%	27.7%	0.4%	93%	0.820	30%	38%	26.9%
Denmark	30,940	93.2%	26.8%	0.2%	87%	0.847	39%	45%	24.7%
New Zealand	21,740	92.6%	22.4%	0.8%	85%	0.772	32%	33%	36.2%
Greece	18,720	90.2%	13.0%	0.7%	83%	0.523	27%	21%	35.4%
South Korea	16,950	88.8%	37.6%	1.1%	86%	0.377	39%	40%	31.6%
Poland	10,560	85.0%	18.0%	0.5%	76%	0.606	31%	28%	31.6%
Hungary	13,400	84.8%	17.5%	–0.2%	75%	0.529	67%	64%	24.4%
Chile	9,820	83.9%	21.4%	1.5%	55%	0.460	32%	36%	57.1%
Costa Rica	8,840	83.4%	14.3%	2.6%	37%	0.664	47%	42%	46.5%
Mexico	8,970	80.2%	17.5%	2.0%	45%	0.563	29%	27%	54.6%
Panama	6,170	79.1%	20.7%	2.1%	50%	0.486	29%	28%	56.4%
Venezuela	5,380	77.8%	20.4%	2.5%	19%	0.444	17%	29%	49.1%
Paraguay	4,610	75.1%	4.2%	2.9%	26%	0.417	43%	31%	56.8%
Bolivia	2,460	68.1%	3.0%	2.2%	29%	0.524	27%	22%	44.7%
Botswana	8,170	58.9%	24.6%	2.8%	29%	0.562	37%	51%	63.0%
Bangladesh	1,700	50.9%	4.5%	2.4%	19%	0.218	19%	14%	31.8%

*The Human Development Index (HDI) is a composite index measuring average achievement in three basic dimensions of human development – a long and healthy life, knowledge and a decent standard of living.

**The Gender Empowerment Measure (GEM) is a composite index measuring gender inequality in three basic dimensions of empowerment – economic participation and decision-making, political participation and decision-making and power over economic resources.

***The Gini index measures inequality over the entire distribution of income or consumption. A value of 0% represents perfect equality, and a value of 100% perfect inequality.

Sources: Human Development Report (2003) and Penn World Table 6.1

What factors affect a country's level of economic development?

correlation coefficients	GDP per cap. at PPP	Human Dev. Index	Saving as % of GDP	annual pop. growth rate	net secondary school enroll.	Gender-Empower Measure	Imports as % of GDP	Exports as % of GDP	Gini Index
	2002	2002	1990-2000	1975-2002	1990-1991		2002	2002	
GDP per cap. at PPP	1.0000								
Human Dev. Index	0.8227	1.0000							
Saving as % of GDP	0.6778	0.6247	1.0000						
annual pop. growth rate	–0.7298	–0.7616	–0.5280	1.0000					
net secondary school enroll.	0.8487	0.8774	0.6824	–0.9011	1.0000				
Gender Empower Measure	0.8057	0.7345	0.4797	–0.5768	0.6580	1.0000			
Imports as % of GDP	0.1513	0.1477	0.1430	–0.1844	0.1082	0.1599	1.0000		
Exports as % of GDP	0.2904	0.2039	0.3832	–0.2348	0.1736	0.2921	0.9312	1.0000	
Gini Index	–0.6134	–0.5406	–0.3995	0.8162	–0.7285	–0.4320	–0.0829	–0.1168	1.0000

- As predicted by the Mankiw-Romer-Weil Model, saving (investment in physical capital) and education (investment in human capital) are positively correlated with a country's level of economic development, as measured both by per capita Gross Domestic Product (GDP) and the Human Development Index (HDI).
- As predicted by the Solow Model and the Mankiw-Romer-Weil Model, higher rates of population growth (which translate to higher rates of labor force growth) are negatively correlated with per capita GDP and HDI.
- The definition of women's empowerment that we used when discussing Homework #6 doesn't match this measure of empowerment. This measure looks at estimates of income based on gender as well as the percentage of female legislators, senior officials, managers, professionals etc. Nonetheless, this measure of women's empowerment is positively correlated with per capita GDP and HDI.
- Notice that women's empowerment is negatively correlated with the population growth rate.
- Larger shares of trade in GDP – as measured by both exports an imports – increases a country's level of economic development.
- Lastly, income inequality – as measured by the Gini Index – is highly correlated with both per capita GDP and HDI. The more unequal is a country's income distribution, the lower is its level of economic development.

Lecture 7

Economic Growth: Transition Dynamics

Eric Doviak

Economic Growth and Economic Fluctuations

Saving

- **In our discussion of the Solow model, we assumed that:**
  - **annual physical capital investment is a fraction, s, of the total output per year, i.e. $I = s \cdot Y$** and
  - **a fraction, δ, of the capital stock depreciates each year**
- **So once again: $\dot{K} = sY - \delta K$**
- **In practice, the saving rate depends on:**
  - **the decisions of individuals within the economy**
  - **government decisions about how much to collect in tax revenue and how much to spend**
- **Government Saving is the difference between Tax Revenues, T, and Government Purchases, G, so we can define the government saving <u>rate</u> as: $s_G \equiv (T - G)/Y$**
- **If the government is running a budget deficit, then s_G is negative**
- **If we define s_P as the "private" saving rate, then the economy's saving rate is: $s \equiv s_G + s_P$**

Saving

- **Now consider the table below.**
  - "Gross Saving" is a measure of s
  - "Federal Gov't Saving" is a measure of s_G
  - the third row gives is the percentage change in private non-residential fixed assets, net of depreciation – a measure of $\dot{K}/K$
  - the last row gives the percentage change in the population aged 20 to 64 – a measure of n, the labor force growth rate

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Gross Saving*	16.2	16.6	17.7	18.2	17.9	17.7	16.2	14.1	13.5	14.0
Federal Gov't Saving*	–2.7	–1.8	–0.7	0.4	1.1	1.9	0.5	–2.4	–3.3	–3.1
%Δ Priv Non-Res Fixed	2.8	3.2	3.6	3.9	3.9	4.0	2.6	1.5	1.5	2.8
%Δ Pop. 20-64	–	0.9	1.1	1.1	1.1	–	1.4	1.4	1.3	1.2
*as a percentage of Gross National Income (Census Bureau and Bureau of Economic Analysis)										

- **Federal government saving has fallen dramatically – a result of:**
  - fluctuations – the 1990's boom temporarily increased tax revenues
  - the wars in Iraq and Afghanistan and
  - **TAX CUTS**
- **Due to the fall in saving, net capital investment is just keeping up with labor force growth – reducing steady state output per worker**

Steady State Income per Worker

- **I don't know how much each of the aforementioned factors contributed to the growth of the federal budget deficit**
- **but the focus of this lecture will be on why we might prefer low saving rates, even though low saving rates lead to low steady state levels of consumption per worker**
- **To illustrate this preference, I'll:**
  - **use the Solow Model without technology,** i.e.: $A = 1$, $g = 0$ and
  - **focus on the effect of tax cuts on the saving rate**

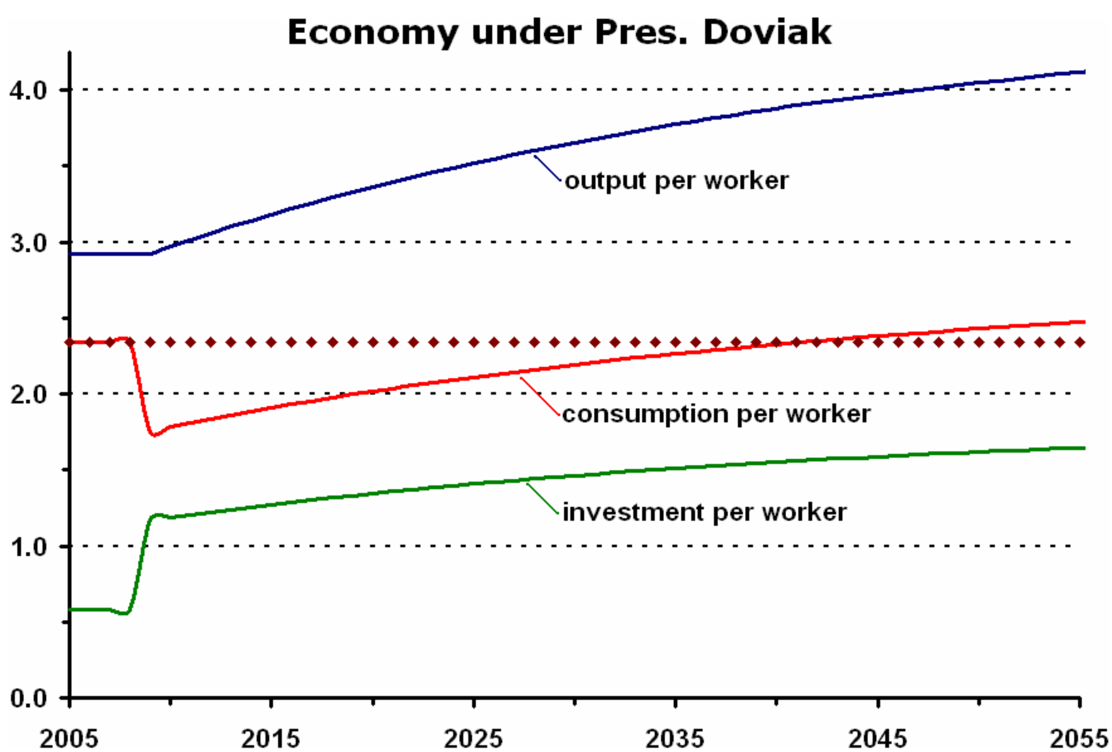
DISCLAIMER

- **Before diving into the discussion, I want to emphasize that empirical evidence suggests that:**
  - **high personal and corporate income tax rates may discourage net capital investment and thus lower steady state output per worker**
  - although potentially beneficial, **major tax reforms designed to increase steady state output per worker will be <u>not</u> be self-financing** and
  - **well-designed government spending can also increase steady state output per worker**

Doviak for President
Investing for the Future

- Currently, our nation saves a mere 20 percent of its output and finds itself at a steady state level of consumption per worker that is 17 percent below the Golden Rule level
- We could now be enjoying a much higher standard of living had budget deficits not crowded out capital investment all these years
- To reach the Golden Rule level however, we need to double our saving rate by repealing Pres. Bush's tax cuts
- The tax increases I propose will immediately reduce your consumption 29 percent, but don't worry …
- the capital investments resulting from the higher saving rate will increase output per worker over time and by the year 2042 your consumption will have returned to its current level
- And it will keep growing over time enabling your great-grandchildren to enjoy the highest possible level of consumption per worker, given the rate at which capital depreciates and our rate of labor force growth

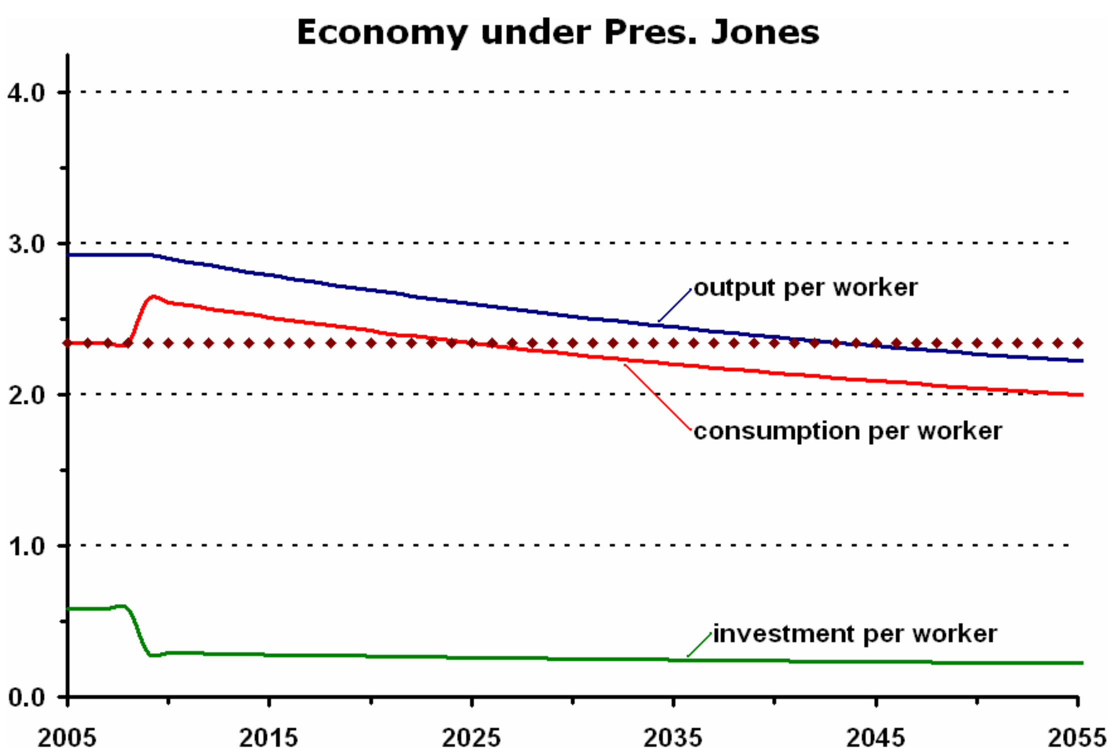
Note: all numbers cited in this mock campaign speech are fictional.



- The increase in the saving rate causes consumption to drop immediately.
- The fall in consumption is matched by an increase in investment.
- Over time output, consumption and investment all increase together.

Jones for President
More Lunches in Every Pail

- Saving Schmaving!
- What's all this gobble-de-gook about a Golden Rule?
- There's only one Golden Rule – the American people need more gold
- Don't listen to this Ivory Tower Elitist! He's outta touch with reality.
- Real people need more consumption now, not 40 years from now!
- Let him scratch Greek letters on a college blackboard, but don't let him run your economy!
- As your next president, my tax cuts will be so deep that the national saving rate will fall by half!
- And when I cut the saving rate in half, you'll consume more than you did before!

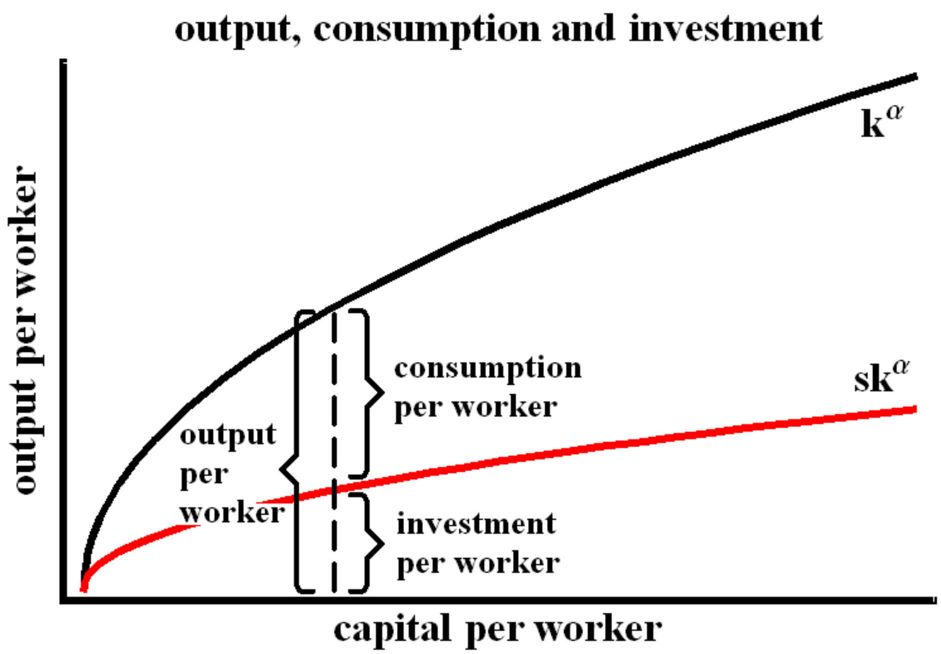


- The decrease in the saving rate causes consumption to rise immediately.
- The increase in consumption is matched by a fall in investment.
- Over time output, consumption and investment all decrease together.

So who would you vote for?

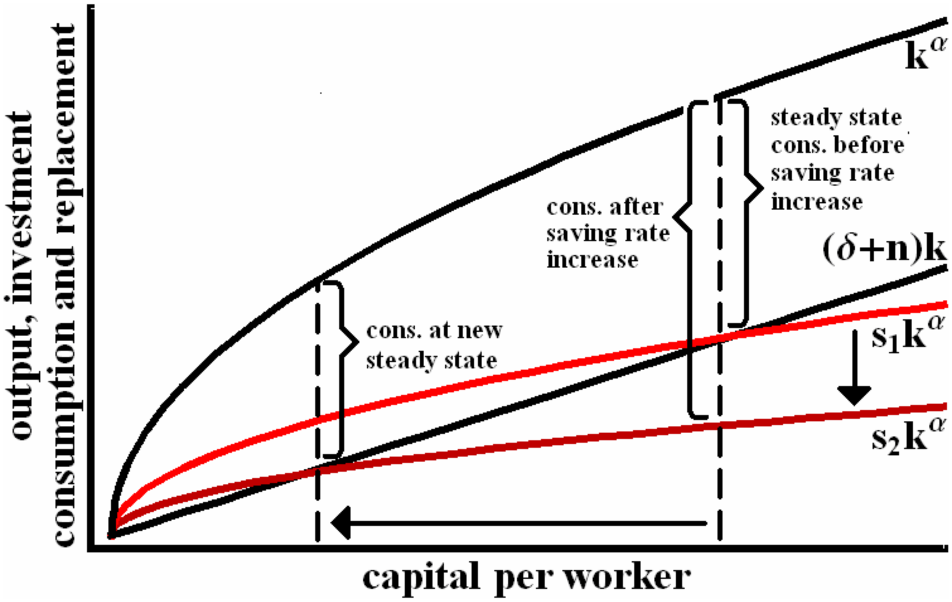
- **You'd vote for Jones.**
- **He immediately increases your consumption 12 percent and your consumption doesn't slip below its original level until 2026.**
- **So how does he get away with it?**
- **Remember from Lecture 5 that there's a tradeoff between consumption and investment.**
- **Each level of capital per worker corresponds to a unique level of output per worker and the saving rate** s **determines the allocation of output between consumption and investment**
- **The level of consumption is free to vary but capital must be accumulated** (or depleted) **<u>over</u> <u>time</u> therefore:**
  - **on any given day, you can decide to consume more or less than you did the day before, but**
  - **to consume more, you save less – which will decrease the rate of capital accumulation**
  - **to consume less, you save more – which will increase the rate of capital accumulation**

Tradeoff between Consumption and Investment



- **the saving rate** s **determines the allocation of output between consumption and investment**

Effect of Jones' Policy



- The decrease in the saving rate causes consumption to rise immediately.
- The increase in consumption is matched by a fall in investment.
- Over time output, consumption and investment all decrease together.

Appendix

- At the beginning of the lecture, I wrote that the economy's saving rate, s, is the sum of the private saving rate, s_P, and the government saving rate, s_G, so that: $s \equiv s_G + s_P$
- Recalling from Lecture 4 that when Net Exports are zero, then:

$$I = Y - C - G$$

$$I = (Y - T - C) + (T - G)$$

$$I = \text{private saving} + \text{government saving}$$

- Now if we define: $s_P \equiv \dfrac{Y - T - C}{Y}$ $\quad s_G \equiv \dfrac{T - G}{Y}$ then $I = (s_P + s_G) \cdot Y$
- These equations give the false impression that taxation has no effect on national saving because I haven't defined the consumption function yet.
- If we assume that: $C = a + b \cdot (Y - T)$ where: $0 \le a$ and $0 < b < 1$

then national saving is an increasing function of Tax Revenues since:

$$s = \frac{Y - T - a - b \cdot (Y - T)}{Y} + \frac{T - G}{Y} \quad \Rightarrow \quad \frac{ds}{dT} = \frac{b}{Y} > 0$$

Homework #7

1. Suppose that an economy was initially in steady state when part of its capital stock is destroyed by war. Assume that none of its residents are killed by the war. Use the Solow Model without technological progress to answer the following questions.

a. What is the immediate impact on total output?

b. What is the immediate impact on output per worker?

c. Assuming that the country's saving rate remains unchanged, what happens to:

- output per worker in the postwar economy?
- investment per worker in the postwar economy?
- consumption per worker in the postwar economy?

Illustrate your answers with diagrams

d. Is the growth rate of output per worker in the postwar economy greater or smaller than it was before the war?

2. Suppose that an economy was initially in steady state when some of its residents are killed by a war. Assume that none of its capital stock is destroyed by the war. Use the Solow Model without technological progress to answer the following questions.

a. What is the immediate impact on total output?

b. What is the immediate impact on output per worker?

c. Assuming that the country's saving rate remains unchanged, what happens to:

- output per worker in the postwar economy?
- investment per worker in the postwar economy?
- consumption per worker in the postwar economy?

Illustrate your answers with diagrams

d. Is the growth rate of output per worker in the postwar economy greater or smaller than it was before the war?